1. Suppose $X_1, \ldots, X_n$ come from a Normal distribution with unknown mean $\mu$ and known variance $\sigma_0^2$.

   a. Consider the test $H_0 : \mu = \mu_0$ vs. $H_A : \mu \neq \mu_0$. Show that the Generalized Likelihood Ratio

   $\Lambda = e^{\frac{n}{2\sigma_0^2}(\bar{X} - \mu_0)^2}.$

   (Hint: You may use the fact that

   \[
   \sum_{i=1}^{n} (X_i - \mu_0)^2 - \sum_{i=1}^{n} (X_i - \bar{X})^2 = n(\bar{X} - \mu_0)^2.
   \]

   \[
   \Lambda = \frac{\max_{\mu = \mu_0} \prod_{i=1}^{n} f(x_i | \mu)}{\max_{-\infty < \mu < \infty} \prod_{i=1}^{n} f(x_i | \mu)}
   = e^{\frac{1}{2\sigma_0^2} \sum_{i=1}^{n} (X_i - \mu_0)^2}
   = e^{\frac{1}{2\sigma_0^2} (\sum_{i=1}^{n} (X_i - \mu_0)^2 - \sum_{i=1}^{n} (X_i - \bar{X})^2)}
   = e^{\frac{-n}{2\sigma_0^2}(\bar{X} - \mu_0)^2}.
   \]

   b. What is the distribution of $-2 \log \Lambda$. (Note that you can still answer this question even if you did not complete part a.)

   \[
   -2 \log \Lambda = \frac{n}{\sigma_0^2} (\bar{X} - \mu_0)^2
   = \left( \frac{X - \mu_0}{\sigma_0^2/\sqrt{n}} \right)^2
   = Z^2,
   \]

   where $Z$ is a standard normal random variable (under the null hypothesis), so $-2 \log \Lambda$ is $\chi^2_1$, i.e. chi-square with 1 degree of freedom.
2. Consider the distribution with density given by

\[ f(x; k, \lambda, \theta) = \frac{k}{\lambda} \left( \frac{x - \theta}{\lambda} \right)^{k-1} e^{-\left( \frac{x - \theta}{\lambda} \right)^k}, \quad x \geq \theta. \]

Using the test \( H_0 : k > 5, \lambda = \lambda_0, \theta > \theta_0 \) vs. \( H_A : k \leq 5, \lambda \neq \lambda_0, \theta \leq \theta_0 \), determine the number of degrees of freedom for the approximate distribution of \(-2 \log \Lambda\) .