Delay-Limited Cooperative Communication with Reliability Constraints in Wireless Networks

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***Abstract*—We investigate optimal resource allocation for delay-limited cooperative communication in time varying wireless networks. Motivated by real-time applications that have stringent delay constraints, we develop dynamic cooperation strategies that make optimal use of network resources to achieve a target outage probability (reliability) for each user subject to average power constraints. Using the technique of Lyapunov optimization, we first present a general framework to solve this problem and then derive quasi-closed form solutions for several cooperative protocols proposed in the literature.**

I. Introduction

There is growing interest in the idea of utilizing cooperative communication [1]–[4] to improve the performance of wireless networks with time varying channels. The motivation comes from the work on MIMO systems [18] which shows that employing multiple antennas on a wireless node can offer substantial benefits. However, this may be infeasible in small-sized devices due to space limitations. Cooperative communication has been proposed as a means to achieve the benefits of traditional MIMO systems using *distributed single antenna* nodes. Much recent work in this area promises significant gains in several metrics of interest (such as diversity gains [1] [2], capacity [3]–[6], energy efficiency [8] etc.) over conventional methods. We refer the interested reader to a recent comprehensive survey [7] and its references.

The main idea behind cooperative communication can be understood by considering a simple 2-hop network consisting of a source s, its destination d and a set of m relay nodes as shown in Fig. 1. Suppose s has a packet to send to d in timeslot t. The channel gains for all links in this network are shown in the figure. In direct communication, s uses the full slot to transmit its packet to d over link $s-d$. In conventional multi-hop relaying, s uses the first half of the slot to transmit its packet to a particular relay node i over link $s-i$. If i can successfully decode the packet, it re-encodes and transmits it to d in the second half of the slot over link $i-d$. In both scenarios, to ensure reliable communication, the source and/or the relay must transmit at high power levels when the channel quality of any of the links involved is poor. However, note that due to the broadcast nature of wireless transmissions, other relay nodes may receive the signal from the transmission by s and can cooperatively relay it to d. The destination now receives multiple copies/signals and can use all of them jointly to decode the packet. Since these signals have been transmitted over independent paths, the probability that all of them have poor quality is significantly smaller. Cooperative communication protocols take advantage of this *spatial diversity gain* by making use of multiple relays for cooperative transmissions to increase reliability and/or reduce energy costs. This is different from traditional multi-hop relaying in which only one node is responsible for forwarding at any time and in which the destination does not use multiple signals to decode a packet.

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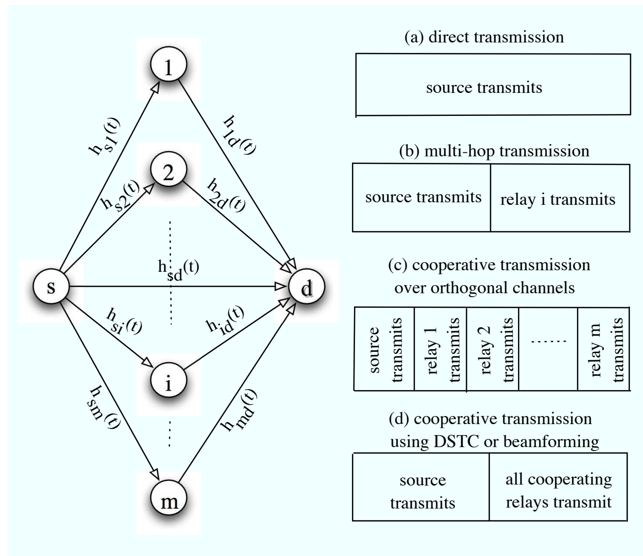

Fig. 1. Example 2-hop network with source, destination and relays. The time slot structures for different transmission strategies are also shown. Due to the half-duplex constraint, cooperative protocols need to operate in two phases. Hence, there is an inherent loss in the multiplexing gain under any such cooperative transmission strategy over direct transmission.

