Multicasting in Time-varying Wireless Networks: Cross-layer Dynamic Resource Allocation

Xijin Yan, Michael J. Neely and Zhen Zhang
Dept. of Electrical Engineering - Systems
University of Southern California
Los Angeles, CA, U.S.A.
{xyan, mjneely, zzhang}@usc.edu

***Abstract*— In this paper, we study the dynamic resource allocation problem for a class of time-varying wireless multicast networks with intra-multicast network coding. We provide distributed and dynamic cross-layer strategy to simultaneously achieve utility optimization and network stability under given power constraints. Our result shows when combined with Lyapunov drift technique for optimal flow control, "one shot" type of network codes, i.e., codes that restrict network coding within packets in a multicast that enter the network in the same timeslot, are sufficient to achieve performance optimality in this class of networks.**

I. Introduction

The dynamic resource allocation problem for time-varying networks has been previously studied under the classical routing setting. In particular, in [1] [2] and [3], a Lyapunov drift technique was introduced to design dynamic control algorithms for system queue stability. Such class of algorithms are shown to achieve network stability and allow decentralized implementation without requiring any knowledge of channel statistics. Subsequent to these works, a further extended Lyapunov drift technique was proposed in [4] and [5] to perform system optimization while achieving network stability simultaneously. However, the recent work by Ahlswede et al [6] has shown, coding, i.e., forwarding a relaxed function of received data at each intermediate node can potentially achieve optimality beyond routing.

As the first step to extend the Lyapunov drift technique into the network coding regime, Ho et. al. in [7] proved a stability region for a class of time-varying multicast networks with intra-session network coding and designed a dynamic algorithm that stabilizes the network for any given input rate vector strictly in this region. On the other hand, an alternative sub-gradient based algorithm was developed in [8] for purpose of utility maximization in static single multicast networks. However, in their performance analysis, the concept of the actual end-to-end throughput at which packets can be successfully decoded is not explicitly defined and no bounds for these actual rates and their related performances were provided.

In this paper, we extend the dynamic resource allocation problem to the class of time-varying wireless multicast networks with intra-multicast network coding. In particular, we provide distributed and dynamic cross-layer strategy that combines flow control, routing, resource allocation and block random network coding to simultaneously achieve utility maximization and network stability subject to given peak and average power constraints. We define the concept of end-to-end throughput in the existence of coding and derive related performance bounds. For networks with ergodic channel processes, our result shows when combined with Lyapunov drift technique for optimal flow control, "one shot" type of network codes, i.e., codes that restrict network coding within packets in a multicast that enter the network in the same timeslot, are sufficient to achieve performance optimality. This clears the previous concern of the loss of optimality for time-constrained network codes [9], [7].

The rest of the paper is organized as follows. In section II, we give a formal definition of the network model. In section III, we provide dynamic cross-layer strategy to solve the utility maximization problem subject to peak and average power constraints. Performance bounds are also derived to justify the conclusions made in section IV.

II. Network Model

Consider an intra-multicast network $G = (\mathcal{N}, \mathcal{A}, \boldsymbol{R}, \mathcal{M})$ with multicast set $\mathcal{C}$, where $\mathcal{N}$ is the set of nodes, $\mathcal{A} = \{(i, Z) : i \in \mathcal{N}, Z \subset \mathcal{N}\}$ is the set of hyperarcs with capacity constraints $\boldsymbol{R} = (R_{iZ} : (i, Z) \in \mathcal{A})$, i.e., each hyperarc (i, Z) represents a broadcast channel from the transmitter i to the receivers Z with capacity constraint R_{iZ}, and $\mathcal{M} = \{M_d^c = S_c : c \in \mathcal{C}, d \in T_c\}$ is the multicast requirements such that only packets from sources S_c in a multicast c are coded and requested at all sinks T_c of c for all $c \in \mathcal{C}$. We assume all sources in all multicasts are mutually independent.

