The Projecting Surface Method for improvement of surface accuracy of large deployable mesh reflectors

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A B S T R A C T

In traditional form-finding of a deployable mesh reflector (DMR), the nodes of the DMR mesh are placed on the desired working surface and the surface accuracy of the DMR is measured either by the deviation of the nodes from the desired working surface or by the deviation of the mesh from its best-fit surface. Placement of nodes on working surface and inaccurate measures of surface accuracy cause non-negligible surface errors that cannot be further reduced. To deal with these issues and to further improve surface accuracy of DMRs, a new mesh geometry design method, called the Projecting Surface Method (PSM), is presented in this paper. The highlight of the PSM is that it purposely places the nodes of a DMR off its working surface, to achieve higher surface accuracy. To this end, a direct RMS error measuring the deviation of a DMR mesh from its desired working surface is introduced and a projecting surface for hosting the nodes of the DMR mesh is defined. By the direct RMS error and projecting surface, an optimization process produces a mesh geometry with its best-fit surface closest to the desired working surface, leading to significant surface error reduction. As shown in numerical examples of DMRs with 37, 271 and 817 nodes, the PSM can reduce surface errors by 50% or more. The proposed method is usable with existing form-finding methods for further improvement of surface accuracy of DMRs.

1. Introduction

Large deployable mesh reflectors (DMRs), with important space applications, have experienced continued R&D interests in the past several decades [1–5]. A typical DMR has a spherical or parabolic surface as a working surface (a required radio-frequency surface), which is often formed by triangular facets of cable members, as shown in Fig. 1. This assembly of triangular facets is usually called a mesh or cable network. In development of such a DMR structure, all the mesh (cable) members must be tensioned, in order to ease the folding procedure and to maintain the required surface accuracy [6]. Thus, DMR meshes generally are tensioned truss structures.

This study is concerned with improvement of surface accuracy of DMRs. There are three main issues that affects DMR surface accuracy: generation of geometric configuration, measure of surface accuracy, and placement of mesh nodes. These issues are discussed in sequel.

One key procedure in design of a DMR is to obtain an initial equilibrium shape for the structure, which involves generation of a geometric configuration (nodal positions) for a given desired working surface and determination of an internal distribution of the internal forces of cable members. This procedure is usually referred to as form-finding. In the past, several methods for generation of mesh geometry have been proposed. In an early study, Nayfeh [7] first generated a mesh of triangular facets on a pyramid, and then projected it onto the designated surface of a DMR. Bush et al. [8] used an arc division approach to achieve unified facet geometry for a DMR surface. Kenner et al. [9] presented a method for design of facet geometries of tetrahedral truss reflectors, which later on was adapted by Mikulas et al. [10]. Tibert [11] generated a DMR geometry by first creating a mesh on a plane (say on the xy-plane in a Cartesian coordinate system) by the force density method and then projecting the xy-plane mesh onto the desired working in the z-direction. In mesh geometry generation, uniform tension distribution for a cable network could be achieved by proper determination of force densities with iterations [12,13]. Mainten ance of surface accuracy for mesh reflectors by active shape adjustment was proposed by Wang et al. [14]. Also, generation of an xy-plane mesh can be done by the dynamic relaxation method [15]. Shi et al. [16] presented a pseudo-geodesic design method, which first creates a mesh on a spherical reference surface and then projects the nodes onto the desired working surface. With this pseudo-geodesic method, the distribution of the internal forces of cable members is determined by a numerical optimization algorithm [17].
Most form-finding methods determine the nodal positions of a DMR in design after a distribution of internal forces is obtained. This stress-first-and-displacement-later approach makes it difficult to place the nodes at the desired locations, especially for complex DMR structures. As a notable exception, the combination of the generation of a pseudo-geodesic mesh [16] and the optimization process for an internal force distribution [17] can keep the designated nodal positions of a DMR unchanged during form-finding, as shown in Ref. [18]. For this reason, the combination shall be called the fixed nodal position method (FNPM).

The surface accuracy of a deployable mesh reflector, which is the closeness of the mesh and a desired working surface, is essentially important to the performance of the DMR. Indeed, a properly designed mesh geometry with high accuracy of a reflecting surface provides important foundation and useful guidance in the subsequent RF design. There exist several methods for measurement of surface accuracy of DMR meshes. One commonly used method proposed by Agrawal et al. [19] measures surface accuracy by root mean square (RMS) error between a mesh that is obtained in form-finding and its best-fit surface, which is a sphere or a parabola with least deviation from the mesh. Because the desired working surface is not considered, the RMS error so defined is not a true measure of the deviation of the mesh from the desired working surface. Another widely used measure of surface accuracy considers RMS error between the nodes of the mesh of a DMR and the desired working surface. This definition is erroneous because a mesh with all its the nodes sitting on the desired working surface would end up with zero RMS error, regardless of the number of nodes, which is misleading. A true measure of surface accuracy in DMR design should be deviation of the surface generated by the facets of a mesh from the desired working surface. Usage of a true measure of surface accuracy in form-finding of deployable mesh reflectors, however, has not been fully explored.

In design of a DMR, traditional thinking is that all the nodes of the mesh must be on the desired working surface. This way of design may not be optimal in reduction of surface error because all the cable members of the DMR would be placed in front of the desired working surface toward the focal point. According to the true measure of surface accuracy as discussed previously, the surface accuracy of a DMR could be significantly improved if the nodes are off the desired working surface and slightly away from the focal point [20]. This approach of purposely placing nodes off the working surface in DMR design deserves further attention.

