ASYMMETRIC FOURIER DESCRIPTOR OF NON-CLOSED SEGMENTS

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ABSTRACT

The Fourier descriptor is an efficient and effective way to describe a closed boundary. However, for a non-closed segment, since the two non-adjacent end points result in signal discontinuity, after eliminating the high-frequency part, the reconstructed segment has large error near the two ends. In this paper, we propose a warping method to connect the two ends and perform odd-symmetric extension to smooth the warped segment around them. With these modifications, the high-frequency components near the two ends can be much reduced and we can obtain the reconstructed segment with accurate end-point locations even when only the low frequency coefficients are preserved. This method could also be used for a closed boundary with a pre-segmentation process, and the experimental result shows that with the same boundary compression rate, our method has better reconstruction quality than directly extracting Fourier descriptors on the closed boundary.

Index Terms— image edge analysis, image line pattern analysis, image coding, discrete Fourier transforms, data compression

1. INTRODUCTION

In image processing and pattern recognition, edge representation and description play important roles not only for shape feature extraction, but also for efficient recording and reconstruction. Among kinds of existing methods, the Fourier descriptor is widely-used because its rotation, scaling, and translation (RST) invariant property as well as its compressibility: only a few coefficients can broadly reconstruct and describe the closed shape boundary [1], [2]. However, this compressibility may not exist when dealing with a **non-closed segment** because the non-adjacent end points result in signal discontinuity and contain high-frequencies. After eliminating these high-frequency components for edge compression, this segment tends to be closed and results in severe distortion around these two ends.

In this paper, we aim at accurately reconstructing the non-closed segment based on low-frequency coefficients and preserving the original end-point locations. We use

(i) linear offset and **(ii) odd-symmetric extension**

to solve the **non-closed segment problem**. Linear offset can convert a non-closed segment into a closed one and odd-symmetric extension can make the first-order differences at the two ends continuous. These two processes reduce the high-frequency components in the original segment, especially around the two end points. Then, the reconstructed segment won't be severely distorted after only preserving low-frequency coefficients, and the end points of it coincide with those of the original segment accurately.

The proposed improved Fourier descriptor could **also be used for closed boundaries.** Generally, the Fourier description process is directly performed on the whole boundary sequence. While at high compression rate, details such as corners and sharp angles will be destroyed. In our method, we first segment the boundary into several non-closed segments based on these points, and then use the proposed Fourier descriptor to record each segment. The experimental result shows that with this method, the reconstructed boundary will have better quality than with the traditional scheme.

This paper is organized as follows: In Section 2, we briefly review the concept of the Fourier descriptor and several previous works against non-closed segments. In Section 3, we propose our method towards the non-closed segment problem, and several simulation results in Section 4 show that this work definitely solves the problem and outperforms other proposed methods. In Section 5, we implement our method on closed boundaries and compare its performance with the traditional scheme. Conclusions are made in Section 6.

2. OVERVIEW OF FOURIER DESCRIPTOR

The Fourier descriptor is generally composed of two stages: shape signature and discrete Fourier transform (DFT). The first stage is to map the 2-D boundary sequence onto a 1-D function, called a shape signature [3]. Then applying the discrete Fourier transform on this 1-D signature, we can get a set of coefficients to represent the boundary. In order to describe the boundary efficiently, we only preserve low-frequency coefficients. There have been many works on designing the shape-signature stage for closed boundaries. For example, Zahn et al. [4] proposed to use cumulative angular function along the boundary, Granlund [5] used complex coordinate, and Zhang et al. [3] gave a comparative study among several existing methods. However, the non-closed segment problem, which frequently occurs in character recognition as well as edge recording, still remained unsolved. It is due to the discontinuity of the 1-D shape signature at the two ends that results in this problem, or we can say if we could design a continuous or nearly continuous 1-D shape signature even deriving from a non-closed segment, the reconstruction error could be much reduced.