For time-varying networks that operate in slotted time, the instantaneous transmission rate on each hyperarc is defined as $\mu_{iZ}(t) = \mu_{iZ}(\boldsymbol{S}(t), \boldsymbol{I}(t))$, where $\boldsymbol{S}(t) = (S_{iZ}(t))$ is a vector of current channel conditions from a finite state space Ω with well defined time average probabilities $\pi_{\boldsymbol{S}}$, and $\boldsymbol{I}(t) = (I_{iZ}(t))$ is a vector of control decisions restricted to a compact set Π of all acceptable resource allocation options. We assume no interference among transmissions to a common receiver and a node can transmit and receive during the same timeslot. Let

$$\Gamma = \sum_{\boldsymbol{S} \in \Omega} \pi_{\boldsymbol{S}} ConvexHull\{\boldsymbol{\mu}(\boldsymbol{S}, \boldsymbol{I}) | \boldsymbol{I} \in \Pi\} \tag{1}$$

be the set of all long-term time average hyperarc rates supportable by the network. Then for any $\boldsymbol{R} \in \Gamma$, $(\mathcal{N}, \mathcal{A}, \boldsymbol{R}, \mathcal{M})$ defines a static time average graph $\tilde{G}$ with $\mathcal{A}$ containing all hyperarcs with positive time average rates

$$R_{iZ} = \lim_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mu_{iZ}(\boldsymbol{S}(\tau), \boldsymbol{I}(\tau)).$$

Thus, it is possible to have different hyperarcs $(i, Z_1), (i, Z_2)$ of different time average rates from the same node i in $\tilde{G}$.

Consider a block intra-multicast coding scheme such that coding is not only restricted within packets of the same multicast but also of packets generated within the same time block. We call these packets commodity (c, k) packets. This block coding scheme was originally raised in [9] to simulate a practical coding scheme in a real packet network.

At any slot t in some block k, let $A_i^{ck}(t)$ and $R_i^{ck}(t)$ be respectively the amount of (c, k) packets that exogenously arrived at the transport layer and that were admitted to the network layer from the transport layer at node i. We assume $A_i^{ck}(t)$ have well defined time average rates and $R_i^{ck}(t)$ are upper bounded for all t. Define $L_i^{ck}(t)$ and $U_i^{cdk}(t)$ as the current backlog sizes of the (uncoded) (c, k) packets queued at the transport layer and the (possibly coded) (c, k) packets intended for the sink d queued at the network layer. Thus, extended from [7], we can prove the stability region Λ_K under the above formulation to be the set of all average input rates (r_i^{ck}) such that there exist flow variables $\{f_{abZ}^{cdk}, g_{aZ}^{ck}\}$ satisfying:

$$r_i^{ck} = \sum_{b,Z} f_{ibZ}^{cdk} - \sum_{a,Z} f_{aiZ}^{cdk}, \quad \forall\, i, c, d, k \tag{2}$$

$$\sum_{i} r_i^{ck} = \sum_{a,Z} f_{adZ}^{cdk}, \quad \forall\, c, d, k \tag{3}$$

$$\sum_{b} f_{abZ}^{cdk} \le g_{aZ}^{ck}, \quad \forall\, (a, Z), c, d, k \tag{4}$$

$$\sum_{c,k} g_{aZ}^{ck} \le R_{aZ}, \quad \forall\, (a, Z), \boldsymbol{R} \in \Gamma \tag{5}$$

$$f_{abZ}^{cdk} \ge 0, \quad \forall\, (a, Z), b, c, d, k \tag{6}$$

where K is the number of blocks, g_{aZ}^{ck} and f_{abZ}^{cdk} are respectively the rate of (c, k) flow on the hyperarc (a, Z) and the rate of (c, k) flow destined for the sink d on the edge $(a, b), b \in Z$. Clearly when $K = 1$, Λ_K reduces to Λ defined in [7].

III. Utility Maximization With Peak and Average Power Constraints

In this section, we design cross-layer dynamic control algorithm to stabilize the network while achieving maximum network utility. Specifically, our goal is to maximize the network utility subject to the peak and average power constraints and the network stability.

