New Methodology of Surface Mesh Geometry Design for Deployable Mesh Reflectors

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Large deployable mesh reflectors are found in many space applications. To warrant the performance of this type of space structure, it is desirable to have a methodology for systematic design of deployable mesh reflector surface geometries that can yield high surface accuracy and produce almost uniform distribution of member lengths and triangular facet sizes while maintaining a minimum total member length. The desired design methodology, however, is currently unavailable. This effort aims to fill the aforementioned technical gap in the literature. Presented in this paper is a new optimal geometry mesh design method that automatically determines the nodal coordinates and member connectivity for a given deployable mesh reflector with high surface accuracy and almost uniform member lengths. This design method also guarantees the pseudogeodesic property of the generated surface mesh geometry, which in many cases gives a minimum total member length. The proposed method is applicable to spherical and parabolic surfaces with either a center-feed configuration or offset-feed configuration, and it can meet the operating frequency requirements of large deployable mesh reflectors.

I. Introduction

Large deployable mesh reflectors (DMRs), due to their important space applications, have experienced continued research and development interest in the past several decades [1–6]. A deployable mesh reflector uses a spherical or parabolic surface as a
working shape (a required radio-frequency surface), which is formed by a network or mesh of tensioned facets. The geometry and formation of the mesh is directly related to the surface accuracy and the total weight of the DMR. Because of this, optimal design of the mesh geometry of the working surface is essentially important to the performance of the DMRs [7–9].

A typical DMR working surface is formed by a mesh of triangular facets, with their edges being considered as structural members. In most design cases, a desired DMR mesh should have a geodesic or pseudogeodesic property, by which the structural members and nodes of the reflector are either on or very close to the geodesic lines of the designated surface curvature (see [8], for instance). A mesh with a geodesic or pseudogeodesic mesh geometry yields a minimal total length of members, which translates into a minimized total weight of the DMR structure.

Early work on mesh geometry design for mesh reflectors was performed by Nayfeh and Hefzy [7] in 1979, in which the triangular facets of a mesh were first generated on a pyramid and then projected onto the designated surface of a DMR. Miura and Miyazaki [10] applied this method in the development of a concept of tension truss reflectors. Bush et al. [11] used an arc division approach to achieve uniform facet geometry for the surface of deployable truss reflectors. Kenner et al. [12] designed the facet geometries of tetrahedral truss reflectors, which later on were adapted by Mikulas et al. [13]. In these investigations, the uniform member length or triangular facet size was not considered, and the mesh geometries generated were not geodesic. Additionally, the methods used in these investigations had difficulty compromising the aperture rim.

Tibert and Pellegrino [9] proposed a useful approach that was similar to Nayfeh and Hefzy’s design [7], which generated triangular facets on the xy plane before the projection in the z direction. In a follow-up work [14], Tibert and Pellegrino [9] used a force density approximation to revise the facet geometry on the xy plane. With uniform force densities being used, a minimal total member length of the mesh was obtained on the xy plane. Morterolle et al. [15] adopted a similar force density approach to generate the nodes on the xy plane and then projected the so-obtained nodes vertically (in the z direction) onto the reflector surface in consideration. With a trial and error process with respect to the force densities, a uniform tension distribution in the mesh was obtained. There are two issues about the aforementioned investigations. First, a geodesic three-dimensional mesh, which renders the minimal total member length, is not guaranteed. Second, the member lengths may vary significantly from the inner to outer layers, as shown in [15], which can reduce the surface accuracy of the DMR in design. Indeed, a mesh with almost uniform triangular facets can help increase its surface accuracy and effective area.

Thomson et al. [16] presented a pseudogeodesic design for a perimeter-truss DMR that was applied to AstroMesh deployable reflectors [5,8], although no detailed description of the design method was given therein. Deng et al. [17] generated the mesh for a DMR with real geodesic curves in a three-dimensional working surface. In theory, the total member length is minimal due to the property of geodesic curves. The relevant numerical simulation presented in [17], however, implied that the reduction of total member lengths was limited. On the other hand, real geodesic curves of a paraboloid can cause irregular regions, as mentioned in [17], which is not acceptable in a DMR design.

In summary of previous research, it is desirable to have a methodology for the systematic design of DMR surface geometries that gives high surface accuracy and warrants almost uniform distribution of member lengths and triangular facet sizes while maintaining a relatively small total member length. Such design methodology, however, is currently unavailable in the literature. This effort aims to fill the aforementioned technical gap.

Developed in this paper is a new method for the optimal design of perimeter-truss DMRs, which are a class of state-of-the-art space structures. As shall be seen, the proposed design method can systematically generate the surface mesh geometries of DMRs with an assured pseudogeodesic property and almost uniformity of the cable element lengths, as well as simultaneously fulfill the operating radio-frequency requirements for the reflectors. Different from some design methods that rely on an xy-plane mesh, the new method makes use of a spherical reference surface for mesh geometry generation, which has certain advantages.

The remainder of this paper is arranged as follows. The problem of the surface mesh geometry design for DMRs is described in Sec. II. A five-step procedure for the proposed surface geometry design method is presented in Sec. III. Addressed in Sec. IV are additional issues in surface mesh geometry design, such as the effective region of a reflector and compensation of the best-fit surface deviation. The proposed method is demonstrated in numerical simulations in Sec. V. And, finally, some conclusions about this research effort are drawn in Sec. VI.

II. Problem Statement

The deployable mesh reflector in consideration is illustrated in Fig. 1; after full deployment, it is supported by a stiff and stable flat truss. The front net (the working surface) in the figure, as well as the rear net, is constructed by a mesh of flat triangular facets. The edges of the facets are elastic cable elements interconnected at facet nodes. The nodes of the front and rear nets are also connected by tension ties, the lengths of which can be adjusted. In setting up the DMR, folded nets are deployed into highly stretched elastic meshes, with the length of the tension ties properly adjusted such that the facets of the front net eventually form a working surface that is approximate to the desired radio-frequency surface. During in-space missions, the tension ties are used to maintain a desired shape of the front net under the influence of thermal loads. A fully deployed DMR surface can be modeled as a tensioned truss, with its members (cable elements) only sustaining axial tensions. Obviously, the modeling and design of the previously described structure must deal with both geometric and material nonlinearities.

The design of the DMRs has two main tasks. The first task is to generate a mesh geometry (i.e., nodal coordinates and member connectivity) for the fully deployed mesh for a given desired working surface. The second task is to determine a profile of the corresponding undeformed reflector such that the surface of the reflector, when fully deployed, matches the mesh geometry as generated in the first task. The current paper is concerned with the first design task. The second task has been addressed in [18].

A desired working surface in design, which can be either a center-feed or offset-feed paraboloid, is shown in Fig. 2, where F and D are the required focal length and aperture diameter for a center-feed parabolic configuration; and Fp, Dp, and ep are the parent focal length, parent aperture diameter, and offset distance for a offset-feed parabolic configuration, respectively. The objective of the current investigation is to develop an optimal design method for systematic generation of the mesh geometry of the working surface. Here, the word "optimal" means that the proposed method 1) guarantees the surface accuracy requirement based on the operating frequency of reflector applications, and 2) is able to generate reflector meshes of the pseudogeodesic property.

![Fig. 1 Typical DMR with deployed working surface.](image-url)
III. New Method for Generation of Mesh Geometry

Before the mesh geometry of a DMR is established, two parameters are to be specified: 1) the allowable root-mean-square (RMS) error \( \delta_{\text{rms}} \) between the mesh facets of the reflector surface and the desired working surface, which gives a measure of the surface accuracy of the reflector; and 2) the allowable (maximum) length \( \bar{l}_f \) of facet edges (cable elements that form facets). It has been known that \( \delta_{\text{rms}} \) is restricted by the operating frequency or, equally, the wavelength \( \lambda \) of the reflector. Depending on applications, different budgets for surface accuracy related to the facet geometry have been suggested as follows [5,19–23]:

\[
\delta_{\text{rms}} = \frac{\lambda}{N}
\]

(1)

where \( N = 50, 75, 100, 150, \) or 200. For a spherical mesh surface with equilateral triangular facets, the allowable length \( \bar{l}_f \) of facet edges is related to \( \delta_{\text{rms}} \) by

\[
\bar{l}_f = 4\sqrt{15} \sqrt{\frac{\delta_{\text{rms}} F}{D^3}}
\]

(2)

where \( F \) and \( D \) are the focal length and diameter of the surface. This formula was proposed in [24] and confirmed in [5,20]. Although the facets considered in the current investigation are not all equilateral triangles, Eq. (2) can still provide an initial guidance in the design of the mesh geometry for DMRs.