In previous works, Uesaka [6] proposed the use of complex-valued exponential function of the cumulative angular function along the boundary. His method did preserve the end-point locations accurately, while the equal-length sampling process is not suitable for digital images. Nijim [7] extended the non-closed segment at the two ends first and then applied the Fourier descriptor. He claimed that after compression, the distortion will only affects the extended ends and the original end points could be preserved, but the extended length is remained undefined. In [8], Weyland et al. proposed the generalized Fourier descriptor using cumulative angular function, while it has been mentioned that this function has slow convergence speed [3], [6] and is not suitable for boundary compression. In this paper, **we aim at boundary compression and reconstruction rather than retrieval purpose, so the shape signature mapping must be reversible.** We adopt the complex coordinate mapping [5] and further modify the 1-D sequence into a closed and smooth one to solve the non-closed problem.

3. PROPOSED METHOD

Fig. 1 shows the processing procedure in our work. At first, we perform the complex coordinate mapping. Suppose that the coordinates of points on a K-point boundary are (x_k, y_k) (k = 0, 1,..., K−1). Each coordinate pair can be represented as a complex number:

$$s(k) = x(k) + jy(k), \text{ for } k = 0,1,2,...,K-1. \tag{1}$$

That is, x-axis is treated as the real part and y-axis as the imaginary part of a sequence of complex numbers. It is obviously that if the boundary is a non-closed segment, then the complex coordinate sequence will be discontinuous at the two ends. In order to make the sequence continuous at the two ends and smoothen it, we propose a five-step procedure described as below:

Step 1. Record the coordinates of the two end points, (x_0, y_0), (x_{K-1}, y_{K-1}) of the non-closed segment.

Step 2. **Linear Offset:** We shift the boundary points linearly to make the non-closed boundary a closed one. If (x_k, y_k) is a point of the original non-closed segment $s_1(k)$ with K points, for k = 0, 1, ..., K–1, it will be shifted to (x_k', y_k'), where:

$$x_k' = x_k - x_0 - (x_{K-1} - x_0) \times k / (K-1), \tag{2}$$

$$y_k' = y_k - y_0 - (y_{K-1} - y_0) \times k / (K-1). \tag{3}$$

Note that, after Step 2, the non-closed segment becomes a closed curve, called $s_2(k)$, as shown in Fig. 2(b). This step can much reduce the high frequency components caused by the discontinuity at the two ends of $s_1(k)$. Also note that with the recorded end points described in step 1, (x_k, y_k) could be perfectly recovered from (x_k', y_k'). However, after Step 2, there are still some high-frequency components around the two ends because the first-order differences at the two ends are not always continuous.

Step 3. **Odd-Symmetric Extension:** We further add a segment which is odd-symmetric to the original one (see Fig. 2(c)). Then the new segment $s_3(k)$ is closed and will have continuous first-order differences and zero second-order differences at the two ends along the curve. The new segment $s_3(k)$ is of length 2K-2 and defined as:

$$s_3(k) = \begin{cases} s_2(k) & \text{, for } k = 0,1,...,K-1 \\ -s_2(2K-2-k) & \text{, for } k = K,...,2K-3 \end{cases}. \tag{4}$$

Note that adding the odd-symmetric segment does not increase the amount of data since the discrete Fourier transform of an odd input is also an odd sequence. Therefore, we only have to record the first half of Fourier coefficients for perfect reconstruction.

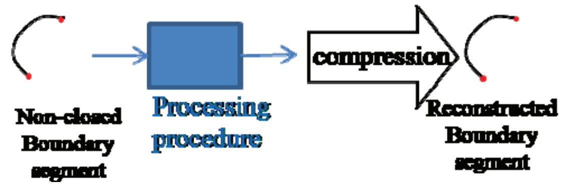


Fig. 1: The flowchart that we follow in this paper to reconstruct a non-closed segment after boundary compression.

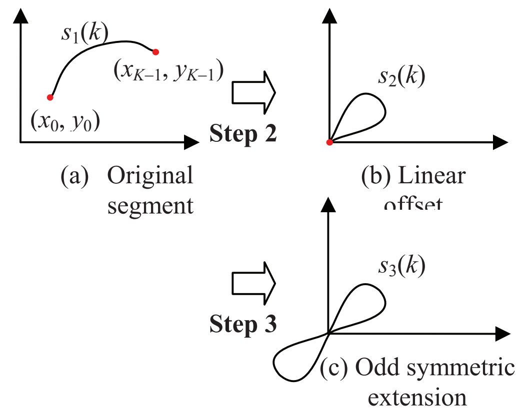


Fig. 2: The process of solving the non-closed segment problem and removing the high-frequency components.

