Chapter 2 Noise

Three important papers:
- *Noise in MRI*, Macovski, MRM 1996
- *Physiological Noise in Oxygenation-Sensitive Magnetic Resonance Imaging*, Kruger and Glover, MRM 2001
- *Phase vs. magnitude information in functional magnetic resonance imaging time series: toward understanding the noise*, Bowtell, MRI

1. What causes thermal noise?
   Brownian motion in
   a. Body (dominant)
   b. coil
   c. Electronics

2. What are its statistical characteristics in k-space?
   In k-space, noise is
   - Ergodic: time average is the same as its average over the probability space
   - Stationary: joint probability doesn’t change over time
   - Uncorrelated:
     - White noise process:
     - Zero mean and std $\sigma_k$

3. In image space:
   1) In image space, noise is
      - Zero mean and (if standard FFT recon) $\sigma_i^2 = \frac{1}{N} \sigma_k^2$
      - (if standard FFT recon), uncorrelated from pixel to pixel

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard deviation</th>
<th>Pixel-pixel Correlation</th>
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<tbody>
<tr>
<td>Direct FFT</td>
<td>0</td>
<td>$\sigma_i^2 = \frac{1}{N} \sigma_k^2$ **</td>
<td>no</td>
</tr>
<tr>
<td>Zero-padding(N-point k-space data $\rightarrow$ M-point image space)</td>
<td>0</td>
<td>$\sigma_i^2 = \frac{1}{N} \sigma_k^2$ ***</td>
<td>yes</td>
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**equal variance in both the real and imaginary parts ($\sigma_i = \frac{1}{N} \sigma_k$)

*** zero-padding doesn’t change noise, N is the # of acquired points.

2) Noise in magnitude image: Rician distribution, $\sigma_{mag} = \sigma_{im} = \sigma_{re}$
3) Noise in phase image (related to QSM): often approximated as Gaussian, $\sigma_p = \frac{1}{\text{SNR}_{mag}}$

Question from pre-qual: why the signal of phase is “$2\pi f t$”? In the case of multi-echo imaging, will signal still be linear with “TE”?

Comments from JH:
We spent a lot of time today talking about phase unwrapping for a single image, but the issue is much more nuanced when estimating field inhomogeneity from multi-TE data. For uniform sampling, the concept of "temporal" Nyquist rate is very useful. In the multi-TE case, the ambiguity by 2pi is associated with the 2pi periodicity of the DTFT and
the concept of aliasing. At the same time, this periodicity problem can be broken by using nonuniform sampling along the echo dimension.

4. When data is acquired from a multi-channel array, what causes there to exist correlation between the noise in different coils?
   No perfect de-coupling between the coils.

5. What are the sources of physiological noise that impact your work?
   In QSM, the signal function is:
   \[ s(r, t) = \rho(r) e^{-t \cdot R^*(r)} e^{-j \cdot 2\pi \cdot t \cdot \Delta f(r)} \]
   Basically anything that gives fluctuation of R2* and Δf will cause physiological noise.
   a. Fluctuation to R2*: BOLD-related, metabolic rate (CMRO2), CBF, CBV
      This part is TE-dependent and signal-dependent.
   b. BOLD-unrelated: brain pulsation
   c. Fluctuation to Δf: cardiac and respiratory motion cause magnetic field modulation

Question from pre-qual: How do you rank the sources? How do you design an experiment to decouple the sources and analyze their significance?

Question from pre-qual: Assume you can have whatever field strength you want for your QSM acquisition, what field strength will you choose?
I need to consider thermal noise (prefer higher field strength) and physiological noise (more noise at higher field strength).

Question from pre-qual: What need to be changed if you do field mapping in a scanner with super high field strength?
   Phase wraps (phase wraps inside one voxel)
   TE need to be shorter to decrease phase wraps

Question from pre-qual: When you see an edge in your phase images, how would you know if it’s phase wraps or natural field variation due to susceptibility?
When your TE is too long, there can be sharp variations in phase (possibly 2pi) around superior sagittal sinus (due to high susceptibility of deoxyhemoglobin). Then you’re unable to tell whether it’s phase wrap or natural field variation.
MRI basics related to this part:

1. Noise(probability) basics
   - What statistical properties are commonly discussed?
     Distribution i.e. PDF
     Mean, variance
     Independence/correlation
   - Why noise model matters?
     Signal estimation method could be chosen wisely based on the noise model/distribution.
   - White noise can be Gaussian, or uniform or even Poisson. It seems to be that the key characteristic of white noise is that each measurement is completely independent of the measurements preceding it. Also, it must be flat on the frequency spectrum. Gaussian, or uniform or even Poisson are about the PDF; “white” is about the spectrum.
   - If a single coil system + fully-sample + IFFT, noise in k-space is Gaussian, and independent→ noise in image-space is also Gaussian; noise in magnitude image space is Rician.

   - Covariance:
     two scalar random variables→ covariance
     two random vectors→ covariance matrix
2. Is it possible to measure true SNR?? How do people measure SNR?
   Impossible to measure the true SNR, because there is no way to measure the true
   signal. If noise is measured using “image-corner”, you need to think whether the noise
   in the corner can represent noise in other regions.

3. Phase noise is proportional to the reciprocal of magnitude SNR. However, phase
   physiological noise cannot be predicted by the reciprocal of magnitude SNR, indicating
   different sensitivity to physiological fluctuation (like B0, respiratory motion) from
   magnitude physiological noise.

4. Signal changes in B0-off imaging can be described by
   \[ \sigma_B = c_1 \cdot \Delta S, \]
   where \( c_1 \) is a constant. Because of that
   analogy and with \( dS/dR_2^* = -TE \cdot S_0 \cdot \exp(-TE \cdot R_2^*), \) \( \sigma_B \)
   can be expressed as:

   \[ \sigma_B = c_1 \cdot S \cdot \Delta R_2^* \cdot TE, \]  \[2\]

   where \( \Delta R_2^* \) represents the physiological fluctuation in \( R_2^* \).
   Note the TE-dependency in \( \sigma_B \) and that \( \sigma_B \) demonstrates a
   maximum at \( 1/R_2^* \). The second contribution arises from
FIG. 3. Anatomy (a) and spatial distribution of raw noise $\sigma_0$ (b), physiological noise contributions $\sigma_{\text{B}}$ (c), and $\sigma_{\text{NR}}$ (d) in a typical image section from one subject. Note that gray scale contrast is identical in b–d. Corresponding maps of $\sigma_0$ and $\sigma_{\text{B}}$ from the phantom are shown in e and f, respectively.
5. SNR dependence: see Macvoski’s paper

\[
\text{SNR} = \frac{M_0}{\sigma_n} = \frac{\omega_0^2 N_r V_b / \gamma}{2 \sqrt{\rho T}} \left[ \begin{array}{c} 2 \sqrt{\rho} \\ \sqrt{\nu_0 \nu_3 \pi^3} \end{array} \right] \left[ \begin{array}{c} \omega_0 V_b / \sqrt{T} \\ \omega_0 V_b / \sqrt{T} \end{array} \right] \left( \omega_0 V_b / \sqrt{T} \right)^{-1} \]  

[24]

6. When parallel imaging is performed, noise is no longer uniform across image space.