One important key in the proposed method is the introduction of a spherical reference surface, on which the geodesic mesh geometry (nodal coordinates and member connectivity) is first generated and then mapped onto the desired working surface of the reflector in consideration. The purpose of using a spherical reference surface is to generate meshes with a pseudogeodesic property, rendering the minimum total lengths of cable elements and the almost uniform cable element lengths and triangular facet sizes.

With the specified parameters \( \delta_{\text{rms}} \) and \( \bar{l}_f \), the proposed mesh design method takes the following five steps:

A. Step 1: Introduction of a Reference Surface

With a given desired working surface of a DMR, the first step is to define its reference surface in the following two cases.

1. Case 1: Center-Feed Reflector

For a spherical desired working surface, which is naturally a center-feed configuration, its reference surface is the same as the desired working surface. On the other hand, for a center-feed parabolic reflector, its working surface is described by

\[
z - H_p = -\frac{1}{4F} (x^2 + y^2)
\]

(3)

where \( H_p \) is the height of the working surface, with \( H_p = D^2/(16F) \). The reference surface of the parabolic reflector is a spherical surface that has the same height and aperture diameter as the desired working surface; see Fig. 3, which shows a side view of the reflector working surface and the reference surface in the \( xz \) plane, with \( O, A, \) and \( R_s \) being the center, vertex, and radius of the reference sphere, respectively. From the figure, the equation of reference surface is given by

\[
x^2 + y^2 + (z + R_s - H)^2 = R^2
\]

(4)

where the height and radius are given by

\[
H = H_p, \quad R_s = (R_s - H)^2 + \frac{D^2}{4}
\]

(5)

As long as the reflector focal ratio \( F/D \) is larger than 0.25, which is often the case in applications, the reference sphere described by Eq. (5) always exists.

2. Case 2: Offset-Feed Parabolic Reflector

When an offset-feed parabolic reflector is designed, its desired working surface or (simply) its working surface is obtained from a parent paraboloid of diameter \( D_p \) and focal length \( F_p \), as shown in Fig. 4, where \( x, y, z \) is the global coordinate system in which the parent surface is defined and \( xz \) is the local coordinate system in which the working surface is described, with both the \( y \) axis and \( z \) axis determined by the right-hand rule. Point \( O \) is the origin of the local coordinate system. The \( x \) axis is in the direction from point \( O \) to point \( C \), and the \( z \) axis is vertical to the plane of the elliptical edge formed by cutting the parent paraboloid with a cylinder. Through intersection of the parent paraboloid with a circular cylinder of radius \( R_s \) that is parallel to the \( z \) axis with an offset distance \( e_{df} \) from the \( z \) axis.
axis, an offset reflector with its working surface having the same focal length as the parent reflector \((F = F_p)\) is obtained.

The focal ratio of the so-obtained offset reflector is defined as \(F/(2R_s)\). Let points \(B\) and \(C\) in Fig. 4 be the intersections of the parent paraboloid and the cylinder in the \(x_gz_g\) plane. Let \(O\) be the middle point of chord \(BC\). The boundary of the working surface is elliptical and coplanar within the local \(xy\) plane, and it has a minimum distance \(e_x\) from the \(x_g\) axis at point \(B\). With the preceding description, the following parameters of the offset-feed reflector are introduced as

\[
R_x = \frac{1}{4}D_p - \frac{1}{2}e_{\text{off}}, \quad H_s = \frac{D_p}{16F_p},
\]

\[
e_{\text{off}} = \gamma_{\text{off}}D_p, \quad \gamma_{\text{off}} = 2 \frac{e_{\text{off}}}{D_p}, \quad \varphi_{\text{off}} = \tan^{-1}\left(\frac{H_s - e_{\text{off}}}{2R_x}\right)
\]

\[
x_o = e_{\text{off}} + R_x, \quad y_o = 0, \quad z_o = -\frac{1}{2}(e_{\text{off}} + H_p)
\]

where \(H_s\) is the height of the parent surface, which is the distance from point \(C\) to the \(x_g\) axis; \(\gamma_{\text{off}}\) is the reflector offset ratio; \(x_o, y_o,\) and \(z_o\) are the global coordinates of point \(O;\) and \(\varphi_{\text{off}}\) is the angle between the \(x_g\) axis and the \(x\) axis.

It can be shown that the working surface of the offset reflector is described in local coordinates \(x, y, \) and \(z:\)

\[
-\sin(\varphi_{\text{off}})x + \cos(\varphi_{\text{off}})z + z_o = -\frac{1}{4F_p}\left\{\cos(\varphi_{\text{off}})x + \sin(\varphi_{\text{off}})z + x_o\right\} + y^2
\]

with its boundary (lying in the \(xy\) plane) given by

\[
\frac{x^2}{a_z^2} + \frac{y^2}{b_z^2} = 1
\]

where \(a_z = R_x/\cos(\varphi_{\text{off}})\) and \(b_z = R_x\). Furthermore, setting \(x = y = 0\) in Eq. (7) yields the following quadratic equation about the reflector height \(H_p\) at the reflector center under the local coordinate system:

\[
\frac{1}{4F_p}\sin^2(\varphi_{\text{off}})H_p^2 + \left(\cos(\varphi_{\text{off}}) + \frac{1}{2F_p}\sin(\varphi_{\text{off}})x_o\right)H_p
\]

\[+ z_o + \frac{x_o^2}{4F_p} = 0 \tag{9}\]

The solution of Eq. (9) gives

\[
H_p = -\frac{2x_o \sin \varphi_{\text{off}} + 4F_p \cos \varphi_{\text{off}} + \sqrt{(2x_o \sin \varphi_{\text{off}} + 4F_p \cos \varphi_{\text{off}})^2 - 4\sin^2 \varphi_{\text{off}}(x_o^2 + 4F_p z_o)}}{2\sin^2(\varphi_{\text{off}})} \tag{10}\]
b) On each subdivision line, properly place \( n_r \) nodes; see Fig. 6b, where nodes 1, 2, \ldots, \( n_r \) correspond to angles \( \phi_1, \phi_2, \ldots, \phi_{n_r} \) that are measured from line \( OA \), with the total spanned angle of the reference surface of \( \phi_{n_r} = \phi_t \). The \( n_r \)th node is on the boundary of the reference surface in the \( xy \) plane. For a smooth mesh, all the subdivision lines should have the same number of nodes with the same angles. The minimum number of the nodes is related to the RMS error \( \delta_{\text{rms}} \) that measures the surface accuracy and allowable element length \( l_f \). By Fig. 6b, the length of the cord between the \( i \)th and \( (i+1) \)th nodes, which is also the length of an element connecting these nodes, is

\[
\tilde{l} = 2R_s \sin \left( \frac{\Delta \phi}{2} \right), \quad \Delta \phi = \phi_{i+1} - \phi_i \quad (13)
\]

and the total spanned angle \( \phi_t \) can be found from

\[
\sin \phi_t = \frac{D}{2R_s} \quad (14)
\]