The main approach we propose involves two components which are performed simultaneously across the transport layer and network layer from slot to slot. First, for any arbitrary exogenous arrival rates, we design transport layer admission control algorithm to locate a feasible source rate vector in the network layer stability region which achieves maximum network utility. Network layer flow control, routing, network coding, and resource allocation schemes are then provided to stabilize the network under this optimal rate vector.

Furthermore, three major techniques are used to facilitate the above design approach. First, virtual queues are introduced to push the arrival rate into the stability region and meet the average power constraint at each transmitting node. Second, an extended Lyapunov drift which includes both queue and performance measures is defined to drive the algorithm decisions to jointly achieve network stability and flow optimality. Third, a block random network coding scheme is integrated with the above flow control scheme to asymptotically arrive at the optimal throughput solutions in the stability region.

We define the *end-to-end throughput* achievable at a sink to be the average rate at which the sink can successfully decode its received data. In the next section, we derive a lower bound on the end-to-end throughput at each sink and analyze the tradeoff between its related performances and other design parameters.

A. An Extended Lyapunov Drift

Consider the following utility maximization problem with peak and average power constraints:

$$\begin{aligned}
\text{Maximize:}\quad & \sum_{i,c} g_i^c(\mathbf{1}_K \boldsymbol{\gamma}_i^c) & \\
\text{Subject to:}\quad & (\bar{P}_i) \le (P_i^{av}) & (7)\\
& (P_{iZ}) \le (P_{iZ}^{peak}) & (8)\\
& \bar{\mathbf{r}} \ge \boldsymbol{\gamma} & \\
& 0 \le \bar{\mathbf{r}} \le \bar{\boldsymbol{\lambda}} & \\
& \bar{\mathbf{r}} \in \Lambda_K, &
\end{aligned}$$

where $\mathbf{1}_K \boldsymbol{\gamma}_i^c = \sum_{k=1}^{K} \gamma_i^{ck}$, P_{iZ} is the power assignment on (i, Z), P_i^{av} and P_{iZ}^{peak} are respectively the average power limit at i and the peak power limit on (i, Z),

$$\bar{P}_i = \lim_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \left[\sum_{Z} P_{iZ}(\tau) \right],$$

$$\bar{r}_i^{ck} = \lim_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{R_i^{ck}(\tau)\},$$

$$\bar{\lambda}_i^{ck} = \lim_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{A_i^{ck}(\tau)\},$$

and $g_i^c(\cdot)$ are nonnegative functions that are continuous, concave, bounded and entry-wise nondecreasing. Note that the auxiliary variables $\boldsymbol{\gamma}$ are introduced to facilitate the algorithm design in cases when $g(\cdot)$ is nonlinear (see [5]).

Consider the following queueing dynamics:

(1) The backlog queues: for all i, c, d, k,

$$U_i^{cdk}(t+1) \le \max[U_i^{cdk}(t) - \sum_{b,Z} \mu_{ibZ}^{cdk}(t), 0] + \sum_{a,Z} \mu_{aiZ}^{cdk}(t) + R_i^{ck}(t), \tag{9}$$

where for all slot t

$$\sum_{c,k,b,Z} \mu_{ibZ}^{cdk}(t) \leq \mu_{max}^{out}, \forall\, i, d,$$
$$\sum_{c,k,a,Z} \mu_{aiZ}^{cdk}(t) \leq \mu_{max}^{in}, \forall\, i, d,$$
$$R_i^{ck}(t) \leq \hat{R}_i^c, \forall\, i, c, k.$$

μ_{max}^{out}, μ_{max}^{in}, $\hat{R}_i^c$ are the given control constants on the maximum input rate, output rate and amount of multicast c data admitted to the network layer at a node for all t.