In this work, we consider a mobile ad-hoc network with *delay-limited* traffic and cooperative communication. Many real-time applications (e.g., voice) have stringent delay constraints and fixed/minimum rate requirements. In slow fading environments (where decoding delay is of the order of the channel coherence time), it may not be possible to meet these delay constraints for every packet. However, these applications can often tolerate a certain fraction of lost packets or outages. A variety of techniques are used to combat fading and meet this target outage probability (including exploiting diversity, channel coding, ARQ, power control, etc.). Cooperative communication is a particularly attractive technique to improve reliability in such delay-limited scenarios since it can offer significant spatial diversity gains in addition to these techniques.

Much prior work on cooperative communication considers physical layer resource allocation for a static network, particularly in the case of a single source. Objectives such as minimizing sum power, minimizing outage probability, meeting a target SNR constraint, etc., are treated in this context [8]–[12]. We draw on this work in the development of *dynamic* resource allocation in a stochastic network with fading channels, node mobility, and random packet arrivals, where *opportunistic cooperation decisions* are required. Dynamic cooperation was also considered in the prior work [13], where queue stability in a multi-user network with static channels and randomly arriving traffic is considered using the framework of Lyapunov drift. Our formulation is different and does not involve issues of queue stability. Rather, we consider a delay limited scenario where each packet must either be transmitted in one slot, or dropped. This is similar to the concept of *delay limited capacity* [14]. We use techniques of both Lyapunov drift and Lyapunov Optimization [17]. Different from most work that applies this theory, our solution involves a 2-stage stochastic shortest path problem due to the cooperative relaying structure. This problem is non-convex and combinatorial in nature and does not admit closed form solutions in general. However, under several important and well known classes of physical layer cooperation models, we develop techniques for reducing the problem exactly to an m-stage set of convex programs. The convex programs themselves are shown to have quasi-closed form solutions and can be computed in real time, often involving simple water-filling strategies that also arise in related static optimization problems.

II. Basic Model and Control Objective

We consider a mobile ad-hoc network with delay-limited communication over time varying fading channels. The network contains a set $\mathcal{N}$ of nodes, all potentially mobile. All nodes are assumed to be within range of each other, and any node pair can communicate either through direct transmission or through a 2-phase cooperative transmission that makes use of other nodes as relays. The system operates in slotted time and the channel coefficient between nodes i and j in slot t is denoted by $h_{ij}(t)$. We assume a block fading model [18] for the channel coefficients so that their value remains fixed during a slot and changes from one slot to the other according to the distribution of the underlying fading and mobility process. The collection of all such channel state information in slot t is represented by $\mathcal{T}(t)$. We assume that this lies in a space of finite but arbitrarily large size and evolves according to an ergodic process with a well defined steady state distribution. This variation in channel quality affects the reliability and power expenditure associated with the direct and the cooperative transmission options. The above scenario extends prior work on 2-phase cooperation in static networks to a mobile environment, and treats the important case where a team of nodes move in a tight cluster but with possible variation in the relative locations of nodes within the cluster.

For simplicity, we assume that the set $\mathcal{N}$ contains a single source node s and its destination node d and that all other nodes act simply as cooperative relays. This is similar to the single-source assumption treated in [9]–[12] for static networks. We derive a dynamic cooperation strategy for this single source problem in Sec. III that optimizes a weighted sum of reliability and power expenditure subject to individual reliability and average power constraints at the source and at all relays. In the following, we denote the set of relay nodes by $\mathcal{R}$ and the set $\{s\} \cup \mathcal{R}$ by $\widehat{\mathcal{R}}$. All nodes $i \in \widehat{\mathcal{R}}$ have both long term average and instantaneous peak power constraints given by P_i^{avg} and P_i^{max} respectively.

Suppose the slot size is normalized to integer slots $t \in \{0, 1, 2, \ldots,\}$. In each slot, the source s receives new packets for its destination d according to an i.i.d Bernoulli process $A_s(t)$ of rate λ_s. Each packet is assumed to be R bits long and has a *strict* delay constraint of 1 slot. Thus, a packet not served within 1 slot of its arrival is dropped. Further, packets that are not successfully received by their destinations due to channel errors are not retransmitted. The source node has a minimum time-average reliability requirement specified by a fraction ρ_s which denotes the fraction of packets that were transmitted successfully. In any slot t, if source s has a new packet for transmission, it has one of the following control options as shown in Fig. 1:

1) Transmit directly to d using the full slot
2) Transmit to d using traditional relaying over two hops
3) Transmit cooperatively with the set $\mathcal{R}$ of relay nodes using the two phase slot structure as described before
4) Stay idle (so that the packet gets dropped)

Let $\mathcal{I}^\eta(t)$ denote the collective control action in slot t under some policy η that includes the choice of the control option and the associated power allocations. Let $P_i^\eta(t)$ denote the resulting power allocation for node $i \in \widehat{\mathcal{R}}$. Note that under any feasible policy η, $P_i^\eta(t)$ must satisfy the instantaneous peak power constraint every slot for all i. The success/failure outcome of the control action is represented by an indicator random variable $\Phi_s(\mathcal{I}^\eta(t), \mathcal{T}(t))$ that depends on the current control action and channel state.[1] Specifically, given a control decision $\mathcal{I}^\eta(t)$ and a channel state $\mathcal{T}(t)$, the outcome $\Phi_s^\eta(t) = \Phi_s(\mathcal{I}^\eta(t), \mathcal{T}(t))$ is defined as follows:

$$\Phi_s^\eta(t) = \begin{cases} 1 & \text{if a packet transmitted by } s \text{ in slot } t \\ & \text{is successfully received by } d \\ 0 & \text{else} \end{cases}$$

Let α_s and β_i for $i \in \widehat{\mathcal{R}}$ be a collection of non-negative weights. Then our objective is to design a policy η that solves

[1]Successful transmission of a packet is usually a complicated function of the control option chosen, the associated power allocations and channel states, as well as physical layer details like modulation, coding/decoding scheme being used etc. In this work, the particular physical layer actions are included in the $\mathcal{I}^\eta(t)$ decision variable. Use of the abstract indicator random variable $\Phi_s(\mathcal{I}^\eta(t), \mathcal{T}(t))$ allows a unified treatment that can include a variety of physical layer models. For notational convenience, in the rest of the paper, we use $\Phi_s^\eta(t)$ instead of $\Phi_s(\mathcal{I}^\eta(t), \mathcal{T}(t))$ noting that the dependence on $(\mathcal{I}^\eta(t), \mathcal{T}(t))$ is implicit.

the following *stochastic optimization problem*:

$$\begin{aligned}
\text{Maximize:} \quad & \alpha_s \bar{r}_s^{\eta} - \sum_{i \in \widehat{\mathcal{R}}} \beta_i \bar{e}_i^{\eta} \\
\text{Subject to:} \quad & \bar{r}_s^{\eta} \geq \rho_s \lambda_s \\
& \bar{e}_i^{\eta} \leq P_i^{avg} \ \forall \ i \in \widehat{\mathcal{R}} \\
& 0 \leq P_i^{\eta}(t) \leq P_i^{max} \ \forall \ i \in \widehat{\mathcal{R}}, \ \forall t \qquad (1)
\end{aligned}$$

where $\bar{r}_s^{\eta}$ is the time average reliability for source s under policy η and is defined as:

$$\bar{r}_s^{\eta} \triangleq \lim_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\left\{\Phi_s^{\eta}(\tau)\right\} \qquad (2)$$

and $\bar{e}_i^{\eta}$ is the time average power usage of node i under η:

$$\bar{e}_i^{\eta} \triangleq \lim_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\left\{P_i^{\eta}(\tau)\right\} \qquad (3)$$

Here, the expectation is with respect to the possibly randomized control actions that policy η might take. The α_s and β_i weights allow us to consider several different objectives.

Problem (1) is similar to the general stochastic utility maximization problem presented in [17]. Suppose (1) is feasible and let r_s^* and $e_i^* \ \ \forall i \in \widehat{\mathcal{R}}$ denote the optimal value of the objective function, potentially achieved by some arbitrary policy. Using the techniques developed in [16], [17], it can be shown that it is sufficient to consider only the class of stationary, randomized policies that take control decisions purely as a (possibly random) function of the channel state $\mathcal{T}(t)$ every slot to solve (1). However, computing the optimal stationary, randomized policy explicitly can be challenging and often impractical as it requires knowledge of arrival and channel probabilities in advance. Further, even in the special case of a static channel (i.e., no mobility), the optimal strategy may involve a mixture of direct transmission, multi-hop, and cooperative modes of operation, and the relaying modes must select different relay sets over time to achieve the optimal time average mixture. In the next section, we present an online algorithm that overcomes these challenges.

