Chapter 5 MRI pipeline
Nuclei's response to a magnetic field:
\[ \vec{J} \Rightarrow \vec{\mu} \Rightarrow \vec{M}_0 \]

Signal Generation:
\[ \vec{M}(r) \Rightarrow \delta(t) \]

Principles:
1. Faraday Induction:
   \[ \text{enf} = -\frac{d\phi}{dt} \]
   \[ \phi = \int_{\text{coil area}} \vec{B} \cdot d\vec{s} \]
2. Reciprocity.

\[ \phi(t) = \int_{\text{Sample}} d^3r \, \vec{B}_{\text{receive}}(r) \cdot \vec{M}(r,t) \]

\[ \text{enf} = -\frac{d}{dt} \phi(t) = -\frac{d}{dt} \int_{\text{Sample}} d^3r \, \vec{M}(r,t) \cdot \vec{B}_{\text{receive}}(r) \]

Ignore B direction.
\[ \vec{M}(r,t) = M_1(r,0) \cos(\omega t) + i M_1(r,0) \sin(\omega t) \]

\[ \vec{B}_{\text{receive}} = B_{1}(r) \cos\Theta_B + i B_{1}(r) \sin\Theta_B \]

\[ S(t) \propto \text{enf} \propto \omega_0 V_s M_1 B_1 \sin(\omega t + \Theta_B - \phi_0) \]

\[ \frac{d}{dt} \int d^3r \, M_0 \sin\Theta_B \text{ receive field coil angle} \]

Signal demodulation:
\[ S(t) \Rightarrow x\sin(\omega t) \]

Real channel

 Imaginary channel

\[ x\cos(\omega t) \]
In OSM, each susceptibility source can be considered as a sphere with a magnetic moment pointing in the z-direction (11Bx).

Assume the source is located at \( \vec{r} \), the observer is located at \( \vec{r}^\prime \), then the magnetic field seen by the observer is:

\[
\vec{B}(\vec{r}) = \frac{1}{4\pi} \frac{3(\vec{m}(\vec{r}) \cdot \vec{e}_{rr}) \vec{e}_{rr} - \vec{m}(\vec{r})}{|\vec{r}^\prime - \vec{r}|^3}
\]

Assume multiple sources, then \( \vec{B}(\vec{r}) \) would be the summation of the contribution from all sources.

\[
\vec{B}(\vec{r}) = \frac{1}{4\pi} \int \frac{3(\vec{m}(\vec{r}) \cdot \vec{e}_{rr}) \vec{e}_{rr} - \vec{m}(\vec{r})}{|\vec{r}^\prime - \vec{r}|^3} d\vec{r}^\prime
\]

If we just look at the z-component of \( \vec{B}(\vec{r}) \)

\[
\vec{B}_z(\vec{r}) = \vec{m}(\vec{r}) \times \vec{d}(\vec{r}) = [\chi(\vec{r}) \times d(\vec{r})] \cdot \vec{B}_0
\]

\[
\vec{m}(\vec{r}) = \iiint \chi \vec{B}_0 d\vec{r} = \chi \cdot \vec{B}_0 \cdot \text{Voxel}
\]

\[
\vec{d}(\vec{r}) = \text{Voxel} \cdot \frac{3\cos^2\theta - 1}{\frac{4\pi}{\rho^3}}
\]

This kernel is special because \( \rho = 0 \) is not defined!

\[
\vec{B}(\vec{r}) \bigg|_{\rho = 0} = \frac{2\mu_0}{3} \vec{m} = \frac{2}{3} \chi \vec{B}_0
\]
Sources of error in each stage:

<table>
<thead>
<tr>
<th>Stage</th>
<th>Description</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s(k,TE) \rightarrow m(r,TE)$</td>
<td>Reconstruction should be phase sensitive. I’m using SENSE R=2</td>
<td>Coil phase</td>
</tr>
<tr>
<td>$m(r,TE) \rightarrow \Delta f(r)$</td>
<td>Phase fitting in low-SNR regions (short R2*, CSF long T1, low signal). I’m using complex fitting.</td>
<td>Nonlinear fitting might suffer from local minimum; choice of echo time;</td>
</tr>
<tr>
<td>Unwrap $\Delta f(r)$</td>
<td>Phase unwrapping in low-SNR regions. I’m using region-growing method.</td>
<td>Phase unwrapping in regions with fast field variation (e.g. superior sagittal sinus)</td>
</tr>
<tr>
<td>$\Delta f(r)$ $\rightarrow \Delta f_{background}(r)$ $+ \Delta f_{local}(r)$</td>
<td></td>
<td>Error near tissue boundaries.</td>
</tr>
<tr>
<td>$\Delta f_{local}(r) \rightarrow \chi(r)$</td>
<td></td>
<td>Ill-posedness $\rightarrow$ streak artifact</td>
</tr>
<tr>
<td>$\chi(r) \rightarrow Y_v(r)$</td>
<td></td>
<td>Reference point</td>
</tr>
</tbody>
</table>

Table 4.1: Representative values of relaxation parameters $T_1$ (see Sec. 4.2) and $T_2$ (see Sec. 4.3), in milliseconds, for hydrogen components of different human body tissues at $B_0 = 1.5 \text{T}$ and 37 °C (human body temperature). These are only approximate values; see Ch. 22.

<table>
<thead>
<tr>
<th>Tissue</th>
<th>$T_1$ (ms)</th>
<th>$T_2$ (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>gray matter (GM)</td>
<td>950</td>
<td>100</td>
</tr>
<tr>
<td>white matter (WM)</td>
<td>600</td>
<td>80</td>
</tr>
<tr>
<td>muscle</td>
<td>900</td>
<td>50</td>
</tr>
<tr>
<td>cerebrospinal fluid (CSF)</td>
<td>4500</td>
<td>2200</td>
</tr>
<tr>
<td>fat</td>
<td>250</td>
<td>60</td>
</tr>
<tr>
<td>blood</td>
<td>1200</td>
<td>100–200</td>
</tr>
</tbody>
</table>

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