Because \( \tilde{l} \leq \tilde{l}_f \), the number of nodes on a subdivision line must satisfy the condition

\[
n_r \geq \frac{\phi_t}{\Delta \phi} \geq \frac{\sin^{-1}\left(\frac{1}{4}(D/F)\right)}{2\sin^{-1}\left(\sqrt{15}\sqrt{\left(\delta_{\text{rms}}/D\right)(D/F)}\right)} \quad (15)
\]

where Eqs. (2), (13), and (14) have been used. For convenience in design, the angles of the nodes are expressed as

\[
\phi_i = \frac{\phi_t}{W_t} \sum_{j=1}^{i} w_j, \quad \text{with} \quad W_t = \sum_{k=1}^{n_r} w_k, \quad i = 1, 2, \ldots, n_r \quad (16)
\]

with \( w_k \) being positive weighting coefficients. The determination of the value of \( w_k \) shall be given in the next section. Note that, in the case of equally spaced nodes, all the \( w_k \) have the same value.

c) According to the previous two substeps, a subdivision is bounded by two adjacent subdivision lines with nodes on them. For each pair of nodes on these two subdivision lines that have the same longitude angle \( \phi_j \), connect them by a curve that is a portion of a great circle of the reference surface. These curves shall be called ring lines, as shown in Fig. 6c. Such a node connection for all the subdivisions yields a cluster of \( n_s \) closed loops or rings. These nodes on the subdivision lines shall be called intersectional nodes because they locate at the intersection between the subdivision lines and the rings. For convenience of analysis, the rings will be labeled according to longitude angles in an ascending order; that is, the \( i \)th ring line is the one passing those nodes with angle \( \phi_i \). Thus, from a top view, the projections of the rings on the \( xy \) plane are cocentered at the vertex \( A \), with the first ring closest to \( A \) and the last one closest to the boundary of the reference surface.
d) In each subdivision, add internal nodes on its ring lines in the following way: on the \((k + 1)\)th ring line, place \(k\) nodes such that all the nodes on the ring line are equally spaced; see Fig. 6d. Note that the first ring does not have any internal node. Accounting intersectional and internal nodes together, the number of nodes on the \(k\)th ring is \(k_{n_{r}}\). We have thus defined a network of nodes on geodesic curves of the spherical reference surface, with the total number of nodes given by

\[
n_{\text{total}} = 1 + \sum_{i=1}^{n_r} k_{n_{r}} = 1 + \frac{n_r}{2} n_r n_s + 1
\]

(17)

e) Connect all the nodes by straight lines, which define members that form triangular facets of the reference surface; see Fig. 6e. Depending on their locations, the members can be divided into two categories: those with ends on two different (adjacent) rings, and those with ends on the same ring.

After the aforementioned five substeps, a mesh of facets for the reference surface is established. For demonstrative purpose, the members that form facets. In Fig. 7, the solid outer circle is the boundary rim of the reference surface and the index pair \((i, j)\) indicates the location of the \(i\)th node on the \(j\)th ring line, place \(B_{0}\).

The first node on a ring is always on the first subdivision line \((y = 0)\), and the rest of the nodes on the ring are labeled in the counterclockwise direction in the \(xy\) plane. In addition, the location of vertex \(A\) is represented by \((0, 1)\).

With the aforementioned setting, the nodal coordinates of the reference surface are obtained in the following two cases.

\textbf{1. Case 1: Intersectional Nodes}

Shown in Fig. 8 is point \(P\) that is the \(j\)th intersectional node on the \(i\)th ring, denoted by index pair \((i, j)\). The coordinates of \(P\) are expressed by

\[
\tilde{x}_{ij} = R_{s} \sin \theta_{ij} \cos \alpha_{ij}
\]

\[
\tilde{y}_{ij} = R_{s} \sin \theta_{ij} \sin \alpha_{ij}
\]

\[
\tilde{z}_{ij} = H - R_{s} (1 - \cos \theta_{ij})
\]

(18)

where \(\theta_{ij}\) is the angle between the \(x\) axis and vector \(\mathbf{B}P\), with \(B\) being the origin of the \(xyz\) coordinate system and \(P\) being the projection of \(P\) on the \(xy\) plane; and \(\alpha_{ij}\) is the angle between the \(z\) axis and vector \(\mathbf{OP}\), with \(O\) being the center of the reference surface (sphere). As a special case, the coordinates of vertex \(A\) are

\[
\tilde{x}_{0,1} = \tilde{y}_{0,1} = 0 \quad \text{and} \quad \tilde{z}_{0,1} = H. \quad \text{The calculation of \(\theta_{ij}\) and \(\alpha_{ij}\) is given in Appendix A.}
\]

\textbf{2. Case 2: Internal Nodes}

Let \(P_{ij}\) be the \(j\)th intersectional node on the \(i\)th ring line and let \(P_{ij+1}\) be the next adjacent intersectional node on the same ring line; see Fig. 2. If \(i > 1\), there exist \(i - 1\) internal nodes between the two adjacent intersectional nodes: \(P_{ij+1}, P_{ij+2}, \ldots, P_{ij+i-1}\). Now, consider a two-dimensional coordinate system \((\xi, \eta)\) in the plane defined by triangle \(\Delta BP_{ij+i}\), with the origin \(B\) being the center of the boundary of the reference surface on the \(xy\) plane and the \(\xi\) axis along line \(BP_{ij+i}\); see Fig. 2.

The coordinates of the internal nodes are obtained as follows. Figure 10 shows the layout of nodes \(P_{ij+1}, P_{ij+2}, \ldots, P_{ij+i-1}\) in the coordinate system \((\xi, \eta)\). Let the coordinates of node \(P_{ij+k}\) be \((\xi_{ij+k}, \eta_{ij+k})\). It can be shown that

\[
\xi_{ij+k} = \cos(\gamma_{k})R_{s}, \quad \eta_{ij+k} = \sin(\gamma_{k})R_{s}
\]

(19)

where \(\gamma_{k} = k \pi / i\) with \(k = 1, 2, \ldots, i - 1\), and \(\gamma\) is the angle between the \(\xi\) axis and line \(P_{ij}P_{ij+i+1}\), which can be easily computed because the coordinates of the intersectional nodes \((P_{ij}\) and \(P_{ij+i})\) are already given by Eq. (18). More details are shown in Appendix B.

The member connectivity is the connection of nodes that forms the triangular facets, which is similar to that in a finite element analysis. To establish member connectivity, nodes and members have to be labeled. A global nodal number system is set up according to Fig. 7, with the following two rules:

\begin{align*}
\text{1. Case 1: Intersectional Nodes} & : \\
\text{2. Case 2: Internal Nodes} & : \\
\end{align*}
Rule 1: Count nodes on the rings from inside to outside, with the first node being the vertex of the reference surface.

Rule 2: On each ring, start with the node on the x axis and label the remaining nodes counterclockwise. This will yield the following results:

Vertex A:

\[ N_1 = (0, 1) \]

Nodes on the first ring:

\[ N_2 = (1, 1), \quad N_3 = (1, 2), \quad \ldots \quad N_{n+1} = (1, n) \]

Nodes on the second ring:

\[ N_{n+2} = (2, 1), \quad N_{n+3} = (2, 2), \quad \ldots \quad N_{3n+1} = (2, 2n) \]

Nodes on the \( n \)th ring:

\[ N_{i+1} = (n_r, 1), \quad N_{i+2} = (n_r, 2), \quad \ldots \quad N_{i+n_n} = (n_r, n_n) \]

where \((i, j)\) is an index pair, \( l = n_{\text{total}} - n_r \). Also, \( n_{\text{total}} \) is the total number of nodes. Note that the subscript \( k \) of \( N_k \) is the node number.

Similarly, a global system of member number is set up as follows. Divide the mesh in Fig. 7 into \( n_r \) layers of facets: the first layer consists of the members within the first ring and those on the first ring; the second layer consists of the members between the first and the second rings, and those on the second ring; and so on. The last layer of facets, which is called the boundary layer, consists of the members between the \((n_r - 1)\)th ring and the boundary (the \( n_r \)th ring). There are no members on the boundary. In labeling members, count the members from the inside layer (the first layer) to the boundary layer. For each layer, start with the member on the x axis (the first subdivision line), and then move on to the members within the layer, followed by those on the outer ring of the layer. It is easy to see that the \( i \)th layer has \((3i - 1)n_r\) members and \((2i-1)n_r\) facets.