Addressing the above-mentioned issues, this paper presents a new method for improvement of surface accuracy for deployable mesh reflectors, namely the Projecting Surface Method (PSM). In the development, a direct RMS error measuring the deviation of a DMR mesh from the corresponding desired working surface is first introduced. With this RMS error, the nodes of the DMR mesh are then generated by the proposed PSM on a projecting surface such that the best-fit surface of the mesh is closest to the desired working surface. Different from a conventional method, the PSM can freely assign the nodes at desired locations in form-finding. Furthermore, in generation of geometric configuration, the traditional restriction of nodes on a desired working surface is lifted, which gives more room for improvement of DMR surface accuracy. The highlight of this effort is that the surface accuracy of DMRs with many nodes can be significantly improved by the proposed method, as seen in numerical examples.

The remainder of this paper is arranged as follows. Three methods for generation of DMR mesh geometry are briefly reviewed in Section 2. A direct RMS error is introduced in Section 3. TheProjecting Surface Method for surface accuracy improvement in generation of DMR mesh geometry is presented in Section 4. And results from numerical simulation by the proposed method are given in Section 5.

2. Brief review of methods for mesh geometry generation

In this section, three existing methods of form-finding, namely, the force density method, the dynamic relaxation method and the fixed nodal position method, are briefly reviewed to prepare for the presentation of the new method of surface accuracy improvement for deployable mesh reflectors.

2.1. The force density method

For the cable net of a DMR as shown in Fig. 1, balance of forces at the ith node in the x-direction can be described by the equilibrium equation

\[ \sum_j T_{ij} (x_i - x_j) = f_{ix} \]  

(1)

where \( T_{ij} \) and \( L_{ij} \) are the tension force and length of the mesh member that connects the ith node and the jth node of the cable net, respectively; \( x_i \) is the x-coordinate of the ith node; and \( f_{ix} \) is the x-direction component of the external load applied at the ith node. The length \( L_{ij} \) is given by

\[ L_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2} \]  

(2)

The force equilibrium equation in the y-direction can be obtained by replacing the x-direction components \( x_i, x_j \) and \( f_{ix} \) in Eq. (1) by the y-direction components \( y_i, y_j \) and \( f_{iy} \).

Consideration of force equilibrium in the x- and y-directions at all the nodes yields a system of nonlinear algebraic equations with the nodal coordinates being unknowns. Solution of these equations is difficult and computationally expensive for large-scale truss structures. One way to deal with this issue is to apply the force density method (FDM), which was first proposed by Schek [21] and later on extended by Linkwitz [22]. The FDM, by defining force densities \( q_{ij} \) as

\[ q_{ij} = \frac{T_{ij}}{L_{ij}} \]  

(3)

transforms the original nonlinear equations like Eq. (1) into

\[ C^T Q C x = f_x \]  

(4)

where \( x \) is the vector of nodal coordinates (unknowns), \( C \) is the incidence matrix, and \( Q \) is a diagonal matrix containing the force densities. In solution, the force density matrix \( Q \) must be determined first. Once \( Q \) is known, Eq. (4) becomes a system of linear algebraic equations, which can be solved by a standard method for the nodal coordinate vector \( x \).

For mesh geometry generation of a deployable mesh reflector, a common practice takes two steps: (i) to determine the nodal
coordinates for a mesh in the xy-plane by the FDM; and (ii) to project the xy-plane nodes vertically in the z-direction onto the desired working surface, in which the z-direction components of the nodal forces are balanced by tension tie forces [11]. In this process, the force densities are required to be positive so that all the cable members are tensioned. For a DMR with uniform tension distribution, force densities are obtained through a try and error process [12].

According to the above description, the force density method is a stress-first-and-displacement approach, which in general cannot freely place mesh nodes at desired locations.

2.2. The dynamic relaxation method

Like the force density method, the dynamic relaxation method can be applied to determine to the nodal positions in the xy-plane for a DMR. To this end, a virtual mass-spring-damper system in the x-direction is described by

\[ m\ddot{x} + c\dot{x} + kx = f_x \]  

(5)

where \( m \) and \( c \) are the fictitious mass and damping coefficient of a node, which can be selected by the methods given in Ref. [23]. The equilibrium nodal positions are the steady state position of Eq. (5), which can be determined by a numerical iteration process. Note that during the iteration, the values internal forces are kept unchanged. Once the mesh geometry is generated in the xy-plane, the nodes are projected in the z-direction onto the desired working surface.

Like the force density method, the dynamic relaxation method is a stress-first-and-displacement approach, which cannot freely place mesh nodes at desired locations.

2.3. The fixed nodal position method

The fixed nodal position method (FNPM) has two key parts: (a) generation of geometric configuration (nodal coordinates) by a pseudo-geodesic method; and (b) determination of internal force distribution of cable members by an optimization algorithm [17]. Different from the force density method and the dynamic relaxation method, the FNPM obtains the geometric configuration and internal force distribution of a DMR separately. In form-finding, the FNPM first assigns mesh nodes at desired locations, and then determines the corresponding internal force distribution without altering the designated nodal coordinates in the solution process [18].