(2) The virtual power queues: for all i,

$$X_i(t+1) = \max[X_i(t) - P_i^{av}, 0] + \sum_Z P_{iZ}(t), \quad (10)$$

where for all slot t,

$$P_{iZ}(t) \leq P_{iZ}^{peak}, \forall\, (i, Z).$$

(3) The virtual flow state queues: for all i, c, k,

$$Y_i^{ck}(t+1) = \max[Y_i^{ck}(t) - R_i^{ck}(t), 0] + \gamma_i^{ck}(t), \quad (11)$$

where for all slot t,

$$\gamma_i^{ck}(t) \leq \hat{R}_i^c, \forall\, i, c, k.$$

Define the network state vector $\boldsymbol{\Theta}(t) \triangleq [\boldsymbol{U}(t), \boldsymbol{X}(t), \boldsymbol{Y}(t)]$, quadratic Lyapunov function

$$L(\boldsymbol{\Theta}) \triangleq \frac{1}{2}\left[\sum_{i,c,d,k} (U_i^{cdk})^2 + \sum_i X_i^2 + \sum_{i,c} (Y_i^{ck})^2\right] \quad (12)$$

and Lyapunov drift

$$\Delta(\boldsymbol{\Theta}(t)) \triangleq \mathbb{E}\{L(\boldsymbol{\Theta}(t+1)) - L(\boldsymbol{\Theta}(t)) | \boldsymbol{\Theta}(t)\}, \quad (13)$$

we have the following lemma.

Lemma 1:

$$\Delta(\boldsymbol{\Theta}(t)) - V\mathbb{E}\{\sum_{i,c} g(\mathbf{1}_K \gamma_i^c(t)) | \boldsymbol{\Theta}(t)\} \leq NB$$
$$- \sum_i X_i(t) P_i^{av} - \Psi(\boldsymbol{\Theta}(t)) - \Phi(\boldsymbol{\Theta}(t)) - \Upsilon(\boldsymbol{\Theta}(t)) \quad (14)$$

where for slot t in block k,

$$\Psi(\boldsymbol{\Theta}(t)) = \sum_{i,c}\left(Y_i^{ck}(t) - \sum_{d,k} U_i^{cdk}(t)\right)\mathbb{E}\left\{R_i^{ck}(t)|\boldsymbol{\Theta}(t)\right\},$$

$$\Phi(\boldsymbol{\Theta}(t)) = \sum_{i,c} \mathbb{E}\{V g_i^c(\gamma_i^{ck}(t)) - Y_i^{ck}(t)\gamma_i^{ck}(t) | \boldsymbol{\Theta}(t)\},$$

$$\Upsilon(\boldsymbol{\Theta}(t)) = \sum_{i,Z} \mathbb{E}\left\{\sum_{c,d,k} U_i^{cdk}(t)\left(\sum_b \mu_{ibZ}^{cdk}(t) - \sum_a \mu_{aiZ}^{cdk}(t)\right) - X_i(t) P_{iZ}(t) \middle| \boldsymbol{\Theta}(t)\right\},$$

$$B = \frac{\max_c |T_c|}{2}\left[(\mu_{max}^{out})^2 + \left(\mu_{max}^{in} + \max_i \sum_c \hat{R}_i^c\right)^2 + \left(\max_i \sum_Z P_{iZ}^{peak}\right)^2 + \left(\max_i \sum_c \hat{R}_i^c\right)^2\right].$$

Remark. Since packets of different blocks are transmitted sequentially at the source nodes, in any slot t in block k, $\sum_k R_i^{ck}(t) = R_i^{ck}(t)$. Same equations hold for $\boldsymbol{Y}(t)$ and $\boldsymbol{\gamma}(t)$.

B. Dynamic Cross-layer Control Algorithm

We design the following cross-layer control algorithm to maximize $\Psi(\boldsymbol{\Theta}(t)), \Phi(\boldsymbol{\Theta}(t)), \Upsilon(\boldsymbol{\Theta}(t))$ in Lemma 1:

1) *Flow Control*: At each slot t in block k, perform

$$\text{Maximize:} \quad \sum_{i,c}\left(Y_i^{ck}(t) - \sum_{d,k} U_i^{cdk}(t)\right) R_i^{ck}(t)$$
$$\text{Subject to:} \quad 0 \leq R_i^{ck}(t) \leq \min[A_i^{ck}(t) + L_i^{ck}(t), \hat{R}_i^c],$$

which implies for each pair (i, c), observe $U_i^{cdk}(t), \forall\, d, k$ and $Y_i^{ck}(t)$ and set:

$$R_i^{ck}(t) = \begin{cases} \min[A_i^{ck}(t) + L_i^{ck}(t), \hat{R}_i^c], & \text{if } \sum_{d,k} U_i^{cdk}(t) < Y_i^{ck}(t) \\ 0, & \text{otherwise} \end{cases}.$$