With the aforementioned numbering system for nodes and numbers, the connectivity of a member is expressed as

\[ C_k = \{ N_{i0}, N_{j0} \} \]

where \( k \) is the member number, and \( N_{i0} \) and \( N_{j0} \) describe the nodes that are connected to the member. For instance, the connectivity of the members in Fig. 6 are obtained as follows: The first layer:

\[
\begin{align*}
C_1 &= \{(1,1) (1.1)\} = \{N_1 N_2\}, \\
C_2 &= \{(1,1) (1.2)\} = \{N_1 N_3\}, \ldots \quad C_{n+1} = \{(1,1) (1.n)\} = \{N_1 N_{n+1}\}, \\
C_{n+2} &= \{(1,1) (1.3)\} = \{N_2 N_3\}, \ldots \quad C_{2n} = \{(1,1) (1.1)\} = \{N_{n+1} N_1\}
\end{align*}
\]

The second layer:

\[
\begin{align*}
C_{2n+1} &= \{(1,1) (2.1)\} = \{N_2 N_{n+2}\}, \\
C_{2n+2} &= \{(1,1) (2.2)\} = \{N_2 N_{n+3}\}, \ldots \ldots
\end{align*}
\]

where the index pair \((i, j)\) denotes node location.

C. Step 3: Geodesic Mesh Geometry Adjustment

Although the geodesic mesh on the reference surface is successfully generated in step 2, the following adjustment is still necessary for further improvement of the surface accuracy of the DMR.

As can be seen from a top view of the reflector structure in Fig. 11, the curvature of an inner-layer geodesic ring is much larger than an outer-layer geodesic ring. This curvature difference leads to larger facet areas near the boundary layer, which eventually yields a larger surface RMS error. To resolve this problem, the curvatures of the outer-layer rings can be adjusted by relocating the internal nodes, which makes the ring lines more flat. The rings with such a relocation of internal nodes shall be called converging rings.

The relocation of internal nodes is described as follows. The \( xy \)-plane radius \( r_n \) of the \( n \)th ring is first calculated in Appendix C, where \( n_r \) is the number of the converging ring that is closest to vertex A. The \( r_n \) is given by

\[ r_n = r_c n_r \]

where \( r_c \) is a preselected design parameter.

Starting from the \( n \)th ring, the outer rings are all consequently projected onto the \( xy \) plane, with their radius in the \( xy \) plane being adjusted by the following power function. Note that the last projected ring is the \((n_r - 1)\)th ring:

\[ r_i = \left( \frac{1}{R_i \sin(\theta_i)} - \frac{1}{r_n} \right) \left( \rho n_r - n_i \right)^\alpha + \frac{1}{r_n} \]

with \( i = n_r, n_r + 1, \ldots, n_r - 1 \) and \( 1 \leq n_r \leq n_r - 1 \), where \( r_i \) is the radius of the \( i \)th ring in the \( xy \) plane; \( \rho \) is the positive parameter that is preselected appropriately; and \( \theta_i \) is the nodal angle given in Eq. (16). With the radius adjustment in Eq. (24), the geodesic ring lines become the converging rings. After the convergent treatment is applied, the internal nodes are equally spaced on the projected ring lines. The nodes are then projected along the \( z \) axis, back to the reference sphere. Note that the nodes on those converging rings are no longer on the geodesic curves, which renders a slight increase of the total length of the mesh elements.

D. Step 4: Projection of Nodes to the Desired Working Surface

1. Center-Feed Spherical or Parabolic Desired Working Surface

For a spherical desired working surface, the reference surface is itself and the nodes projection are not needed.

For a center-feed parabolic desired working surface, once the nodal coordinates are properly determined on the reference surface, the nodes are projected vertically (along the \( z \) axis) from the reference sphere onto the desired working surface, as shown in Fig. 12. Because the projection is along the \( z \) axis, the nodal coordinates in the \( xy \) plane remain unchanged. The coordinates of the nodes on the desired working surface are thus given by

\[
\begin{align*}
x_{i,j} &= \tilde{x}_{i,j}, \quad y_{i,j} = \tilde{y}_{i,j}, \quad z_{i,j} = -\frac{1}{4F} (\tilde{x}_{i,j}^2 + \tilde{y}_{i,j}^2) + H_p
\end{align*}
\]

where \( \tilde{x}_{i,j} \) and \( \tilde{y}_{i,j} \) are coordinates of the \( j \)th node in the \( i \)th layer on the reference surface, which were obtained in steps 2 and 3.
desired working surface along the $z$ axis. The ellipsoidal intermediate surface has the same origin as the spherical reference surface, and it has the same boundary aperture on the $xy$ plane as the working surface. As such, the intermediate surface can be described by

$$\frac{x^2}{a_x^2} + \frac{y^2}{b_x^2} + \left(\frac{z + R_x - H_y}{a_x^2}\right)^2 = 1$$  \hspace{1cm} (26)$$

with $a_x = R_x$ and $b_x = 2R_xb_x'/D_x$, where $b_x'$ is defined in Eq. (8).

The node mapping from the reference surface to the intermediate surface is illustrated in Fig. 13, where a node $\tilde{N}_{i,j}$ on the spherical reference surface is mapped onto $\tilde{N}_{i,j}$ on the ellipsoidal intermediate surface horizontally (on the $xy$ plane). Let the coordinates of $\tilde{N}_{i,j}$ and $\tilde{N}_{i,j}$ be $(\tilde{x}_{i,j}, \tilde{y}_{i,j}, \tilde{z}_{i,j})$ and $(\tilde{x}_{i,j}, \tilde{y}_{i,j}, \tilde{z}_{i,j})$, respectively. Since the mapping is from a circle to an ellipse in the horizontal direction, the equation of the ellipse on the intermediate ellipsoid corresponding to a given circle on the spherical reference is

$$\frac{x_{i;j}^2}{a^2} + \frac{y_{i;j}^2}{b^2} = 1, \quad \text{with} \quad a = R, \quad b = \frac{b_x'}{a_x'}$$  \hspace{1cm} (27)$$

where $R$ is the radius of the ring line (circle in the $xy$ plane), and $b_x'$ and $a_x'$ are defined in Eq. (8). Let the nodes be spaced on the ellipse such that the length ratio of a curve between two adjacent nodes remains unchanged before and after the mapping. The coordinates of the nodes on the intermediate elliptical surface are given by

$$\tilde{x}_{i;j} = \sqrt{a_x^2 - (\tilde{z}_{i;j} + R_x - H)^2 \cos \tilde{\alpha}_{i;j}}$$
$$\tilde{y}_{i;j} = \frac{b_x}{a_x} \sqrt{a_x^2 - (\tilde{z}_{i;j} + R_x - H)^2 \sin \tilde{\alpha}_{i;j}}$$
$$\tilde{z}_{i;j} = \tilde{z}_{i;j}$$  \hspace{1cm} (28)$$

with $(\tilde{x}_{0,1}, \tilde{y}_{0,1}, \tilde{z}_{0,1}) = (0, 0, H)$ where

$$\tilde{\alpha}_{i;j} = \tan^{-1}\left(\frac{\tilde{y}_{i;j}}{\tilde{x}_{i;j}}\right)$$  \hspace{1cm} (29)$$

Once the nodes on the elliptical intermediate surface are determined, the second node projection yields the coordinates $(\tilde{x}_{i,j}, \tilde{y}_{i,j}, \tilde{z}_{i,j})$ of the nodes on the desired working surface, which is calculated in Eq. (30):

$$x_{i;j} = \tilde{x}_{i;j}, \quad y_{i;j} = \tilde{y}_{i;j}, \quad z_{i;j} = -\frac{b_1 + \sqrt{b_1^2 - 4b_2b_0}}{2b_2}$$  \hspace{1cm} (30)$$

where

$$b_2 = \frac{1}{4F_p} \sin^2(\varphi_{off})$$
$$b_1 = \frac{1}{2F_p} (\cos(\varphi_{off}) \tilde{x}_{i;j} + x_o) \sin(\varphi_{off}) + \cos(\varphi_{off})$$
$$b_0 = \frac{1}{4F_p} ((\cos(\varphi_{off}) \tilde{x}_{i;j} + x_o)^2 + \tilde{y}_{i;j}^2) - \sin(\varphi_{off}) \tilde{x}_{i;j} + z_o$$  \hspace{1cm} (31)$$

with $\varphi_{off}$ shown in Fig. 4.