In generation of geometric configuration for a DMR by the pseudo-geodesic method, a spherical reference surface that has the same height and aperture diameter as the desired working surface is first defined; see Fig. 2, where \( H \) and \( R_s \) are the height and radius of the reference surface, and \( H_p \) and \( D \) are the height and apertuer diameter of the desired working surface. These parameters have the following relationships

\[ H = H_p, \quad R_s^2 = (R_s - H)^2 + \frac{D^2}{4} \]  

(6)

On the spherical reference surface, a mesh of geodesic curves (segments of big circles) is then generated; see Fig. 3 for the five steps in mesh generation [16]. Afterwards, the mesh on the reference surface is vertically projected to a designated surface, resulting in the nodal coordinates of a pseudo-geodesic mesh. In a conventional form-finding method, the designated surface is usually the desired working surface of the DMR. In form-finding by the FNPM, mesh nodes can be placed off the desired working surface for improved surface accuracy, as shall be shown in the subsequent sections.

With the so generated nodal coordinates, the corresponding tension distribution among the members and the tension tie forces are obtained by a numerical optimization algorithm as described in Ref. [17]. The optimization problem can be described as

\[ \min |T - T_{des}|^2 \]  

subject to \( MT = T_{des} \) and \( T > 0 \)  

(7)

where \( T \) is a vector of internal forces; \( T_{des} \) describes a desired tension distribution of cable members; and \( MT = T_{des} \) is the force equilibrium equation of DMR in design, where \( M \) is an equilibrium matrix consisting of direction cosines and \( T_{des} \) is a vector of tension tie forces. Here, the constraint \( T > 0 \) indicates that all the cable members must be tensioned. Throughout the optimization process, the nodal coordinates are unchanged. This special feature of fixed nodal positions in form-finding allows the FNPM to have guaranteed high surface accuracy in design of deployable mesh reflectors [18].

3. Direct RMS error

Surface deviation of a DMR may be caused by geometric difference between the triangular facets and the desired working surface (say, a parabolic or spherical surface), variation of tension tie loads, member length imperfection, ring structure distortion and thermal strains. Because this work is concerned with optimal design of cable networks for improved surface accuracy, only geometric difference between the triangular facets of a DMR and the desired working surface is taken into consideration and all other factors that cause surface deviation are
neglected.

The geometry surface error of a DMR can be estimated by either rough estimation [24] or analytical calculation [19]. For a large DMR, rough surface error estimation is only used for a preliminary design; more accurate estimation involving in analytical calculation should be used in the complete design.

The geometry surface error of a DMR in general can be measured by three approaches: nodal deviation RMS error, the best-fit surface RMS error and the direct RMS error. The nodal deviation RMS error and best-fit surface RMS error usually work under the assumption that all the DMR nodes are placed on the desired working surface. These estimates do not measure the real deviation of the mesh geometry from the desired working surface. If stringent requirement on high surface accuracy in DMR design is implemented or if the nodes of a mesh can be placed off the desired working surface, a direct RMS error is necessary for more accurate evaluation.

In this section, based on quick review of nodal derivation RMS error and best-fit surface RMS error, a direct RMS error is defined for design of DMRs with high surface accuracy.

3.1. Nodal deviation RMS error

One commonly used measure of surface accuracy is to calculate the RMS error due to the deviation of the nodes of a mesh from the desired working surface. For instance, such an RMS error \( \delta_{nd,\text{rms}} \) can be expressed by Ref. [25]
### 3.3. Direct RMS error

The best-fit RMS error described in Section 3.2 is not a true measure of geometry deviation of a DMR mesh from its desired working surface. In this work, for design of DMRs of high surface accuracy, a direct RMS error that truly measures the area deviation of a DMR mesh geometry from its desired working surface is proposed. A comparison of these two types of RMS errors are shown in Fig. 7.

Consider a typical triangular facet in Fig. 8, where the desired working surface is also shown. To define the direct RMS error, a local coordinate system (ξ, ω) is established on the plane of the facet; see Fig. 9, where the origin can be any one of the nodes of the facet. Let μ (ξ, ω) be the normal distance between a point on the facet plane and the desired working surface. The squared deviation of the triangular facet from the desired working surface is

$$\phi = \iint S \mu^2 \, d\xi d\omega$$

(13)

with S being the facet area. By summing the deviations of all the facets, the direct RMS error of the DMR mesh is defined as follows

$$\delta_{d-rms} = \frac{1}{\sqrt{S_{mesh}}} \sum_i \phi_i$$

(14)

where $S_{mesh} = \sum S_i$, which is the total area of all the facets of the mesh.

It should be noted that the distance $\mu (\xi, \omega)$ in Eq. (10) and the distance $\mu (\xi, \omega)$ are not the same. In calculation of $\mu (\xi, \omega)$, many approximations, including the assumption of shallow parabolic working surface, have been made. Because of this, the formulas for computing the best-fit surface RMS error [19] are not applicable in calculation of the direct RMS error defined herein. In this effort, for design of DMRs with high surface accuracy, exact analytical formulas for computing the direct RMS error are derived, which are given in the Appendix.