At each i, $\gamma_i^{ck}(t)$ for all c are computed as follows:

$$\text{Maximize:} \quad V g_i^c(\gamma_i^{ck}) - Y_i^{ck}(t)\gamma_i^{ck}$$
$$\text{Subject to:} \quad 0 \leq \gamma_i^{ck} \leq \hat{R}_i^c. \quad (15)$$

$\boldsymbol{Y}(t+1)$ is then updated according to equations (11).

2) *Routing and Resource Allocation*: Since

$$\sum_{i,c,d,k} U_i^{cdk}(t)\left(\sum_{b,Z} \mu_{ibZ}^{cdk}(t) - \sum_{a,Z} \mu_{aiZ}^{cdk}(t)\right) = \sum_{a,b,Z,c,d,k} \mu_{abZ}^{cdk}(t)\left(U_a^{cdk}(t) - U_b^{cdk}(t)\right),$$

we do the following maximization:

$$\text{Maximize:} \quad \sum_{a,Z}\left(\sum_{c,b,d,k} W_{ab}^{cdk}(t)\mu_{abZ}^{cdk}(t) - X_a(t) P_{aZ}\right)$$
$$\text{Subject to:} \quad P_{aZ} \leq P_{aZ}^{peak}, \forall\, a, Z$$

where

$$W_{ab}^{cdk}(t) = \max[U_a^{cdk}(t) - U_b^{cdk}(t), 0].$$

Further note that,

$$\sum_{c,b,k} W_{ab}^{cdk}(t)\mu_{abZ}^{cdk}(t) \leq R_{aZ}(t) \max_{c,k}\left[\max_{b\in Z} W_{ab}^{cdk}(t)\right].$$

Let $R_{aZ}(t) = \mu_{aZ}(\boldsymbol{P}, \boldsymbol{S}(t))$. Then for the case that $\boldsymbol{\mu}_{aZ}(\boldsymbol{P}, \boldsymbol{S}(t)) = (\mu_{aZ}(P_{aZ}, S_{aZ}(t)))$, the above maximization reduces to the following scheduling policy: for each (a, Z) at a, select optimal commodity

$$(c^*, k^*) = \arg\max_{c,k}\left[\sum_d \max\left[\max_{b\in Z} W_{ab}^{cdk}(t), 0\right]\right]$$

and compute

$$W_{aZ}^* = \sum_d \max\left[\max_{b\in Z} W_{ab}^{c^*dk^*}(t), 0\right].$$

A power vector $\boldsymbol{P}_a^* = (P_{aZ}^*)$ is then allocated such that

$$\boldsymbol{P}_a^* = \arg\max_{P_{aZ}\le P_{aZ}^{peak}} \sum_Z \Big[\mu_{aZ}\left(P_{aZ}, S_{aZ}(t)\right) W_{aZ}^* - X_a(t)P_{aZ}\Big].$$

$\boldsymbol{X}(t+1)$ is then updated according to the equations (10).

3) Network Coding: For each hyperarc (a, Z), define

$$T_{aZ}^{c^*k^*} = \left\{d \in T_{c^*} : \max_{b\in Z} W_{ab}^{c^*dk^*} > 0\right\}$$

for the optimal (c^*, k^*) determined in step 2). Then node a transmits a random linear combination of the packets from all queues indexed by $(c^*, d, k^*), d \in T_{aZ}^{c^*_{}}$ at the optimal rate P_{aZ}^* obtained in step 2) and updates the related virtual backlog queues according to (9). For each $d \in T_{aZ}^{c^*k^*}$, randomly select a receiver from the set $\{b \in Z : b = \arg\max_{b\in Z} W_{ab}^{c^*dk^*}\}$, puts the received packets in the queue corresponding to (c^*, d, k^*) and update the related virtual backlog queues according to (9). Iterate this for all $d \in T_{aZ}^{c^*k^*}$. All other receivers in Z reject the received packets and remain silent.