E. Step 5: Further Improvement of Surface Accuracy

In the previous steps, it is required that all the nodes be placed on the working surface. This is in fact consistent with the conventional methods (see [14, 15, 25]) of DMR mesh generation. However, this requirement may not be optimal because it pushes all the cable elements of a DMR away from the desired working surface toward the focal point, as shown in Fig. 14. In other words, the best-fit surface of the generated mesh, which was defined in [24], is different from the desired working surface. For instance, see Appendix D for the evaluation of the best-fit surface of an offset-feed parabolic mesh geometry. In addition, refer to Appendix E for correction of some errors in geometric calculations in the literature.
The surface accuracy of a DMR can be further improved by placing the nodes off the desired working surface. Indeed, elimination of the restriction of nodes on the desired working surface gives more freedom to further improve surface accuracy for DMRs. Two new methods were proposed in [26] for improvement of the surface accuracy for DMRs in form finding. In the development, the RMS error of the DMR was minimized through iterations that allowed the DMR nodes to be away from the desired working surface. The first method is to project the nodes onto a parabolic or spherical projecting surface, which is determined via an iterative procedure such that the best-fit surface of the designed DMR is close to the desired working surface. The second method is to properly adjust the coordinates of the DMR nodes, in which the nodes are first projected onto the desired working surface and adjusted in the \( z \) direction by a numerical optimization algorithm that minimizes the RMS error of the DMR. The numerical simulation in [26] showed that the surface accuracy of a DMR could be significantly improved by these two methods.

### IV. Effective Region and Boundary Nodes Reduction Technique

Based on the concept of effective region, which is introduced in this section, a boundary nodes reduction technique is applied for further DMR design improvement, such as reducing the total member lengths, as well as simplifying the supporting structure and the deployment procedure in space.

#### A. Effective Region

In application, only a major portion of a DMR’s working surface can be used for signal transmission. This portion of the working surface shall be called the effective region, and the mesh facets within the effective region are called effective facets. Therefore, the design objective is to meet the operating frequency requirements in the effective region. For a center-feed parabolic reflector as shown in Fig. 15, the effective diameter is specified as \( D_{\text{eff}} \). For an offset-feed parabolic reflector as seen in Fig. 16, the effective diameter is specified as \( D'_{\text{eff}} \).

To integrate this consideration in the design process, the effective portion, which is defined to be the first \( (n_r - 1) \) layers of the mesh geometry, shall be designed by the procedure introduced in this paper. The \( n_r \)-th layer (namely, the boundary layer) shall be designed separately by the boundary nodes reduction technique. To this end, the weighting coefficients in Eq. (16) are set as

\[
\begin{align*}
& w_1 = \frac{\varsigma + 1}{2} \\
& w_i = 1 + (1 - \varsigma) \left( \frac{\rho_i - 1}{\rho_i - \rho_{i-1}} \right)^\theta + \frac{\varsigma + 1}{2} \\
& w_{B_r} = \frac{\rho_i - \rho_{i-1}}{2} \\
& \text{for } i = 1, 2, \ldots, n_r - 1 \\
& \text{for } i = n_r \\
& \text{for } i = n_r - 1
\end{align*}
\]  

(32)

where \( \varsigma \) is a design parameter that controls the thickness of the first \( (n_r - 1) \)th layers, and \( w_B \) is the thickness of the boundary layer. The value of \( \varsigma \), which is usually around one, is adjusted via nonlinear programming so as to improve the surface accuracy. If \( \varsigma = 1 \), the thickness of the first \( (n_r - 1) \) layers are the same. The \( w_B \) is the thickness of the boundary layer.

The layer angles are given in

\[
\sin(\phi_{n_r-1}) = \frac{D_{\text{eff}}}{2R_s}, \quad \sin(\phi_{n_r}) = \frac{D}{2R_s} \tag{33}
\]

From Eqs. (16), (32), and (33), the thickness of the boundary layer is then calculated as

\[
w_B = \left( \frac{\phi_{n_r} - \phi_{n_r-1}}{\phi_{n_r-1}} \right) \sum_{i=1}^{n_r-1} w_i \tag{34}
\]

#### B. Node Number Reduction on Boundary Layer

To reduce the complexity of the supporting structure and the difficulty in reflector deployment, certain nodes on the boundary (aperture rim) of a DMR are removed (for instance, see [27]). To this end, assume that, after the nodes number reduction, there are \( n_B \) nodes on the boundary (the \( n_r \)-th ring or the aperture ring) of the reflector. Each node on the boundary is connected to \( c_B \) nodes on the adjacent inner ring [the \((n_r - 1)\)-th ring] by cable elements. The \( c_B \) in...
method in \[15\]. In general, a good mesh geometry design method for DMRs should achieve three goals: 1) small variation of member lengths and triangular facet sizes; 2) small total member length; and 3) high surface accuracy, which is measured by a small RMS error of the mesh reflector. Thus, the comparison parts shall be focused on the member length ratio, total member length, and the RMS error of the DMRs in consideration.

Given a number of nodes or cable elements for a mesh, it is important to keep the member length ratio as small (toward unity) as possible. This is because a long cable element leads to poor local surface accuracy. Almost uniformity of member lengths, on the other hand, renders smaller variations of the local surface accuracy, which eventually yields a larger effective region for the DMR in design.

In design of the surface mesh geometry for a DMR by the proposed method, seven parameters are to be selected: \(\lambda\) and \(N\) in Eq. (1), the number \(n_r\) of rings, the relative thickness \(w_B\) of the boundary layer, the number \(n_s\) of subdivisions, the index \(n_c\) of the first converging ring, and the power coefficient \(\rho\) in Eq. (24).

In this work, DMRs with shallow and deep reflecting surfaces are considered: both of which are of great importance in deployable antenna designs. In the literature, besides relatively large focal ratios, paraboloid reflecting surfaces of small \(F/D\) values have been investigated. For instance, center parabolic DMRs with an \(F/D\) ratio of 0.4 were examined in \[23\]. For offset-feed DMRs, studies have been reported on Aerospatiale with an \(F/D\) of 0.167 in \[28\], AstroMesh with an \(F/D\) of 0.286 in \[29\], and BAE/Surrey with an \(F/D\) of 0.273 in \[28\]. These studies provided a background for the numerical simulations in the current effort.

A. Quick View of Design Results

The proposed mesh design method is illustrated on two sets of deployable mesh reflectors: a set of three center-feed spherical DMRs, and a set of three offset-feed parabolic DMRs. First, consider three center-feed spherical reflectors with the same diameter and focal length: \(D = 4\,\text{m}\) and \(F = 2\,\text{m}\). Figure 18 shows the top view (in the \(xy\) plane) of three mesh reflectors that are generated by the proposed method, where \(n_r\) is the number of subdivisions. Except for \(n_r\), the other six design parameters as defined in Sec. IV are assumed to be the same for all the DMRs, and they have the following values:

\[
\lambda = 2 \times 10^{-5}, \quad N = 50, \quad n_c = 7, \quad w_B = 1.2, \quad n_s = 4, \quad \rho = 2.5
\]

Next, consider three offset-feed parabolic DMRs with the diameter and focal length of the same parent parabola: \(D_p = 30\,\text{m}\), and \(F_p = 8\,\text{m}\). The offset distance is the same for all the reflectors: \(e_{off} = 2\,\text{m}\). Shown in Fig. 19 is the top view (in the \(xy\) plane) of the offset-feed parabolic DMRs that are generated by the proposed method, where \(c_B\) is the number of connections at a node on the boundary, as discussed in Sec. IV, and the solid think curve in each plot is the boundary of the effective region. Note that \(c_B\) is three for the first two DMRs and five for the third DMR. All the other design parameter are the same as in those of the center-feed spherical DMRs, which are given in Eq. (40).