Step 4. Compute the Fourier descriptor of the new curve $s_3(k)$:

$$a(u) = \frac{1}{2K-2} \sum_{k=0}^{2K-3} s_3(k)\, e^{-j2\pi uk/(2K-2)} . \tag{5}$$

Since $s_3(k)$ is odd symmetric, we get $a(u)$ = -a(2K–2–u), and a(0) and a(K-1) = 0. Thus, we only have to record $a(u)$ for u = 1, 2, …., K-2.

Step 5. Boundary Compression: We preserve the first P coefficients (u=1 to u=P, where $0 \leqq P \leqq K\text{-}2$) of $a(u)$ and truncate the other coefficients.

The new segment $s_3(k)$ and its discrete Fourier transform $a(u)$ have several good properties. First, no matter how many coefficients we preserved, the end points of the reconstructed segment will coincide with those of the original segment accurately. This property is shown as:

$$\begin{aligned} s_{3Rec}(0) &= \sum_{u=1}^{P} a(u)\, e^{j2\pi u0/(2K-2)} + \sum_{u=2K-2-P}^{2K-3} a(u)\, e^{j2\pi u0/(2K-2)} \\ &= \sum_{u=1}^{P} a(u) + \sum_{u=2K-2-P}^{2K-3} a(u) = 0, \end{aligned} \tag{6}$$

$$\begin{aligned} s_{3Rec}(K-1) &= \sum_{u=1}^{P} a(u)\, e^{j2\pi u(K-1)/(2K-2)} + \sum_{u=2K-2-P}^{2K-3} a(u)\, e^{j2\pi u(K-1)/(2K-2)} \\ &= \sum_{u=1}^{P} a(u)\, e^{j\pi u} + \sum_{u=2K-2-P}^{2K-3} a(u)\, e^{j\pi u} \\ &= \sum_{u=1}^{P} [a(u) + a(2K-2-u)]\, e^{j\pi u} = 0 \ , \end{aligned} \tag{7}$$

where $s_{3\text{Rec}}(k)$ is the reconstructed segment of $s_3(k)$. The reconstruction process is achieved by first regenerating the $a(u)$ with length 2K-2, and then performing the inverse-DFT to get $s_{3\mathrm{Rec}}(k)$. The original sequence of length K is recovered by eliminating the odd-symmetric extension part of $s_{3\mathrm{Rec}}(k)$ and performing inverse operations of (2) and (3). Second, it is obviously from $s_3(k)$ that the first-order differences around the two ends ($s_3(0)$ and $s_3(K\text{-}1)$) are continuous, and the second-order differences at these points are zero. Since the discontinuous complex-coordinate sequence is made continuous and smoothed in Steps 2 and 3, our algorithm will have much smaller reconstruction error than the original complex coordinate method under the same compression rate.

4. SIMULATION RESULTS

In this section, we perform several simulations to test our method. First, we check that using our method on a non-closed boundary could achieve a slight-distorted reconstruction even with high compression rate. The compression rate R used in our paper is:

$$R = \frac{\text{Number of coefficients preserved}}{\text{Number of points on the processing segment}}, \qquad (8)$$

where the preserved coefficients include end points and the remained coefficients. Fig. 3 is an originally non-closed segment, and Fig. 4 shows the result after linear offset operation, now we can see the warped segment becomes closed. We further perform other steps mentioned in Sec. 3 and keep the compression rate R=0.1. The result shown in Fig. 5 denotes that our method is effective to compress non-closed segments.