C. Performance Analysis

Let $r_i^{ck}(T)$ be the T-slot average input rate of multicast-c data from block k at node i. Define a modified stability region $\tilde{\Lambda}_K$ under the additional average power constraint, i.e., $\tilde{\Lambda}_K$ is the set of all (r_i^{ck}) such that there exist variables $\{f_{abZ}^{cdk}, g_{aZ}^{ck}, P_{aZ}\}$ for which constraints (2)-(8) are satisfied. For any $\epsilon > 0$, further define a region

$$\tilde{\Lambda}_K^\epsilon = \left\{(r_i^{ck})\middle|(r_i^{ck} + \epsilon 1_i^{ck}) \in \tilde{\Lambda}_K, r_i^{ck} \ge 0, \forall\ i, c, k\right\}. \quad (16)$$

Let $\mathbf{1}_K$ be the all-one length-K row vector, we have:

Lemma 2: For nonnegative and concave utility functions $g_i^c(\cdot)$ with bounded first-order derivatives,

$$\sum_{i,c} g_i^c(\mathbf{1}_K \mathbf{r}_i^{c*}(\epsilon)) \to \sum_{i,c} g_i^c(\mathbf{1}_K \mathbf{r}_i^{c*}) \text{ as } \epsilon \to 0,$$

where $\mathbf{r}^*(\epsilon)$ and $\mathbf{r}^*$ are respectively the optimal solutions to the optimization problem with constraints $\tilde{\Lambda}_K^\epsilon$ and $\tilde{\Lambda}_K$.

Lemma 3: For $\mathbf{r}^*(\epsilon) \in \tilde{\Lambda}_K^\epsilon$, there exists a static randomized scheme such that for all i, c, k

$$\mathbb{E}\left\{\sum_{b,Z} \mu_{ibZ}^{cdk}(t) - \sum_{a,Z} \mu_{aiZ}^{cdk}(t)\middle|\boldsymbol{\Theta}(t)\right\} = r_i^{*ck}(\epsilon) + \epsilon 1_i^{ck},$$

$$\mathbb{E}\left\{\sum_Z P_{iZ}(t)\middle|\boldsymbol{\Theta}(t)\right\} \le P_{av}^i.$$

Now, fixing $\gamma_i^{ck}(t) = r_i^{*ck}(\epsilon)$ for all t, and admit new arrivals independently in each timeslot with probability $p_i^{ck} = r_i^{*ck}(\epsilon)/\lambda_i^{ck}$, thus $\mathbb{E}\left\{R_i^{ck}(t)|\boldsymbol{\Theta}(t)\right\} = p_i^{ck}\mathbb{E}\{A_i^{ck}(t)\} = r_i^{*ck}(\epsilon)$. Plugging these terms into (14) and canceling the common terms, we obtain

$$\begin{aligned}&\Delta(\boldsymbol{\Theta}(t)) - V\mathbb{E}\{g(\boldsymbol{\gamma}(t))|\boldsymbol{\Theta}(t)\} \\ \le\ & NB - \epsilon \sum_{i,c,d,k} U_i^{cdk}(t) - V\sum_{i,c} g_i^c(\mathbf{1}_K \mathbf{r}_i^{c*}(\epsilon)). \quad (17)\end{aligned}$$

Lemma 4: Suppose the network starts empty. Since the Lyapunov drift satisfies (17) for all slots t, the T-slot time average sum congestion and utility satisfy:

$$\frac{1}{T}\sum_{\tau=0}^{T-1}\left[\sum_{i,c,d,k} \mathbb{E}\{U_i^{cdk}(\tau)\}\right] \le \frac{NB + Vg_{max}}{\epsilon_{max}},$$

$$\sum_{i,c} g_i^c\left(\mathbf{1}_K \boldsymbol{\gamma}_i^c(T)\right) \ge \sum_{i,c} g_i^c\left(\mathbf{1}_K \mathbf{r}_i^{c*}(\epsilon)\right) - \frac{NB}{V},$$

where g_{max} is an upper bound on $\sum_{i,c} g_i^c(\cdot)$ and ϵ_{max} is the maximum positive constant such that $(\epsilon_{max} 1_i^{ck}) \in \tilde{\Lambda}_K$.