B. Design of Center-Feed DMRs

The proposed method and the uniform tension shape design method in \[15\] are compared in the design of three center-feed parabolic DMRs with \(D = 12\,\text{m}\) and two different focal ratios of \(F/D = 0.26\) and 0.40. The number of nodes of the designed DMRs are 37 and 217, respectively. It should be pointed out that the proposed method, when combined with a technique of placing the nodes off the desired working surface as introduced in \[26\], can significantly improve the surface accuracy. For pure comparison of the proposed method and the uniform tension method, however, the technique given in \[26\] shall not be used. The design parameters for the simulations are assigned as follows:

V. Numerical Simulations

This section has four subsections: Sec. VA shows generation mesh geometries by the proposed method, and Secs. VB–VD compare the proposed mesh design method with the uniform tension shape design
37-node DMR:
\[ \lambda = 5.89 \quad N = 50 \quad w_B = 1 \quad n_x = 3, \quad n_y = 6, \quad n_z = 2, \quad \rho = 2.5 \]

217-node DMR:
\[ \lambda = 2.30 \quad N = 50 \quad w_B = 1 \quad n_x = 8, \quad n_y = 6, \quad n_z = 5, \quad \rho = 2.5 \] (41)

Here, the design parameters are selected rather arbitrarily. Indeed, proper selection of the design parameters via optimization can further improve the performance of the designed DMRs, which will be pursued in the future. Nonetheless, even with arbitrarily selected values of the design parameters, the proposed method can deliver good enough results, as will shown subsequently.

The numerical results for the aforementioned reflectors with focal ratios of \( F/D = 0.26 \) and 0.4 are given in Tables 1 and 2, respectively, where \( L_{\text{total}} \), \( r_{\text{length}} \), \( r_{\text{tension,in}} \), \( r_{\text{tension,b}} \), \( r_{\text{tension,tie}} \), and \( \delta_{\text{rms}} \) are the total member length, the member length ratio, the member tension ratio in the effective region as defined in the previous section, the boundary member tension ratio, the tension tie force ratio, and the RMS error, respectively. The \( r_{\text{length}} \) is a measure of the uniformity of member lengths and is defined as the ratio of the maximum member length to the minimum member length of the designed DMR:

\[ r_{\text{length}} = \frac{L_{\text{max}}}{L_{\text{min}}} \] (42)

The members of a DMR are of uniform length when \( r_{\text{length}} \) is one. Similarly, \( r_{\text{tension,in}} \), \( r_{\text{tension,b}} \), and \( r_{\text{tension,tie}} \) are measures of the uniformity of internal (tension) forces as follows:

\[ r_{\text{tension,in}} = \frac{T_{\text{in,max}}}{T_{\text{in,min}}} \]
\[ r_{\text{tension,b}} = \frac{T_{\text{b,max}}}{T_{\text{b,min}}} \]
\[ r_{\text{tension,tie}} = \frac{T_{\text{tie,max}}}{T_{\text{tie,min}}} \] (43)

where \( T_{\text{in,max}} \) and \( T_{\text{in,min}} \) are the maximum and minimum forces in the effective area; \( T_{\text{b,max}} \) and \( T_{\text{b,min}} \) are the maximum and minimum forces of the boundary members; and \( T_{\text{tie,max}} \) and \( T_{\text{tie,min}} \) are the maximum and minimum forces of the tension ties, respectively.

In the proposed method, the tension distribution among members and tension tie forces are obtained by a numerical optimization algorithm given in [18]. The objective function and constraints in the optimization are as follows:

\[ \text{min}(\| T - T_{\text{des}} \|^2) \]

subject to

\[ MT = T_{\text{tie}} \quad \text{and} \quad T > 0 \] (44)

where \( T_{\text{des}} \) is the desired tension of the cable elements; and \( MT = T_{\text{tie}} \) is the force equilibrium equation, with \( M \) being the tension direction matrix and \( T_{\text{tie}} \) a vector of tension tie forces. Because all the cable members can only sustain tensions, the constraint of \( T > 0 \) is imposed. The RMS \( \delta_{\text{rms}} \) that is caused by geometry faceting is computed by a method given in [26].

It is seen from Tables 1 and 2 that the proposed method produces better results in minimization of the member length ratio \( r_{\text{length}} \) and tension tie force ratio \( r_{\text{tension,tie}} \), as well as reduction of the RMS error \( \delta_{\text{rms}} \), as compared to the uniform tension method. For instance, for a...

Fig. 18 Quick view design of center-feed parabolic shape without boundary node reduction applied.

Fig. 19 Quick view design of offset-feed parabolic shape with boundary node reduction applied.
A nonshallow DMR with a small focal ratio (say $F/D = 0.26$), the proposed method can keep $r_{\text{length}}$ under 1.54, as shown in Table 1, indicating a relatively small variation of member lengths. The uniform tension method, on the other hand, gives a member length ratio ($r_{\text{length}} = 6.05$) that is almost four times as large as that by the proposed method. Also, the proposed method gives a better tension force ratio: $r_{\text{tension}, \text{in}}$ is 1.2 by the proposed method, and $r_{\text{tension}, \text{in}}$ is 2.1 by the uniform tension method. Nevertheless, the uniform tension method always gives better tension ratios for cable members in both the effective region and boundary layer. For instance, for the 217-node DMR with $F/D = 0.26$, $r_{\text{tension}, \text{in}}$ is 1.59 by the proposed method; whereas $r_{\text{tension}, \text{in}}$ is almost one by the uniform tension method. This is understandable because the optimization problem $\min(\|F - T_{\text{geo}}\|^2)$ does not necessarily give a uniform distribution of the cable tension forces. Additionally, although the total member length by the proposed method is almost less than that by the uniform tension method, they do not differ much.

### C. Design of Offset-Feed DMRs

Consider offset-feed parabolic DMRs with $D_p = 30$ m, $e_{\text{off}} = 1$ m, and focal ratios of $F_p/D_p$ = 0.2 and 0.4, where $F_p$ and $D_p$ are the focal length and diameter of the parent parabola, as shown in Fig. 4. Like in Sec. V.B, the meshes of the DMRs are generated with 37 and 217 nodes. Choose two values of $\lambda$ as 3.22 and 1.70. All other design parameters are the same, as given in Eq. (41).

Tables 3 and 4 list the design results obtained by the proposed method and the uniform tension shape design method in [15]. In general, offset-feed parabolic DMRs are much shallower than center-feed DMRs; as such, the uniform tension method delivers relatively better results in these cases. It is seen from the simulation results that the two design methods in comparison deliver almost the same total member length. Theoretically, the member length of the geodesic curves should be minimal. However, due to the vertical projection as described in Sec. III, the members are no longer on exact geodesic curves. Hence, the total member length is not strictly minimal. In the case of $F_p/D_p = 0.4$ and 217 nodes, the total member length by the proposed method is slightly larger than that by the uniform tension method. With a focal ratio of 0.2, the RMS error by the proposed method is smaller than the uniform tension method. With a focal ratio of 0.4, the RMS errors by the two methods are almost the same. Both methods have relatively large variations in tension force ratios, which should be further modified in future research. Additionally, the proposed method can maintain a relatively smaller member length variation.

The uniform tension method achieves almost equal tension distributions in both effective areas and boundary layers, which is clearly seen in all the numerical simulations. The proposed method, on the other hand, gives decent tension force ratios ranging from 1.29 to 1.69. Thus, if uniform tension distributions are a crucial factor in the design of DMRs, the uniform tension method is preferred. On the other hand, if the requirement of uniform tension can be relaxed, the proposed method has other advantages, including reduced RMS errors, as shown in Tables 1–4, and increased effective regions, as shall be shown in Sec. V.D.