III. Optimal Control Algorithm

In this section, we present a dynamic control algorithm that achieves the optimal solution r_s^* and $e_i^* \ \ \forall i \in \widehat{\mathcal{R}}$ to the stochastic optimization problem presented earlier. This algorithm is similar in spirit to the backpressure algorithms proposed in [16], [17] for problems of throughput and energy optimal networking in time varying wireless ad-hoc networks.

The algorithm makes use of the following "reliability" queue for source s:

$$Z_s(t+1) = \max[Z_s(t) - \Phi_s(t), 0] + \rho_s A_s(t) \qquad (4)$$

Additionally, it also uses the following virtual power queues $\forall i \in \widehat{\mathcal{R}}$:

$$X_i(t+1) = \max[X_i(t) - P_i^{avg}, 0] + P_i(t) \qquad (5)$$

All these queues are initialized to 0. We note that these queues are virtual in that they do not represent any real backlog of data packets. Rather, they facilitate the control algorithm in achieving the time average reliability and energy constraints of (1) as follows. If a policy η stabilizes (4), then we must have that its service rate is no smaller than the input rate, i.e.,

$$\bar{r}_s^{\eta} = \lim_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\left\{\Phi_s^{\eta}(\tau)\right\} \geq \lim_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\left\{\rho_s A_s(\tau)\right\} = \rho_s \lambda_s$$

Similarly, stabilizing (5) yields the following:

$$\bar{e}_i^{\eta} = \lim_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\left\{P_i^{\eta}(\tau)\right\} \leq P_i^{avg}$$

where we used definitions (2), (3). This technique of turning time-average constraints into queueing stability problems was first used in [16]. To stabilize these virtual queues and optimize the objective function in (1), the algorithm operates as follows. Let $Q(t)$ denote the collection of these queues in timeslot t. Every slot, given $Q(t)$ and any channel state $\mathcal{T}(t)$, it chooses a control action $\mathcal{I}_s(t)$ that minimizes the following stochastic metric every slot (for a given control parameter $V \geq 0$):

$$\begin{aligned}
\text{Minimize:} \quad & (X_s(t) + V\beta_s)\mathbb{E}\left\{P_s(t)|Q(t), \mathcal{T}(t)\right\} \\
& + \sum_{i \in \mathcal{R}} (X_i(t) + V\beta_i)\mathbb{E}\left\{P_i(t)|Q(t), \mathcal{T}(t)\right\} \\
& - (Z_s(t) + V\alpha_s)\mathbb{E}\left\{\Phi_s(t)|Q(t), \mathcal{T}(t)\right\} \\
\text{Subject to:} \quad & 0 \leq P_i(t) \leq P_i^{max} \ \forall i \in \widehat{\mathcal{R}} \qquad (6)
\end{aligned}$$

The above optimization is a 2-stage *stochastic shortest path* problem [19] where the two stages correspond to the two phases of the underlying cooperative protocol. Specifically, when s decides to use the option of transmitting cooperatively, the cost incurred in the first stage is given by the first term $(X_s(t) + V\beta_s)\mathbb{E}\left\{P_s(t)|Q(t), \mathcal{T}(t)\right\}$. The cost incurred during the second stage is given by $\sum_{i \in \mathcal{R}} (X_i(t) + V\beta_i)\mathbb{E}\left\{P_i(t)|Q(t), \mathcal{T}(t)\right\}$ and at the end of this stage, we get a reward of $(Z_s(t) + V\alpha_s)\mathbb{E}\left\{\Phi_s(t)|Q(t), \mathcal{T}(t)\right\}$. The transmission outcome $\Phi_s(t)$ depends on the power allocation decisions in *both* phases which makes this problem different from greedy strategies (e.g., [13], [16]). Note that this problem is unconstrained since the long term time average reliability and power constraints do not appear explicitly as in the original problem. These are implicitly captured by the virtual queue values. Further, solving (6) does not require knowledge of the system parameters (like channel statistics, mobility patterns etc.). Thus, the control strategy involves implementing the solution to a sequence of such unconstrained problems every slot and updating the queue values according to (4), (5).