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**Fig. 4.** Triangular facet and desired working surface. (a) nodes are placed off the desired working surface; (b) nodes are placed on the desired working surface.

$$\delta_{d-rms} = \frac{1}{\sqrt{S}} \sum (\Delta x^2 + \Delta y^2 + \Delta z^2)$$

(8)

where $\Delta x$, $\Delta y$ and $\Delta z$ are the normal distances between the nodes and the desired working surface in the $x$, $y$ and $z$ directions; and $N$ is the total number of nodes. While being simple and easy to use, the formula in Eq. (8) is not accurate enough because it fails to consider the geometry difference between the facet plane and the working surface. For instance, the surface deviation of a triangular facet from the desired working surface in Fig. 4 (a) should be smaller than that in Fig. 4 (b), but the $\delta_{d-rms}$ by Eq. (8) gives an opposite result simply because the facet nodes in Fig. 4 (a) are off the desired working surface. Furthermore, Eq. (8) concludes zero surface error if all the nodes of a mesh are on the desired working surface, regardless of the number of nodes, which is misleading.

Another type of nodal deviation RMS error, proposed in Ref. [12], is to compare the values of two parameters: $Z'$ and $Z''$, where $Z'$ is the position of the gravity center of every computed triangular facet on the $z$-axis (i.e., the vertical axis in Fig. 1), and $Z''$ is the $z$-coordinate of this point when projected vertically onto the desired working surface. The RMS error is then given by

$$\delta_{n-rms} = \left( \frac{1}{n} \sum_{i=1}^{n} S_i (Z_i - Z_0)^2 / \sum_{i=1}^{n} S_i \right)^{1/2}$$

(9)

with $S_i$ being the projected area of the triangular facet on the $xy$-plane.

This calculation generally provides rough estimation of surface error because only one point (gravity center) is used for each facet. To achieve more accurate estimation of surface error, the geometry difference between the facet plane and the desired working surface must be considered.

### 3.2. Best-fit surface RMS error

After a DMR mesh is generated, it is natural to find out what surface (spherical or parabolic) the mesh geometry best represents. The concept of best-fit sphere is thus introduced. The best-fit surface of a DMR mesh geometry is the sphere or paraboloid, which, among all the possible spherical or parabolic surfaces, has the least deviation from the mesh [26,27]. A schematic of the DMR mesh, its best-fit surface and the desired working surface is shown in Fig. 5.

The best-fit surface RMS error is defined as follows. As shown in Fig. 6, for a given triangular facet of the generated mesh, a plane $P$ is defined by containing the $z$-axis and the centroid of the triangular facet. A local coordinate system $(\xi, \eta)$ is generated by having the triangular facet node with the largest $z$-coordinate be the origin. The $\xi$ axis is parallel to the intersection between plane $P$ and the plane of the triangular facet. The $\eta$ axis is in the plane of the triangular facet, perpendicular to the $\xi$ axis. Denote the normal distance between a point on the facet plane and the best-fit surface by $\omega (\xi, \eta)$, which was calculated in Ref. [19]. A squared deviation of the facet plane from the best-fit surface is

$$\phi = \iint S \omega^2 d\xi d\eta$$

(10)

with $S$ being the facet area. The best-fit surface RMS error of the entire mesh then is defined as follows

$$\delta_{bf-rms} = \frac{1}{\sqrt{S_{mesh}}} \sum_i \phi_i$$

(11)

where $S_{mesh} = \sum S_i$, which is the total area of all the facets of the mesh.

In a conventional method, all the nodes are placed on the desired working surface. The best-fit parabolic or sphere surface is then found by properly determining its focal length $F_{bf}$ and the vertex height $H_{bf}$, such that $\delta_{bf-rms}$ in Eq. (11) is minimized. This procedure can be done by a nonlinear optimization algorithm.

$$\text{min} [\delta_{bf-rms}(F_{bf}, H_{bf})]$$

(12)

The minimized $\delta_{bf-rms}$ is the best-fit surface RMS error.
4. Surface accuracy improvement by the Projecting Surface Method

With the direct RMS defined in Section 3.3, a new mesh design method for improvement of DMR surface accuracy, which shall be called the Projecting Surface Method (PSM), is developed. One key in this new method is to project the nodes of the mesh of a DMR on a projecting surface, which is created such that the best-fit surface of the generated mesh is close to the desired working surface. The projecting surface in general is not the same as the desired working surface, which indicates that the nodes can be placed off the desired working surface in design of the DMR.

Ideally, surface accuracy improvement is achieved by finding a mesh geometry with its best-fit surface closest to the desired working surface of the DMR in consideration. This thought, however, is not practical because the projecting surface is unknown in the first place. Under the circumstances, an iteration process for determination of the projecting surface is developed. For the convenience of subsequent discussion, the \((i-1)\)th step in the iteration shall be called the previous step and the \(i\)-th step the current step. Let \(F_{pr}^{i-1}\) and \(D_{pr}^{i-1}\) be the focal length and diameter of the projecting surface in the current step, respectively. The process takes the following six steps.

Step 1. Make an initial guess of the projecting surface

The desired working surface of the DMR in design is taken as an initial guess of the projecting surface; that is \(F_{pr}^{0} = F_{des}\) and \(D_{pr}^{0} = D_{des}\), where \(F_{des}\) and \(D_{des}\) are the focal length and diameter of the desired working surface.

Step 2. Generate the mesh geometry on a reference surface

The mesh geometry of the DMR is generated on a reference surface by one of the form-finding methods described in Section 2. For the force density method and the dynamic relaxation method, the reference surface is the \(xy\)-plane. For the FNPM, the reference surface is a sphere with the same height and aperture diameter as the projecting surface obtained in the previous step.

