1 Recognizing Rules

For each of the following arguments, say whether it is an instance of repetition, modus ponens, modus tollens, or double negation, or none of the above (the argument has to instantiate one of these rules in a single step; if you need to apply two of the rules, then the argument will not be an instance of either.) (worth 5pts)

(1) \(\sim (\sim P \to W)\)

\[\vdash P \to W\]

(2) \(\sim Q \to R\)

\[R\]

\[\vdash Q\]

(3) \((\sim Q \to S) \to P\)

\(\sim Q \to S\)

\[\vdash P\]

(4) \(\sim \sim P\)

\[\vdash \sim \sim P\]

(5) \(P \to Y\)

\(\sim P\)

\[\vdash \sim Y\]

2 Direct Derivations

1. Check through each line of the following direct derivations to determine whether it can be constructed by means of the provisions for direct derivations given above, where the set P is taken to be the premises of the displayed arguments. (When assessing a given line, assume that all previous lines are correct.) (worth 3pts)

- Argument:

\[R \to (S \to \sim Q)\]

\(\sim R \to \sim S\)
S

\therefore \sim Q

1. Show \sim Q
   2. S pr
   3. \sim \sim S 3 dn
   4. \sim R \rightarrow \sim S pr
   5. \sim \sim R 3 4 mt
   6. R 5 dn
   7. R \rightarrow (S \rightarrow \sim Q) pr
   8. S \rightarrow \sim Q 6 7 mp
   9. \sim Q 2 8 mp dd

- Argument:

   R \rightarrow (S \rightarrow \sim Q)
   \sim R \rightarrow \sim Q
   Q
   \therefore \sim S

1. Show \sim S
   2. Q pr
   3. \sim R \rightarrow \sim Q 2 pr
   4. \sim \sim Q 2 dn
   5. \sim \sim R 3 4 mt
   6. R 5 dn
   7. R \rightarrow (S \rightarrow \sim Q) pr
   8. S \rightarrow \sim Q 6 7 mp
   9. \sim S 4 8 mt dd

- Argument:

   P \rightarrow (Q \rightarrow \sim R)
   \sim \sim P
   \sim R
   \therefore Q
1. Show Q
2. P → (Q → ∼R) pr
3. ∼∼P pr
4. P 3 dn
5. Q → ∼R 2 4 mp
6. ∼R pr
7. Q 5 6 mp dd

2. Construct direct derivations to validate each of the following arguments (worth 2pts):

i. P → ∼Q
   
   Q
   ∼P → R
   ∴ R

ii. P → (Q → ∼ R)
   
   ∼∼P
   R
   ∴ ∼ Q