We further compare our proposed method with several existing techniques, including the methods in [5], [6], and [7]. The shape signature of modified Uesaka's method is defined as:

$$p(k) = s(k+1) - s(k) \quad , \text{for } k = 0,1,...,K-2, \qquad (9)$$

where $s(k)$ is defined in (1), and the first point of it should be recorded. This modification lifts the equal-length sampling constraint and makes Uesaka's method applicable on digital images. We do not compare with other methods introduced in [3] because those shape signatures are not reversible, which means we can't recover the boundary sequence from those shape signatures. In Fig. 7, we show the comparison of the five methods with R = 0.1, and 0.05 on a 213-point segment in Fig. 6. We can see that only our proposed method and the modified Uesaka's method can definitely preserve the end-point locations, while the proposed method has lower L1 distance between the original segment and the reconstructed segment, which is defined as:

$$L = \sum_{k=0}^{K-1} \{ \ |real(s(k) - s_{\mathrm{Rec}}(k))| + |imag(s(k) - s_{\mathrm{Rec}}(k))| \ \} / K, \quad (10)$$

where $s_{\mathrm{Rec}}(k)$ denotes the reconstructed segment, and K is the number of points on it. Note that the number of points on the original segment is equal to the number of point on the reconstructed segment by Fourier descriptors.

We further test the averaged L1 distance of the above five methods on 21 non-closed segments and their length are 225 pixels in average. The results in Fig. 8 show that the linear offset operation much reduces the reconstruction error, and the reconstruction quality could be further improved by the odd-symmetric extension. Based on the purpose of reducing reconstruction error and the end points preservation, our proposed method outperforms other existing Fourier descriptors.

Fig. 3: The original non-closed segment.

Fig. 4: The closed boundary after linear offset operation.

Fig. 5: Reconstructed boundary using our proposed method with compression rate R = 0.1.

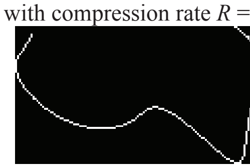


Fig. 6: The original segment used in Fig. 7 with 213 points. The two ends are at (17, 66) and (122, 70).

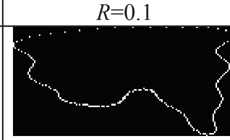
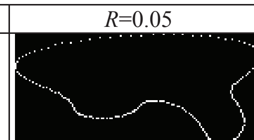
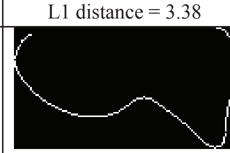
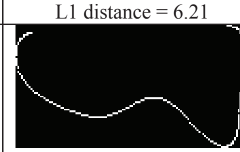
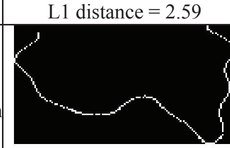
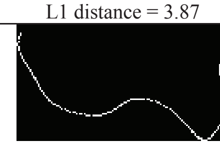
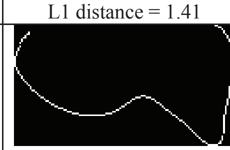
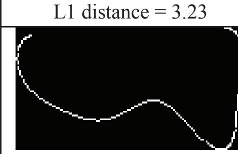
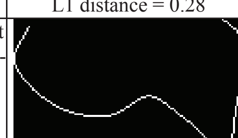
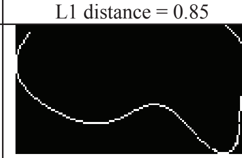
	R=0.1	R=0.05
Granlund's Method [5]	 Ends: (65, 67), (76, 67) L1 distance = 3.38	 Ends: (67, 65), (73, 65) L1 distance = 6.21
Modified Uesaka's Method [6]	 Ends: (17, 66), (122, 70) L1 distance = 2.59	 Ends: (17, 66), (122, 70) L1 distance = 3.87
Nijim's Method [7] with 10% extension on each side	 Ends: (20, 66), (120, 70) L1 distance = 1.41	 Ends: (8, 67), (130, 72) L1 distance = 3.23
Linear offset only **(Proposed)**	 Ends: (17, 65), (123, 69) L1 distance = 0.28	 Ends: (17, 63), (123, 67) L1 distance = 0.85
Linear offset with odd-symmetric extension **(Proposed)**	 Ends: (17, 66), (122, 70) **L1 distance = 0.17**	 Ends: (17, 66), (122, 70) **L1 distance = 0.66**

Fig. 7: The simulation results of five different methods. The reconstructed end points and the reconstruction error calculated by the L1 distance are listed under each simulation.