Assuming i.i.d $\mathcal{T}(t)$ states, the following theorem characterizes the performance of this dynamic control algorithm.[2]

Theorem 1: (Algorithm Performance) Suppose all queues are initialized to 0. Then, implementing the dynamic algorithm (6) every slot stabilizes all queues, thereby satisfying the minimum reliability and time-average power constraints, and guarantees the following performance bounds (for some $\epsilon > 0$

[2]A similar statement can be made for more general Markov modulated $\mathcal{T}(t)$ [17]. For simplicity, here we consider the i.i.d case.

that depends on the slackness of the feasibility constraints):

$$\lim_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t-1}\mathbb{E}\left\{Z_s(\tau)\right\} \leq \frac{B+V(\alpha_s+\sum_{i\in\widehat{\mathcal{R}}}\beta_i P_i^{max})}{\epsilon}$$

$$\lim_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t-1}\sum_{i\in\widehat{\mathcal{R}}}\mathbb{E}\left\{X_i(\tau)\right\} \leq \frac{B+V(\alpha_s+\sum_{i\in\widehat{\mathcal{R}}}\beta_i P_i^{max})}{\epsilon}$$

Further, the time average utility achieved for any $V \geq 0$ satisfies:

$$\lim_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t-1}\mathbb{E}\left\{\alpha_s\Phi_s(\tau) - \sum_{i\in\widehat{\mathcal{R}}}\beta_i P_i(\tau)\right\} \geq \alpha_s r_s^* - \sum_{i\in\widehat{\mathcal{R}}}\beta_i e_i^* - \frac{B}{V}$$

where $B \triangleq \frac{1+\lambda_s^2\rho_s^2+\sum_{i\in\widehat{\mathcal{R}}}(P_i^{avg})^2+(P_i^{max})^2}{2}$.

Proof: Omitted for brevity. □

IV. 2-Stage Resource Allocation Problem

In this section, we present solutions to the basic 2-stage resource allocation problem (6) for two widely studied co-operative protocols proposed in the literature: Decode-and-Forward (DF) and Amplify-and-Forward (AF) [1], [2]. These protocols differ in the way the transmitted signal from the first phase is processed by the cooperating relays. In DF, a relay fully decodes the signal. If the packet is received correctly, it is re-encoded and transmitted in the second phase. In AF, a relay simply retransmits a scaled version of the received analog signal. We refer to [1] [2] for further details on the working of these protocols as well as derivation of expressions for the mutual information achieved by them.

Let $m = |\mathcal{R}|$. In the following, we assume a Gaussian channel model with a total bandwidth W and unit noise power per dimension. We use the information theoretic definition of a transmission failure (an outage event) as discussed in [14], [15]. Here, an outage occurs when the total instantaneous mutual information is smaller than the rate R at which data is being transmitted. For brevity, we only consider the case when all channel gains are known to s.[3] In this scenario, (6) becomes a *deterministic* shortest path problem because the outcome $\Phi_s(t)$ due to any control decision and its power allocation can be computed beforehand. Specifically, $\Phi_s(t) = 1$ when the resulting total mutual information exceeds R and $\Phi_s(t) = 0$ otherwise. In order to determine the optimal solution, we must choose between the four modes of operation as discussed in Sec. II. Let c_i and I_i denote the optimal cost of the metric (6), and the corresponding action that achieves that metric, assuming that mode i in $\{1, 2, 3, 4\}$ is chosen. The algorithm computes c_i and I_i for each mode, then chooses the mode i (and the resulting action I_i) that minimizes cost. Note that the cost c_1 for the idle mode is trivially 0. The cost of modes 2 and 3 are also easy to compute. Below we concentrate on computing the cost for the cooperative mode. In what follows, we drop the time subscript (t) for notational convenience.

[3]The case where only statistics of the channel gains are known can also be treated in our framework.