Step 3. Project nodes on the projecting surface

The nodes of the DMR mesh generated in Step 2 are projected on the projecting surface obtained in the previous step, with focal length \(F_{pr}^{i-1}\) and diameter \(D_{pr}^{i-1}\).

Step 4. Determine the best-fit surface

With the projected mesh obtained in Step 3, determine its best-fit surface by the method described in Section 3.2. Denote the focal length...
and diameter of the best-fit surface by $F_{bf}$ and $D_{bf}$, respectively.

Step 5. Compare the difference between the best-fit surface and the desired working surface

The difference between the best-fit surface in Step 4 and the desired working surface is compared. To this end, a square root error parameter $\epsilon$ is defined as follows

$$
\epsilon = \sqrt{(F_{bf} - F_{des})^2 + (D_{bf} - D_{des})^2}
$$

(15)

If $\epsilon$ is acceptably small (below a prescribed threshold value), the iteration process is terminated and the mesh geometry of the DMR obtained in the previous step is the final structure. Otherwise, go to Step 6.

Step 6. Adapt the projecting surface

The projecting surface is adapted by

$$
F_{pro} = F_{pro}^{-1} + \kappa (F_{bf} - F_{des}), \quad D_{pro} = D_{pro}^{-1} + \lambda (D_{bf} - D_{des})
$$

(16)

where $\kappa$ and $\lambda$ are positive coefficients that are properly selected for the iteration process. After the adaptation, go back to Step 2 for the next round of the iteration.

The formulas given in Eq. (16) adjust the size of the projecting surface. If the best-fit surface in the previous step is smaller than the desired working surface (see Fig. 7 (a) for an example), the projecting surface updated in the current step should be slightly larger than that in the previous step, to generate a mesh geometry with its best-fit surface being larger and closer to the desired working surface (see Fig. 7 (b) for an example). On the other hand, if the best-fit surface in the previous step is larger than the desired working surface, the projecting surface updated in the current step should be slightly smaller than that in the previous step.

A flowchart of the six-step Projecting Surface Method is given in Fig. 10. The PSM is different from conventional methods in that the nodes are projected onto a properly selected projecting surface instead of the desired working surface and that the node of the final DMR mesh generally are off the desired working surface. This way, the surface accuracy of a DMR is improved through reduction of the deviation of the mesh facets from the desired working surface. The proposed PSM is easy to implement and it can significantly reduce the surface error of DMRs, as shall be seen in the numerical examples given in the next section.

5. Numerical examples

The Projecting Surface Method is applied in design and analysis of large deployable mesh reflectors. The DMR mesh geometries under investigation are a class of AstroMesh nets [28]. Without loss of generality, only the front cable net of a DMR (the reflecting surface) is considered and the boundary nodes are assumed fixed. For illustrative purposes, a 37-node front cable net is shown in Fig. 11. DMRs with more nodes are also considered.

5.1. Geometric difference between the best-fit surface and desired working surface

Before application of the proposed PSM, the geometric parameters (focal length and aperture diameter) of the best-fit surface of a DMR...
(designed by a traditional method) and those of the desired working surface are compared. The purpose is to investigate the limitation of traditional form-finding, which requires that all the nodes of a DMR be on the desired working surface. As shall be seen, the difference between the desired working surface and the best-fit surface of the mesh generated by a traditional method is non-negligible. To this end, the 37-node DMR in Fig. 11 is designed by the force density method with the nodes being placed on the desired working surface. For comparison, two sets of desired working surfaces are considered. In the first set, the desired working surfaces are center-feed paraboloids with aperture diameter $D_{des}$ being 12 m and focal length $F_{des}$ being 14, 16 and 18 m, respectively. In the second set, the desired working surfaces are center-feed paraboloids with $F_{des}$ being 12 m and $D_{des}$ being 14, 16 and 18 m, respectively.

Listed in Tables 1 and 2 are the geometric differences between the best-fit surface in the traditional sense and the desired working surface for the 37-nodes DMR. In the tables, $\varepsilon_F$ and $\varepsilon_D$ are the focal length error and aperture diameter error that are defined as follows:

$$\varepsilon = \sqrt{(F' - F_{des})^2 + (D' - D_{des})^2}$$

As can be seen from the tables, the focal length error $\varepsilon_F$ varies from 1.22% to 1.59% and the aperture diameter error $\varepsilon_D$ is larger than 2.1%, which are not negligible in design of a DMR with high surface accuracy requirements. Obviously, traditional form-finding with the requirement of nodes on desired working surface cannot avoid such errors. Indeed, to obtain DMRs of higher surface accuracy, a new method that allows the nodes of a DMR to be placed off the desired working surface becomes necessary. The improvement of surface accuracy by the proposed PSM shall be seen in the following numerical examples.

5.2. Surface accuracy improvement for center-feed DMRs

The proposed Projecting Surface Method, which places mesh nodes on the projecting surface, is illustrated on further improvement of surface accuracy in design of large DMRs. The surface accuracy of a DMR is measured by the direct RMS error defined in Section 3.3. The direct RMS errors of DMRs with and without the application of the PSM are compared. In generation of mesh geometries, the three form-finding methods described in Section 2 are used.

In numerical simulation, the desired working surface is a center-feed paraboloid with diameter $D_{des}$ being 12 m and focal ratio $F_{des}/D_{des}$ being 0.26. Three meshes with 37, 271 and 817 nodes respectively are considered.