### D. Effective Regions

The effective region of a DMR has been described in Sec. VI. The numerical simulations in this study indicate that a DMR designed by the proposed method has a larger effective region than the uniform tension method. To show this, consider a 217-node center-feed parabolic reflector with an aperture diameter $D$ of 12 m and a focal ratio $F/D$ of 0.26. The values of the other design parameters are the same as given in Eq. (41). Figure 20a shows an effective region of 32.3 m$^2$ obtained by the uniform tension method; and Fig. 20b shows an effective region of 84.9 m$^2$ obtained by the proposed method, where the boundaries of the effective regions are marked by the solid thick curves. Thus, with the proposed method, the effective region gains 162.8%. The reason for this discrepancy in effective regions is explained as follows. Because the desired working surface is not a shallow parabola, a much wider boundary layer of the DMR is needed if uniform tension is required, which in turn reduces the area of the effective region.

The comparison of effective regions is extended to a 217-node offset-feed parabolic DMR, as shown in Fig. 21. The parent parabola of the desired working surface has a diameter $D_p$ of 30 m and a focal ratio of 0.1. The offset distance $e_{\text{off}}$ is 5 m, and $\lambda$ is 0.42 m. All other design parameters are the same as given in Eq. (41). The uniform tension method produces an effective region of 127.8 m$^2$, as shown in Fig. 21a; and the proposed method gives an effective region of 149.8 m$^2$, as shown in Fig. 20b. The latter is 20% larger than the former. The increase in the effective region in Fig. 21 is not as much as in Fig. 20 because the offset-feed DMRs are shallow.

It should be noted that the results presented in Figs. 20 and 21 are about deep DMRs with focal ratios of 0.26 and 0.1, respectively. For a shallow DMR, such as one with $F/D = 0.6$, the uniform tension method can produce an almost equal tension distribution and a good

### Table 1 Design results for the center-feed parabolic DMRs with $F/D = 0.26$

<table>
<thead>
<tr>
<th>37-node DMR</th>
<th>217-node DMR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform tension method</td>
<td>Proposed method</td>
</tr>
<tr>
<td>$L_{\text{total}}$</td>
<td>220.3052</td>
</tr>
<tr>
<td>$r_{\text{length}}$</td>
<td>2.6901</td>
</tr>
<tr>
<td>$r_{\text{tension}, \text{in}}$</td>
<td>1.0005</td>
</tr>
<tr>
<td>$r_{\text{tension}, \text{out}}$</td>
<td>1.0004</td>
</tr>
<tr>
<td>$\delta_{\text{rms}}$</td>
<td>0.1178</td>
</tr>
</tbody>
</table>

### Table 2 Design results for the parabolic shape with $F/D = 0.40$

<table>
<thead>
<tr>
<th>37-node DMR</th>
<th>217-node DMR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform tension method</td>
<td>Proposed method</td>
</tr>
<tr>
<td>$L_{\text{total}}$</td>
<td>205.2527</td>
</tr>
<tr>
<td>$r_{\text{length}}$</td>
<td>1.5111</td>
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<tr>
<td>$r_{\text{tension}, \text{in}}$</td>
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<tr>
<td>$r_{\text{tension}, \text{out}}$</td>
<td>1.0007</td>
</tr>
<tr>
<td>$\delta_{\text{rms}}$</td>
<td>0.0630</td>
</tr>
</tbody>
</table>

### Table 3 Design results for the offset-feed parabolic DMRs with $F_p/D_p = 0.2$

<table>
<thead>
<tr>
<th>37-node DMR</th>
<th>217-node DMR</th>
</tr>
</thead>
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<td>Uniform tension method</td>
<td>Proposed method</td>
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<tr>
<td>$L_{\text{total}}$</td>
<td>257.5019</td>
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<tr>
<td>$r_{\text{length}}$</td>
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</tr>
<tr>
<td>$r_{\text{tension}, \text{in}}$</td>
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</tr>
<tr>
<td>$r_{\text{tension}, \text{out}}$</td>
<td>1.9987</td>
</tr>
<tr>
<td>$\delta_{\text{rms}}$</td>
<td>0.0643</td>
</tr>
</tbody>
</table>

### Table 4 Design results for the offset-feed parabolic DMRs with $F_p/D_p = 0.4$

<table>
<thead>
<tr>
<th>37-node DMR</th>
<th>217-node DMR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform tension method</td>
<td>Proposed method</td>
</tr>
<tr>
<td>$L_{\text{total}}$</td>
<td>237.0752</td>
</tr>
<tr>
<td>$r_{\text{length}}$</td>
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<tr>
<td>$r_{\text{tension}, \text{in}}$</td>
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<td>$r_{\text{tension}, \text{out}}$</td>
<td>1.2756</td>
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<tr>
<td>$\delta_{\text{rms}}$</td>
<td>0.0339</td>
</tr>
</tbody>
</table>
element length ratio, as shown in [15]. Thus, it can be claimed that the proposed method can be more useful in the design of deep DMRs with small $F/D$ ratios.

VI. Conclusions

A new method for the optimal design of surface mesh geometries of deployable mesh reflectors has been developed. Through the selection of seven design parameters, a five-step design procedure generates a pseudogeodesic mesh geometry for the DMR in consideration. As found from numerical simulations and comparison studies, the proposed design method can systematically produce mesh geometries with almost uniform mesh facets (small member length ratio), a minimum total element length, competitive surface root-mean-square errors, and large effective regions, for spherical and parabolic working surfaces and with center-feed and offset-feed configurations. Additionally, certain errors in mesh geometry calculations in the literature have been corrected. In future research, it is worth addressing practical issues in DMR designs, including application of a technique for boundary node reduction and consideration of a radio-electrical function. With further developments, the proposed mesh design method is believed to be a useful tool for systematic development of large deployable mesh reflectors.

Appendix A: Derivations of $\theta_{i,j}$ and $\alpha_{i,j}$ to Obtain the Intersectional Nodal Coordinates

This part of the appendix presents the detailed derivation of determining the coordinates of the intersectional nodes that are introduced in step 2 of Sec. III. After step 1 and first two substeps of step 2 of the design, $D$ and $F$ of the reference sphere $\left[w(i) and \phi_i\right]$ are available and will be used to calculate the nodal coordinates in this section.

The indices of the intersectional nodes satisfy the following equation:

$$\tilde{j} = 1 + (q - 1)i$$

in which $q = 1, 2, \ldots, n_s$ and $i = 1, 2, \ldots, n_r - 1$. Therefore, all the intersectional nodes are denoted as $P_{i,j}$. From the geometry in Figs. 6 and 8,

$$\theta_{i,j} = \phi_i$$

$$\alpha_{i,j} = (q - 1) \frac{2\pi}{n_s}$$
Appendix B: Calculation of Internal Nodal Coordinate

A two-dimensional coordinate system \((\xi, \eta)\) is defined in step 2 of Sec. III. The plane equation of this coordinate system in global coordinates is given by

\[ Ax + By + Cz + D = 0 \quad (B1) \]

This equation can be obtained from the global coordinates of the points \( O, P_{i,j}, \) and \( P_{i,j+1} \). In the global coordinate, the unit vector of the \( \xi \) axis is given as \( e_1 \); the unit vector that is normal to the plane [Eq. (B1)] is given as \( e_2 \); and the unit vector of the \( \eta \) axis is given as \( e_3 \). With the coordinate transformation matrix \( T \) in Appendix B, the plane equation of the converging ring in the converging treatments introduced in step 3.