The next theorem bounds the decoding error probability.

Theorem 1: Let $P_e^{ck}(T)$ be the probability that not all (c, k) packets are decodable, then for $N = |\mathcal{N}|$ and T large enough,

$$P_e^{ck}(T) \le 1 - \left(1 - \frac{|T_c|}{q}\right)^{\frac{NBT}{K}+\epsilon} \quad (18)$$

where B is a bounded constant and $\epsilon \to 0$ as $T \to \infty$.

Proof: We introduce an equivalent graph for each commodity (c, k) such that the (c, k) packets are transmitted in the same shot as in a wired directed graph. Such a graph can be constructed by making n_e copies of each edge e that had been activated n_e times for the transmissions of (c, k) packets. The capacity of each edge copy equals the actual assigned rate. Each of such generated edge corresponding to a timeslot t is further decomposed into a set of parallel unit capacity edges with equal sum capacity. Then, all these time-indexed edges are connected according to the real packet transmissions. This results in a directed graph G_c^k with edges of unit capacity.

Since the given algorithm only codes data in queues across different destinations, thus each of the received packets at sink d corresponds to a different (c, k) source packet admitted at some source i. By assigning 1 to the random coding coefficients for all packets destined for d and 0 else, the coding matrix received at d for this trunk of data equals the identity matrix. Thus the determinant of the coding matrix received at d is not identically zero. And this is true for all sink nodes $d \in T_c$. Thus

$$\sum_{c,k}\left(\sum_e \sum_{i=1}^{n_e} \mu_e^{ck}(i)\right) \le N\mu_{max}^{out}T.[1] \quad (19)$$

By the Weak Law of Large Number, for T large enough,

$$\left|\sum_e \sum_{i=1}^{n_e} \mu_e^{c_l k_l}(i) - \sum_e \sum_{i=1}^{n_e} \mu_e^{c_j k_j}(i)\right| < \epsilon, \quad (20)$$

[1]Note, there is a residual time τ for all the (c, k) packets to clear from the network after T and it is bounded by Lemma 4. We ignore it for the space limit. However this does not change the essence of the proof.

for all (c,k). Plugging (20) into (19), we obtain

$$\sum_e \sum_{i=1}^{n_e} \mu_e^{ck}(i) \le \frac{NBT}{K} + \epsilon, \quad (21)$$

where $B = \mu_{max}^{out}/|\mathcal{C}|$. Thus, by Theorem 2 in [10],

$$\begin{aligned} P_e^{ck}(T) &\le 1 - \left(1 - \frac{|T_c|}{q}\right)^{\sum_e \sum_{i=1}^{n_e} \mu_e^{ck}(i)} \\ &\le 1 - \left(1 - \frac{|T_c|}{q}\right)^{\frac{NBT}{K}+\epsilon}. \end{aligned}$$

And this is true for all (c,k). □

Lemma 5: Let $g_K^*, K > 1$ and g^* be the maximum utilities for the utility optimization problems with stability regions Λ_K and Λ_1 respectively, then we have $g_K^* = g^*$.

Proof: Since the block coding scheme is a special case of the general case with coding across packets generated at any time, we have $g_K^* \le g^*$. On the other hand, Λ_K can be regarded as K symmetric subgraphs whose optimal utility is obtained by time-sharing them according to the capacity constraint (5). Thus $g^* \le g_K^*$. □

Now we prove the performance of the given algorithm.