A. Example 1: Regenerative DF, Orthogonal Channels

Here, the source and relays are each assigned an orthogonal channel of equal size. An example slot structure is shown in Fig. 1(c) in which the entire slot is divided into $m+1$ equal mini-slots. In the first phase of the protocol, s transmits the packet in its slot using power P_s. In the second phase, a subset $\mathcal{U} \subset \mathcal{R}$ of relays that were successful in reliably decoding the packet, re-encode it using the *same* code book and transmit to the destination on their channels with power P_i (where $i \in \mathcal{U}$). Given such a set $\mathcal{U}$, the total mutual information $\mathcal{I}_{tot}$ under this protocol is given by [1]:

$$\mathcal{I}_{tot} = \frac{W}{m}\log\left(1+\frac{mP_s}{W}|h_{sd}|^2+\sum_{i\in\mathcal{U}}\frac{mP_i}{W}|h_{id}|^2\right)$$

Define binary variables x_i to be 1 if relay i can reliably decode the packet after the first stage and 0 else. Then, for this protocol, (6) is equivalent to the following optimization problem:

$$\text{Minimize: } (X_s+V\beta_s)P_s+\sum_{i\in\mathcal{R}}(X_i+V\beta_i)P_i - Z_s - V\alpha_s$$

Subject to:

$$\frac{W}{m}\log\left(1+\frac{mP_s}{W}|h_{sd}|^2+\sum_{i\in\mathcal{R}}x_i\frac{mP_i}{W}|h_{id}|^2\right) \geq R$$
$$\frac{W}{m}\log\left(1+\frac{mP_s}{W}|h_{si}|^2\right) \geq x_i R$$
$$0 \leq P_s \leq P_s^{max}, 0 \leq P_i \leq P_i^{max}, x_i \in \{0,1\}\ \forall i \in \mathcal{R} \tag{7}$$

The variables x_i capture the requirement that a relay can cooperatively transmit in the second stage only if it was successful in reliably decoding the packet using the first stage transmission. A similar setup is considered in [9] but it treats the limiting case when W goes to infinity. Because of the integer constraints on x_i, (7) is non-convex. However, we can exploit the structure of this protocol to reduce the above to a set of $m+1$ subproblems as follows. We first order the relays in decreasing order of their $|h_{si}|^2$ values. Define $\mathcal{U}_k$ as the set that contains the first k (where $0 \leq k \leq m$) relays from this ordering. Let $P_s^{\mathcal{U}_k}$ denote the minimum source power required to ensure that all relays in $\mathcal{U}_k$ can reliably decode the packet after the first stage. We note that for all values of P_s in the range $(P_s^{\mathcal{U}_k}, P_s^{\mathcal{U}_{k+1}})$, the relay set that can reliably decode remains the same, i.e., $\mathcal{U}_k$. Thus, we need to consider only $m+1$ subproblems, one for each $\mathcal{U}_k$. It turns out that the subproblem for any set $\mathcal{U}_k$ can be expressed as an LP:

$$\text{Minimize: } (X_s+V\beta_s)P_s+\sum_{i\in\mathcal{U}_k}(X_i+V\beta_i)P_i - Z_s - V\alpha_s$$

$$\text{Subject to: } P_s|h_{sd}|^2+\sum_{i\in\mathcal{U}_k}P_i|h_{id}|^2 \geq \alpha$$
$$P_s^{\mathcal{U}_k} \leq P_s \leq P_s^{max}$$
$$0 \leq P_i \leq P_i^{max} \quad \forall i \in \mathcal{U}_k \tag{8}$$

where $\alpha = \frac{W}{m}(2^{Rm/W}-1)$. The solution to the above has a greedy structure where we start by allocating increasing power to the nodes (including s) in decreasing order of the value of $\frac{|h_{id}|^2}{(X_i+V\beta_i)}$ where $i \in \mathcal{U}_k \cup \{s\}$ until any constraint is met.

Therefore, for this protocol, the optimal solution to (6) can be computed by solving (8) for each $\mathcal{U}_k$ and picking the one with the least cost.