The tension distribution of a DMR is required to show the overall performance of the form-finding methods. For the force density and the dynamic relaxation methods, the mesh geometry and tension

![Flowchart of the projecting surface method](image)
distribution are determined simultaneously, no further design techniques are required. For the fixed nodal position method, the tension distribution is determined by a numerical optimization algorithm as described in Section 2.3. A desired uniform tension distribution is specified as follows

\[ T_{\text{new}} = \begin{pmatrix} 10 & 10 & \cdots & 10 \end{pmatrix} \text{in newton (N)} \]  

Table 3 lists the RMS errors of mesh geometries obtained by the force density method (FDM), the dynamic relaxation method (DRM) and the fixed nodal position method (FNPM), with and without the application of the proposed Projecting Surface Method. The unit of the RMS errors is millimeter (mm). In the numerical simulation, the FDM and DRM produce the same results. As such, the results by the FDM and DRM are put in the same columns of the table.

As seen from Table 3, when the proposed PSM (placing nodes on the projecting surface) is applied in mesh generation by the FDM/DRM, the RMS errors are reduced by 63.6%, 52.5% and 49.2% for the meshes of 37, 217 and 817 nodes, respectively. When the PSM is applied in mesh generation by the FNPM, the RMS errors are reduced by 72.8%, 73.6% and 71.4%, respectively. Furthermore, even without the PSM, the FNPM results already are much better than those by the FDM/DRM, especially when the number of nodes is large.

The tension distributions of the 271-node DMRs are given in Table 4, where average member tension \( T_{\text{ave}} \), tension ratio \( \frac{T_{\text{max}}}{T_{\text{min}}} \), tension tie forces \( T_{\text{tie,ave}} \) and tension tie ratio \( \frac{T_{\text{tie, max}}}{T_{\text{tie, min}}} \) are compared. While the FDM and DRM deliver almost uniform tension distributions for the cable net, the FNPM yields non-uniform tension distributions with a tension ratio about 2.2. The areas of effective region of reflection generated by FDM and DRM are much smaller than those by the FNPM through. According to Table 4, with the application of the Projecting Surface Method, the tension distributions are maintained at a similar level without much changes. This observation indicates a special and useful feature of the

Table 1
Focal length difference between the best-fit surface and the desired working surface for the 37-nodes DMR.

<table>
<thead>
<tr>
<th>( F_{\text{des}} )</th>
<th>12</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon_F )</td>
<td>5.9269</td>
<td>7.8822</td>
<td>9.8408</td>
<td></td>
</tr>
<tr>
<td>( \varepsilon_D )</td>
<td>1.22%</td>
<td>1.47%</td>
<td>1.59%</td>
<td></td>
</tr>
</tbody>
</table>

Table 2
Aperture diameter difference between the best-fit surface and the desired working surface for the 37-nodes DMR.

\[ \varepsilon_F = \left| \frac{F_{\text{bf}} - F_{\text{des}}}{F_{\text{des}}} \right| , \quad \varepsilon_D = \left| \frac{D_{\text{bf}} - D_{\text{des}}}{D_{\text{des}}} \right| \]  

Table 3
RMS error comparison of center-feed DMRs designed (mm).

<table>
<thead>
<tr>
<th></th>
<th>FDM/DRM (nodes on the working surface)</th>
<th>FDM/DRM (nodes on the projecting surface)</th>
<th>FNPM (nodes on the working surface)</th>
<th>FNPM (nodes on the projecting surface)</th>
</tr>
</thead>
<tbody>
<tr>
<td>37 nodes</td>
<td>117.84</td>
<td>42.84</td>
<td>85.28</td>
<td>23.19</td>
</tr>
<tr>
<td>271 nodes</td>
<td>43.06</td>
<td>20.44</td>
<td>9.51</td>
<td>2.51</td>
</tr>
<tr>
<td>817 nodes</td>
<td>36.43</td>
<td>18.50</td>
<td>3.01</td>
<td>0.86</td>
</tr>
</tbody>
</table>

Table 4
Comparison of tension distribution of a 271 nodes center-feed DMR.

<table>
<thead>
<tr>
<th></th>
<th>FDM/DRM (nodes on the working surface)</th>
<th>FDM/DRM (nodes on the projecting surface)</th>
<th>FNPM (nodes on the working surface)</th>
<th>FNPM (nodes on the projecting surface)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{\text{ave}} )</td>
<td>9.9958</td>
<td>9.9962</td>
<td>9.7982</td>
<td>9.7972</td>
</tr>
<tr>
<td>( T_{\text{max}}/T_{\text{min}} )</td>
<td>1.0048</td>
<td>1.0046</td>
<td>2.1911</td>
<td>2.1917</td>
</tr>
<tr>
<td>( T_{\text{ave}} )</td>
<td>2.8961</td>
<td>2.8786</td>
<td>3.1516</td>
<td>3.1571</td>
</tr>
<tr>
<td>( T_{\text{tie, max}}/T_{\text{tie, min}} )</td>
<td>2.2620</td>
<td>2.1848</td>
<td>1.2500</td>
<td>1.2493</td>
</tr>
</tbody>
</table>
proposed PSM in form-finding: the method can be applied to improve the surface accuracy of a DMR without significantly altering its tension distribution.

In summary of the results of this example, with nodes placed on the projecting surface that is determined by the PSM, form-finding by any existing method (the FDM or the DRM or the FNPM) can significantly reduce the surface error of DMRs. Furthermore, for the best results on surface accuracy improvement, the combination of the PSM and FNPM is recommended.

