1. Compute the limits
   
   a. \( \lim_{x \to \infty} \frac{2x + 5}{3x - 4} = \lim_{x \to \infty} \frac{2 + 5/x}{3 - 4/x} = \frac{2}{3} \lim \frac{5}{4} = \frac{2}{3} \)
   
   b. \( \lim_{x \to \infty} \frac{x + 2}{\sqrt{9x^2 + 1}} = \lim_{x \to \infty} \frac{1 + 2/x}{\sqrt{9 + 1/x^2}} = \lim \frac{1}{\sqrt{9}} = \frac{1}{3} \)
   
   c. \( \lim_{x \to \infty} x^2 - x^4 = \lim_{x \to \infty} x^2 \left(1 - x^2\right) = -\infty. \)

2. Is there a number that is exactly 1 more than its cube? (Hint: Use the Intermediate Value Theorem to show that there is a solution to \( x = x^3 + 1. \))

   In order to apply the Intermediate Value Theorem we must determine a continuous function \( f \) with which to work with. Let us choose

   \( f(x) = x^3 - x + 1, \)

   which is continuous because it is a polynomial. Then we need to find two numbers, \( a \) and \( b \), such that \( f(a) < 0 \) and \( f(b) > 0 \). Let us start with the easiest number to plug into a polynomial, \( x = 0 \). We obtain

   \( f(0) = 1 > 0. \)

   The second easiest number to plug into a polynomial is 1, so let’s try

   \( f(1) = 1 - 1 + 1 = 1 > 0, \)

   which doesn’t help us, so now let’s try \( x = -1 \), and we get

   \( f(-1) = -1 + 1 + 1 = 1 > 0 \)

   which is still no help. At this point we should stop and think. This polynomial has highest degree odd, so as \( x \to -\infty \), \( f(x) \to -\infty \), so we just need to take a small enough \( x \) to make \( f(x) \) negative. After 0, -1, and 1 the next easiest point(s) to evaluate is ±2. Plugging in \( x = -2 \), we get

   \( f(-2) = -8 + 2 + 1 = -5 < 0, \)

   which gives us our final point! We thus conclude by writing the following:

   **We have \( f(0) > 0 \) and \( f(-2) < 0 \), so by the Intermediate Value Theorem (IVT) there exists a \( c, -2 < c < 0 \), such that \( f(c) = 0 \), i.e. there is indeed a number that is exactly 1 more than its cube.**
3. Find an equation of the tangent line to the curve \( y = \frac{2x}{(x + 1)^2} \) at the point \((0, 0)\). (Hint: Use the limit definition of the derivative. You will get no points for this problem if you use a shortcut or rule.)

The derivative of \( f(x) = \frac{2x}{(x + 1)^2} \) at \( x = 0 \) is given by

\[
\begin{align*}
  f'(0) &= \lim_{h \to 0} \frac{f(0 + h) - f(0)}{h} \\
  &= \lim_{h \to 0} \frac{2h/(h + 1)^2 - 0}{h} \\
  &= \lim_{h \to 0} \frac{2}{h + 1)^2} \\
  &= 2.
\end{align*}
\]

The equation for a line is given by \( y - y_0 = m(x - x_0) \), where \((x_0, y_0)\) is a point on the line and \( m \) is the slope. For our problem we are using \((x_0, y_0) = (0, 0)\) and \( m = f'(0) \), hence the tangent line to \( f(x) \) at the point \((0, 0)\) is given by \( y - 0 = 2(x - 0) \), or more simply

\[
y = 2x.
\]