Here we will use Matlab to artificially simulate from a population. We will use the following commands: for, if, mean, std, rand, randn, hist. To understand how to use them simply type for example ”help mean” (without the quotes) in the terminal, and some text explaining how to use it will appear.

1. First, recall how to sample from a given distribution if all you have is a uniform(0,1). To sample from a distribution $X$ with distribution function $F$ use $X = F^{-1}(U)$. So for example, an exponential random variable with mean 1 has distribution function

$$F(x) = 1 - e^{-x},$$

so to sample from the distribution one would compute

$$F(x) = u$$
$$1 - e^{-x} = u$$
$$e^{-x} = 1 - u$$
$$-x = \log(1 - u)$$
$$x = -\log(1 - u).$$

So to generate a sample of $n$ exponentials one would simulate $n$ uniform(0,1) and compute $-\log(1 - u)$.

a. Compute a sample of 10000 exponentially distributed random variables with mean 1 and plot a histogram. Sketch the shape of the graph.

```matlab
u = rand(1,10000);
x = -log(1-u);
hist(x)
```

b. Estimate the mean of this distribution by computing the sample mean.

```matlab
mean(x)
```

c. Compute a (2-sided) 95% confidence interval for the true mean.

```matlab
[mean(x)-2*std(x)/sqrt(10000),mean(x)+2*std(x)/sqrt(10000)]
```

2. Plot the distribution of $\overline{X}$ from above (i.e., compute 10000 values of $\overline{X}$ and create a histogram of those values). Sketch the histogram. What distribution does it look like?

```matlab
for R=1:10000
    u=rand(1,10000);
x=-log(1-u);
end
```
y(R) = mean(x);
end

hist(y)

3. Sketch the histogram of 1000 standard normal random variables squared. How is it distributed?

n = randn(1,1000).^2;
hist(n);

4. Create 100 (2-sided) 95% confidence intervals for the true mean of the standard exponential from problem 1. How many of them contain the mean?

for R=1:100
u=rand(1,10000);
x=-log(1-u);
lower = mean(x)-2*std(x)/sqrt(10000);
upper = mean(x)+2*std(x)/sqrt(10000);
if(lower > 1 | upper < 1)
p(R) = 1;
else
p(R) = 0;
end
end

This code is designed to speed up the process. You must still type it in yourself into Matlab and answer the questions. Please do NOT print anything out, simply sketch the shape of the graph and answer all of the questions.

It is Due on Tuesday, 16 by 2:50pm handed to me in class.