Define the unit vectors of the \( x \) axis, \( y \) axis, and \( z \) axis as \( e_1^x, e_1^y, \) and \( e_3^z \) in Eq. (B5):

\[ e_1^x = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad e_1^y = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad e_3^z = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (B5) \]

The coordinate transformation matrix \( T \) from the global coordinate to local coordinate is given as follows:

\[ T = \begin{bmatrix} e_1^x & e_1^y & e_1^z \\ e_2^x & e_2^y & e_2^z \\ e_3^x & e_3^y & e_3^z \end{bmatrix} \quad (B6) \]

The global coordinates of the internal nodes are obtained by Eq. (B7), with the local coordinate calculated in Eq. (19):

\[ \begin{pmatrix} \tilde{\xi}_{i,j+k} \\ \tilde{\eta}_{i,j+k} \\ \tilde{\zeta}_{i,j+k} \end{pmatrix} = T^{-1} \begin{pmatrix} \xi_{i,j+k} \\ \eta_{i,j+k} \\ 0 \end{pmatrix} \quad (B7) \]

Appendix C: Calculation of the Curvature of the First Converging Ring

This appendix describes the curvature calculation of the first converging ring in the converging treatments introduced in step 3. Figure C1 presents the \( xy \)-plane projection of the first converging ring, which is the first converging ring from inside. \( A' \) is projected vertex A, and two adjacent intersectional nodes are projected as \( P_{n,i,j} \) and \( P_{n,i,j+n} \) with the coordinates \((\tilde{\xi}_{n,i,j}, \tilde{\eta}_{n,i,j}, \tilde{\zeta}_{n,i,j})\) and \((\tilde{\xi}_{n,i,j+n}, \tilde{\eta}_{n,i,j+n}, \tilde{\zeta}_{n,i,j+n})\) respectively. Implement the coordinate transformation introduced in Appendix B [Eqs. (B1–B7)]; the \( xy \) coordinate of the midpoint of the geodesic curve is obtained as \( \tilde{\xi}_{n,i,j}, \tilde{\eta}_{n,i,j} \). From the geometry shown in Fig. C1, the following is obtained:

\[ P_{n,i,j} = \sqrt{\left( \tilde{\xi}_{n,i,j} - \tilde{\xi}_{n,i,j+n} \right)^2 + \left( \tilde{\eta}_{n,i,j} - \tilde{\eta}_{n,i,j+n} \right)^2} \quad (C1) \]

\[ P_{n,i,j} = \frac{1}{2} \sqrt{\left( \tilde{\xi}_{n,i,j} - \tilde{\xi}_{n,i,j+n} \right)^2 + \left( \tilde{\eta}_{n,i,j} - \tilde{\eta}_{n,i,j+n} \right)^2} \quad (C2) \]

\[ P_{n,i,j} = \sqrt{P_{n,i,j}^2 + r_c^2} \quad (C3) \]

and

\[ r_c^2 = P_{n,i,j}^2 + (r_c - P_{n,i,j}B)^2 \quad (C4) \]

Appendix D: Evaluation of the Best-Fit Surface for an Offset-Feed Parabolic Reflector

The evaluation of the best-fit surface of the offset-feed parabolic mesh geometry, which is needed in Sec. III (step 5), is presented in this appendix. As shown in Fig. D1, the parent paraboloid and its
best-fit surface are in the global coordinates. Here, $D_{c,ab}$ is the diameter of the circular aperture of the reflector’s best-fit working surface, which is the portion of the parent best-fit surface within the offset aperture; $F_{p,bt}$ and $ΔH_{p,bt}$ are the parent best-fit surface and the vertical deviation obtained in [24]; and $ϕ$ is the slope of the best-fit parabola at the point intersecting with the parent aperture in the $x_5z_5$ plane. From Fig. D1,

$$D_{p,bt} = 4\sqrt{F_{p,bt}(H - ΔH_{p,bt})}$$ \hspace{1cm} (D1)

$$ϕ = \sin^{-1}\left(\frac{D_{p,bt}}{2R_c}\right)$$ \hspace{1cm} (D2)

$$q_{off} = \tan^{-1}\left(\frac{H}{2R_c}\right)$$ \hspace{1cm} (D3)

If $ΔH_{p,bt}$ is always sufficiently small, we can assume that

$$R_s = R_s' \hspace{0.5cm} ϕ = ϕ'$$ \hspace{1cm} (D4)

From the geometry in Fig. D1,

$$\frac{AC}{BC} = 1 - \frac{\tan(q_{off})}{\tan(ϕ)}$$ \hspace{1cm} (D5)

where

$$AC = \frac{1}{2}(D_{s,a} - D_{c,ab}) \hspace{0.5cm} BC = \frac{1}{2}(D_p - D_{p,bt})$$ \hspace{1cm} (D6)

Hence,

$$D_{c,ab} = D_{s,a} - \frac{\tan(ϕ)}{\tan(q_{off})}(D_p - D_{p,bt})$$ \hspace{1cm} (D7)

**Appendix E: Correction of Error in the Literature**

There are two errors in the appendix of [24], which are corrected as follows:

1) Equation A.4 in [24] gave the deviation $w$ of the desired working surface from the $ξη$ plane:

$$w = a + bξ + cη + \frac{ξ^2}{2R_s} + \frac{η^2}{2R_q}$$ \hspace{1cm} (E1)

Assume $a = 0$ and the desired working surface passes the three points $(0, 0), (ξ_2, η_2)$, and $(ξ_3, η_3)$ of the facet, which means that

$$w(0, 0) = 0 \hspace{1cm} w(ξ_2, η_2) = 0 \hspace{1cm} w(ξ_3, η_3) = 0$$ \hspace{1cm} (E2)

Combining Eqs. (E1) and (E2), the remaining constants are then obtained as

$$b = \frac{-(ξ_2d_2 - η_2d_3)}{4S} \hspace{0.5cm} c = \frac{(ξ_2d_3 - ξ_3d_2)}{4S}$$

$$S = \frac{1}{2}(ξ_2η_2 - ξ_3η_3) \hspace{0.5cm} d_i = \frac{ξ_i^2}{R_ξ} + \frac{η_i^2}{R_η}$$ \hspace{1cm} (E3)

which is different from equation A.5 in [24], where

$$b = \frac{(ξ_3d_2 - η_3d_3)}{4S}$$ \hspace{1cm} (E4)

2) In the appendix of [24], $a$ is described as the normal distance from the points $ζ_i$ and $η_i$ to the surface representing the effective reflector surface. However, this description of $a$ could be misleading because, according to equations B7–C2 in [24] (although there is a mistake in equation C2 that will be discussed later), $a$ is actually the normal distance between the effective surface and the desired working surface for a given mesh facet.

The geometries of a general triangular mesh facet, the desired working surface, and the effective working surface are given in Fig. E1. The equation of the desired parabolic working surface and the effective working surface are

$$y^2 + z^2 = 4F(x_0 - x)$$ \hspace{1cm} (E5)

$$y^2 + z^2 = 4F'(x_0 - δ - x)$$ \hspace{1cm} (E6)

A line $PQ$ is created by passing through the centroid of the facet normal to its plane. The equation of the line in its plane created by $x$ and $r$ is given in equation A.10 of [24], which is represented here:

$$r^2 = y^2 + z^2 \hspace{1cm} x = x_c + \frac{r - r_c}{\sqrt{B^2 + C^2}}$$ \hspace{1cm} (E7)

where $x_c$ and $r_c$ are the coordinates of the centroid of the facet. $r_c$ is given as

$$r_c^2 = y_c^2 + z_c^2$$ \hspace{1cm} (E8)

where $r_p, r_q, x_p$, and $x_q$ are shown in Fig. E1; and their formulas of calculation are obtained by substituting Eq. (E7) into Eqs. (E5) and (E6):

$$r_q = \frac{-2F}{\sqrt{B^2 + C^2}} + 2\sqrt{\frac{F^2}{B^2 + C^2} + F\left(x_0 - x_c + \frac{r_c}{\sqrt{B^2 + C^2}}\right)}$$ \hspace{1cm} (E9)

$$r_p = \frac{-2F'}{\sqrt{B^2 + C^2}} + 2\sqrt{\frac{F'^2}{B^2 + C^2} + F'\left(x_0 - δ - x_c + \frac{r_c}{\sqrt{B^2 + C^2}}\right)}$$ \hspace{1cm} (E10)

$$x_q = x_0 - r_q^2/4F$$ \hspace{1cm} (E11)

$$x_p = x_0 - δ - r_p^2/4F'$$ \hspace{1cm} (E12)

$$a = \sqrt{(x_p - x_q)^2 + (r_p - r_q)^2}$$ \hspace{1cm} (E13)
where Eq. (E12) is different from the incorrect form in equation A.12 of [21], which is

\[ x_p = x_0 - \frac{r_0^2}{4F} \]  

(E14)

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References


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