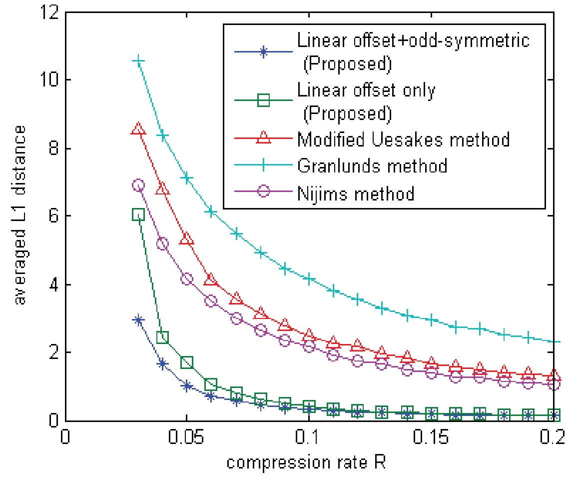


Fig. 8: The averaged L1 distance of the above five different methods on 21 non-closed segments whose length are 225 pixels in average. It shows that our method has the lowest reconstruction error and outperforms other methods.

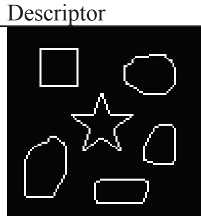
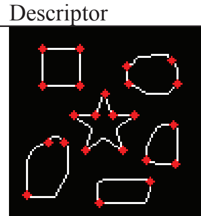
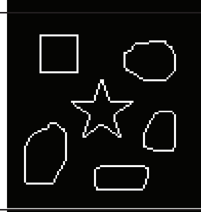
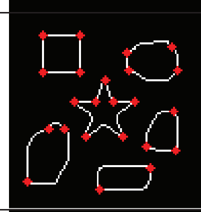
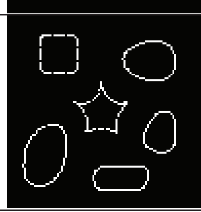
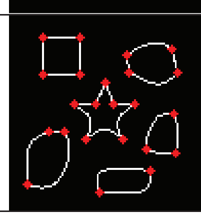
	Original Fourier Descriptor	Proposed Fourier Descriptor
Origin image		
Reconstructed image when compression rate is 20%		
Reconstructed image when compression rate is 10%		

Fig. 9: Comparisons between the traditional Fourier description and the proposed framework for describing closed boundaries. It is obviously that our method can preserve more details at high compression rate.

5. USING FOR CLOSED BOUNDARIES

In addition to non-closed segments, our proposed method could also be used for describing closed boundaries. Since corners and sharp angels usually consist of many high frequency components, after performing the original Fourier description and truncation the high frequency components, the reconstructed boundary will have severe distortion in the corner regions, as the simulations in the 2nd column of Fig. 9.

By contrast, in our work, we first cut the closed boundary at corners and divide it into several non-closed segments. Then we use the proposed algorithm to describe each non-closed segment. In our implementation, we compare the original boundary and the reconstructed one with high compression rate by the traditional Fourier descriptor, and the locations with large distance between these two boundaries are denoted as corners. With the pre-segmentation process, locations of corners are preserved and high-frequency components in each segment could be much reduced, which is advantageous for boundary compression. From the simulations in Fig. 9, our method achieves much better reconstruction qualities than the traditional scheme at the same compression rate.

6. CONCLUSIONS

Most of the existing Fourier descriptors have severe distortion when dealing with non-closed segments. In this paper, we proposed a method composed of "linear offset" and "odd symmetric extension" to solve the problem. With this method, the reconstructed segment won't be severely distorted after only preserving low-frequency coefficients, and the end-point locations of it coincide with those of the original segment accurately. The proposed method could be further used for closed boundaries with a pre-segmentation process and has better reconstruction quality than the traditional scheme.

In addition to used for boundary compression and reconstruction, we also expect that the proposed method could also be used for shape retrieval such as Chinese and Arabic character recognition, and this application is remained for future works.

7. REFERENCES

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