1. Four married couples are arranged in a row. Compute the probability that no wife is next to her husband.

This was a homework problem. Let \( E_i \) = \( i^{th} \) couple sitting next to each other, \( i = 1, 2, 3, 4 \). Then

\[
P(E_i) = \frac{2(N-1)!}{N!} = \frac{2}{N} = \frac{2}{8} = \frac{1}{4},
\]

\[
P(E_i \cap E_j) = \frac{6!2!2!}{8!} = \frac{1}{14},
\]

\[
P(E_i \cap E_j \cap E_k) = \frac{5!2!3}{8!} = \frac{1}{42},
\]

\[
P(E_1 \cap E_2 \cap E_3 \cap E_4) = \frac{4!2!4}{8!} = \frac{1}{105}.
\]

The answer we are looking for is

\[
P((E_1 \cup E_2 \cup E_3 \cup E_4)^C) = 1 - P(E_1 \cup E_2 \cup E_3 \cup E_4).
\]

\[
= 1 - \left( \sum_i P(E_i) - \sum_{i<j} P(E_i \cap E_j) + \sum_{i<j<k} P(E_i \cap E_j \cap E_k) - P(E_1 \cap E_2 \cap E_3 \cap E_4) \right)
\]

\[
= 1 - \left( \frac{4}{1} \cdot \frac{1}{4} + \frac{4}{2} \cdot \frac{1}{14} - \frac{4}{3} \cdot \frac{1}{42} + \frac{4}{4} \cdot \frac{1}{105} \right)
\]

\[
= \frac{12}{35}
\]

2. Two fair dice are rolled. For \( i = 2, 3, \ldots, 12 \), compute the probability that the first one shows 1 given that the sum is \( i \). (Hint: There is a simple answer for \( i = 8, 9, 10, 11, 12 \).)

For \( i = 8, 9, 10, 11, 12 \) the answer is 0 since there is no way one of the dice could be 1 and still add up to a number greater than 7.

For \( i = 2, \ldots, 7 \), consider the events \( A = \) the first die shows 1, \( B_i = \) the sum of the two dice is \( i \). We want to know \( P(A|B_i) \). Let \( D_1 = \) the value of the first die and \( D_2 = \) the value of the second die. Using Bayes’ Theorem, we have
\begin{align*}
P(A|B_i) &= \frac{P(A \cap B_i)}{P(B_i)} \\
&= \frac{P(D_1 = 1 \cap D_2 = i - 1)}{P(D_1 + D_2 = i)} \\
&= \frac{\frac{1}{6} \times \frac{1}{6}}{P(D_1 + D_2 = i)} \\
&= \frac{1}{i - 1}.
\end{align*}