5.3. Surface accuracy improvement for offset-feed DMRs

An offset-set DMR shown in Fig. 12 is designed. For the parent paraboloid, the diameter $D_p$ is 30 m and the focal ratio $F_p/D_p$ is 0.2. The offset-distance $e_{off}$ is 1 m. As in the previous section, offset-feed parabolic DMRs of 37, 271 and 817 nodes are considered and DMR mesh geometries are generated by the three form-finding methods (the FDM, the DRM and the FNPM).

The direct RMS errors (in millimeter) and tension distributions (in newton) of the mesh geometries so generated are compared in Tables 5 and 6. Again, the FDM and the DRM give almost identical results in all the cases. The results from the tables show that the proposed Projecting Surface Method is equally efficient in design of offset-feed DMRs. By Table 5, with the PSM applied to FDM/DRM, the RMS errors are reduced by 58.0%, 44.7% and 36.8% for the meshes of 37, 217 and 817 nodes, respectively. With the combination of the PSM and the FNPM, the RMS errors are reduced by 47.9%, 46.3% and 52.2%, respectively. From Table 6, the application of the PSM in form-finding with any of the three methods hardly changed the tension distribution. This feature of almost no influence on tension distribution makes the proposed Projecting Surface Method desirable in design of DMRs.

6. Conclusions

A new mesh design method, namely the Projecting Surface Method (PSM), has been developed for improvement of surface accuracy of large deployable mesh reflectors. The main results from this investigation are summarized as follows.

(i) A direct RMS error for measurement of surface accuracy of DMRs is introduced and the relevant analytical formulas are derived. Compared to the existing measures of RMS errors, the direct RMS error provides a better estimation of surface accuracy because it directly computes the deviation of a DMR mesh from its desired working surface, without making approximations. The direct RMS error is applicable to both deep and shallow deployable mesh reflectors.

(ii) A projecting surface is defined to achieve better surface accuracy in form-finding of DMRs. The projecting surface, which is not the same as the desired working surface of a DMR, hosts the nodes of the DMR mesh such that the best-fit surface of the mesh is closest to the desired working surface.

(iii) Based on the projecting surface and an optimization process, the PSM for form-finding is developed, which is applicable to both center-feed and offset-feed DMRs. The highlight of this new mesh design method is that it purposely places the nodes of a DMR mesh off the desired working surface. It is this removal of the requirement of mesh nodes on working surface in conventional form-finding that makes more room for further improvement of surface accuracy for large deployable mesh reflectors.

(iv) The Projecting Surface Method can be used with existing form-finding methods, including the force density method (FDM), the dynamic relaxation method (DRM) and the fixed nodal position method (FNPM), for further reduction of surface errors of deployable mesh reflectors. One nice feature about the proposed PSM is that it can significantly reduce surface error of a DMR without largely altering the distribution of pretension forces of the DRM.

(v) The proposed PSM is illustrated on center-feed and offset-feed parabolic DMRs, with three meshes of 37, 271 and 817 nodes, respectively. The numerical results show that the PSM, when used either the FDM or DRM, reduces surface errors by 36.8–63.6%. The combination of the PSM and the FNPM delivers the best results:

![Fig. 12. The desired working surface of an offset-feed DMR.](Image)

### Table 5

<table>
<thead>
<tr>
<th></th>
<th>FDM/DRM (nodes on the working surface)</th>
<th>FDM/DRM (nodes on the projecting surface)</th>
<th>FNPM (nodes on the working surface)</th>
<th>FNPM (nodes on the projecting surface)</th>
</tr>
</thead>
<tbody>
<tr>
<td>37 nodes</td>
<td>64.34</td>
<td>27.04</td>
<td>65.27</td>
<td>33.99</td>
</tr>
<tr>
<td>271 nodes</td>
<td>8.89</td>
<td>4.92</td>
<td>7.87</td>
<td>4.23</td>
</tr>
<tr>
<td>817 nodes</td>
<td>3.34</td>
<td>2.11</td>
<td>2.53</td>
<td>1.21</td>
</tr>
</tbody>
</table>

### Table 6

Comparison of tension distribution of a 271 nodes offset-feed DMR.

<table>
<thead>
<tr>
<th></th>
<th>FDM/DRM (nodes on the working surface)</th>
<th>FDM/DRM (nodes on the projecting surface)</th>
<th>FNPM (nodes on the working surface)</th>
<th>FNPM (nodes on the projecting surface)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{tie_{avg}}$</td>
<td>9.9999</td>
<td>9.9999</td>
<td>9.9419</td>
<td>9.9419</td>
</tr>
<tr>
<td>$T_{tie_{ave}}/T_{tie_{max}}$</td>
<td>1.0008</td>
<td>1.0008</td>
<td>2.0118</td>
<td>2.0118</td>
</tr>
<tr>
<td>$T_{tie_{av}}/T_{tie_{max}}$</td>
<td>1.6007</td>
<td>1.6017</td>
<td>1.5971</td>
<td>1.5971</td>
</tr>
<tr>
<td>$T_{tie_{av}}/T_{tie_{max}}$</td>
<td>2.7985</td>
<td>2.8017</td>
<td>3.0194</td>
<td>3.0194</td>
</tr>
</tbody>
</table>
reducing surface errors by 47.9–72.8% and obtaining the minimum RMS errors in all the cases of simulation. For the center- 
feed DMR of 817 nodes (Table 3), the PSM with the FNPM leads to an RMS error of 0.86 mm, compared with 3.34 mm by the 
FDM or the DRM alone, which is a 63.8% reduction in RMS error. 
The proposed Projecting Surface Method can be generalized to ad-
dress the issue of reflecting surface distortions of DMRs that are caused 
by thermal loads or member length imperfection in structure assembly. 
This is a topic of future research effort.