B. Example 2: AF, Orthogonal Channels

In this protocol, the source and relays are again assigned an orthogonal channel of equal size. An example slot structure is shown in Fig.1(c). However, instead of decoding the packet, the relays amplify and forward the received signal from the first stage. The total mutual information under this protocol is given by [10] [11]:

$$\mathcal{I}_{tot} = \frac{W}{m} \log \left(1 + \frac{mP_s}{W} \left[|h_{sd}|^2 + \sum_{i \in \mathcal{R}} \psi_i \right] \right)$$

where $\psi_i \triangleq \frac{P_i |h_{si}|^2 |h_{id}|^2}{P_s |h_{si}|^2 + P_i |h_{id}|^2 + W/m}$. Using this, we can express (6) for this model as follows.

$$\begin{aligned} \text{Minimize: } & (X_s + V\beta_s)P_s + \sum_{i \in \mathcal{R}} (X_i + V\beta_i)P_i - Z_s - V\alpha_s \\ \text{Subject to: } & \frac{W}{m} \log \left(1 + \frac{mP_s}{W} \left[|h_{sd}|^2 + \sum_{i \in \mathcal{R}} \psi_i \right] \right) \geq R \\ & 0 \leq P_s \leq P_s^{max}, 0 \leq P_i \leq P_i^{max} \; \forall i \in \mathcal{R} \qquad (9) \end{aligned}$$

This problem is non-convex. However, if we fix the source power P_s, then it becomes convex in the other variables. This reduction has been used in [11] as well, although it considers a static scenario with the objective of minimizing instantaneous outage probability. After fixing P_s, we can compute the optimal relay powers for this value of P_s by solving the following:

$$\begin{aligned} \text{Minimize: } \quad & \sum_{i \in \mathcal{R}} (X_i + V\beta_i)P_i - Z_s - V\alpha_s \\ \text{Subject to: } \quad & P_s |h_{sd}|^2 + \sum_{i \in \mathcal{R}} P_s \psi_i \geq \alpha \\ & 0 \leq P_i \leq P_i^{max} \qquad \forall i \in \mathcal{R} \qquad (10) \end{aligned}$$

where $\alpha = \frac{W}{m}(2^{Rm/W} - 1)$. The constraint can be simplified as:

$$\begin{aligned} P_s |h_{sd}|^2 + \sum_{i \in \mathcal{R}} P_s \psi_i &= P_s (|h_{sd}|^2 + \sum_{i \in \mathcal{R}} |h_{si}|^2) \\ & - \sum_{i \in \mathcal{R}} \frac{P_s^2 |h_{si}|^4 + P_s |h_{si}|^2 W/m}{P_s |h_{si}|^2 + P_i |h_{id}|^2 + W/m} \end{aligned}$$

Since we have fixed P_s, we can express (10) as:

$$\begin{aligned} \text{Minimize: } \quad & \sum_{i \in \mathcal{R}} (X_i + V\beta_i)P_i - Z_s - V\alpha_s \\ \text{Subject to: } \quad & \sum_{i \in \mathcal{R}} \frac{P_s^2 |h_{si}|^4 + P_s |h_{si}|^2 W/m}{P_s |h_{si}|^2 + P_i |h_{id}|^2 + W/m} \leq \alpha' \\ & 0 \leq P_i \leq P_i^{max} \qquad \forall i \in \mathcal{R} \qquad (11) \end{aligned}$$

where $\alpha' = P_s(|h_{sd}|^2 + \sum_{i \in \mathcal{R}_s} |h_{si}|^2) - \alpha$. Using the KKT [20] conditions, the solution the above convex optimization problem is given by:

$$P_i^* = \left[\sqrt{\frac{\nu^* (P_s^2 |h_{si}|^4 + P_s |h_{si}|^2 W/m)}{(X_i + V\beta_i)|h_{id}|^2}} - \frac{P_s |h_{si}|^2 + W/m}{|h_{id}|^2} \right]_0^{P_i^{max}}$$

where $\nu^* \geq 0$ is chosen so that the second constraint is met with equality. We note that this solution has a water-filling type structure as well. Therefore, to compute the optimal solution to (6) for this protocol, we would have to solve the above for each value of $P_s \in [0, P_s^{max}]$. In practice, this computation could be simplified by considering only a discrete set of options for P_s.

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