Theorem 2: For any transport layer rate vector (λ_i^{ck}) (possibly outside the stability region), the given algorithm stabilizes the network, conforms to the peak and average power constraints and yields the average congestion and utility bounds:

$$\limsup_{T\to\infty} \frac{1}{T} \sum_{\tau=0}^{T-1} \left[\sum_{i,c,d,k} \mathbb{E}\{U_i^{cdk}(\tau)\} \right] \le \frac{NB_1 + V g_{max}}{\epsilon_{max}}, \quad (22)$$

$$\sum_{i,c} g_i^c \left(\mathbf{1}_K \boldsymbol{\nu}_i^{c*}\right) \ge g^* - \frac{NB_1}{V} - \frac{N^2 B_2 \Delta T}{q}, \quad (23)$$

where ΔT is the length of each block.

Proof: (22) is clearly obtained by Lemma 4. Meanwhile we have the T-slot average utility bound

$$\sum_{i,c} g_i^c \left(\mathbf{1}_K \boldsymbol{\gamma}_i^c(T)\right) \ge \sum_{i,c} g_i^c \left(\mathbf{1}_K \mathbf{r}_i^{c*}(\epsilon_1)\right) - \frac{NB_1}{V}. \quad (24)$$

Using (11) and the fact $g_i^c(\cdot)$ has bounded first-order derivatives, it is not difficult to prove

$$\sum_{i,c} g_i^c(\mathbf{1}_K \mathbf{r}_i^c(T)) \ge \sum_{i,c} g_i^c \left(\mathbf{1}_K \boldsymbol{\gamma}_i^c(T)\right) - \frac{NB_3 V}{T}. \quad (25)$$

By Theorem 1, the lower bound on the actual T-slot end-to-end throughput is

$$\begin{aligned} \nu_i^{ck}(T) &\ge \frac{\Delta T}{T} r_i^{ck}(T) P_e^{ck}(T) \\ &= \frac{1}{K} r_i^{ck}(T) \left(1 - \frac{|T_c|}{q}\right)^{\frac{NBT}{K}+\epsilon_2} \\ &\ge \frac{1}{K} r_i^{ck}(T) \left(1 - \frac{|T_c|}{q}\Big(\frac{NBT}{K} + \epsilon_2\Big)\right). \end{aligned}$$

Thus we can bound

$$\begin{aligned} \sum_{i,c} g_i^c \left(\mathbf{1}_K \boldsymbol{\nu}_i^{c*}(T)\right) \ge \sum_{i,c} g_i^c \left(\mathbf{1}_K \mathbf{r}_i^c(T)\right) \\ - \frac{N^2 B_2 T}{Kq} - \frac{NB_4}{q}\epsilon_2. \end{aligned} \quad (26)$$

Combining (24), (25) and (26), we obtain

$$\begin{aligned} \sum_{i,c} g_i^c \left(\mathbf{1}_K \boldsymbol{\nu}_i^{c*}(T)\right) \ge \sum_{i,c} g_i^c \left(\mathbf{1}_K \mathbf{r}_i^{c*}(\epsilon_1)\right) - \frac{NB_1}{V} \\ - \frac{NB_3 V}{T} - \frac{N^2 B_2 \Delta T}{q} - \frac{NB_4}{q}\epsilon_2. \end{aligned}$$

Letting $T \to \infty$ and $\epsilon_1 \to 0$, we have

$$\sum_{i,c} g_i^c \left(\mathbf{1}_K \boldsymbol{\nu}_i^{c*}\right) \ge g_K^* - \frac{NB_1}{V} - \frac{N^2 B_2 \Delta T}{q}.$$

Thus (23) is obtained by applying Lemma 5. □

Remark: 1) Clearly, (23) is a function only of V, q if $\Delta T = 1$, but a function of V, q, T if $\Delta T = T$. 2) $\Delta T = 1$ also minimizes the average decoding delay.

IV. Conclusion

In this paper, we provided a distributed and dynamic cross-layer strategy to simultaneously achieve network stability and performance optimality. We showed when combined with Lyapunov drift technique for optimal flow control, "one shot" type of network codes are sufficient to achieve optimal performances. This suggests a framework for the design of dynamic algorithms to perform optimal resource allocations for time-varying networks with intra-multicast network coding.

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