Appendix. Formulas for Calculation of the Direct RMS Error Defined in Section 3.3

To precisely evaluate the surface accuracy of a DMR, it is desirable to have a methodology for directly evaluating the deviation of the mesh 
geometry of the DMR from its desired working surface. Such methodology is currently unavailable in the literature. The exact analytical forms in 
this appendix fill the technical gap.

For a typical triangular facet, a local coordinate system (τ, υ, μ) is defined in Fig. 9, where the origin is at one of the facet nodes; the τ-axis is in the 
direction from (τ1, υ1, μ1) to (τ2, υ2, μ2); the υ-axis is normal to the τ-axis, in the facet plane; and the μ-axis is normal to the facet plane. The equation of the 
facet plane in the global coordinate system (x, y, z) is

$$a_τx + b_τy + c_τz + d_τ = 0$$

which can be obtained from the coordinates of the three nodes, namely (x1, y1, z1), (x2, y2, z2) and (x3, y3, z3). Here for convenience, it is assumed 
that the node (x1, y1, z1) is the origin of the local coordinate system (τ, υ, μ). In the global coordinate system, let the unit vectors of the τ-, υ- and μ-
axes be e1, e2 and e3, respectively. These unit vectors are given by

$$
e_1 = \frac{[x_2-x_1, y_2-y_1, z_2-z_1]^T}{\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}}$$

$$
e_2 = \frac{1}{\sqrt{d_1^2 + d_2^2 + d_3^2}} [a_τ, b_τ, c_τ]^T, \quad e_3 = \frac{n \times e_1}{|n \times e_1|}$$

(A.2)

Denote the unit vectors of the global coordinate system xyz by as e1′, e2′ and e3′, which are be expressed by

$$e_1' = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad e_2' = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad e_3' = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

(A.3)

The coordinate transformation matrix A from local coordinate system (τ, υ, μ) to global coordinated system (x, y, z) is

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\
A_{21} & A_{22} & A_{23} \\
A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} e_1^T e_1' & e_1^T e_2' & e_1^T e_3' \\
e_2^T e_1' & e_2^T e_2' & e_2^T e_3' \\
e_3^T e_1' & e_3^T e_2' & e_3^T e_3' \end{bmatrix}$$

(A.4)

The global to local coordinates are related by

$$\begin{bmatrix} ς \\ υ \\ μ \end{bmatrix} = A^{-1} \begin{bmatrix} x-x_1 \\ y-y_1 \\ z-z_1 \end{bmatrix}$$

(A.5)

Because A is an orthogonal matrix,

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1} \begin{bmatrix} ς \\ υ \\ μ \end{bmatrix} = A^T \begin{bmatrix} ς \\ υ \\ μ \end{bmatrix} + \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$$

(A.6)

It follows that the global coordinates can be expressed by

$$x = A_{11} \tau + A_{12} \upsilon + A_{13} \mu + x_1$$

$$y = A_{21} \tau + A_{22} \upsilon + A_{23} \mu + y_1$$

$$z = A_{31} \tau + A_{32} \upsilon + A_{33} \mu + z_1$$

(A.7)

Recall that the equation of the desired parabolic working surface is

$$z - H = -\frac{1}{4F}(x^2 + y^2)$$

(A.8)

Substitute Eq. (A.7) into Eq. (A.8) and rearrange the resulting equation with respect to μ, to obtain

$$a\mu^2 + b\mu + c = 0$$

(A.9)

with

$$a = (A_{11}^2 + A_{12}^2)\frac{1}{4F}$$

$$b = A_{13} + A_{31}(x_1 + A_{11}\tau + A_{12}\upsilon)^2 + A_{32}(y_1 + A_{21}\tau + A_{22}\upsilon)^2 + \frac{1}{4F}$$

$$c = -H + x_1 + A_{13}\upsilon + A_{31}\upsilon + (x_1 + A_{11}\tau + A_{12}\upsilon)^2 + (y_1 + A_{21}\tau + A_{22}\upsilon)^2$$

(A.10)

According to Eq. (A.9), μ is a function of τ and υ, namely, μ = μ(τ, υ). For a point (τ*, υ*) on the facet, μ(τ*, υ*) is the normal distance from the 
point on the facet to the parabola as described by Eq. (A.8). Solution of Eq. (A.9) gives
As shown in Fig. A.1, for a line that is normal to the facet plane and passes through one point on the facet, there are two intersections between the line and the parabola. For the calculation of the direct RMS error, only the intersection with smaller distance from the point on the facet represent the deviation of the point from the desired working surface. Thus, out of the two roots given by Eq. (A.11), only the one with smaller absolute value is the true solution. With such selected μ, the direct RMS error can be computed by Eqs. (13) and (14).

Fig A.1. Two intersections between the line normal to the facet plane and the desired parabolic working surface.

Appendix A. Supplementary data

Supplementary data related to this article can be found at https://doi.org/10.1016/j.actaastro.2018.07.005.
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