Numerical Methods

GOALS

The primary goal of the course is to equip students with the numerical tools necessary to tackle interesting questions in macroeconomics. The course has two main focuses. The first is the study of numerical methods and algorithms pertinent to solving and analyzing macro models. The second is the study of examples of their application by macroeconomists. While this is not a computer programming course, the course work will be computational in nature. Students should be familiar with some programming language and all the course work can be completed with Matlab.

One pervasive aspect of economic modeling is its essential nonlinearity, sometimes made even less amenable by the presence of uncertainty. In the absence of closed form solution, most economic problems have to be solved using numerical schemes. In other words, many interesting economic models cannot be solved analytically using the standard mathematical techniques of algebra and calculus. Models that cannot be solved in this way are often applied economic models that attempt to capture the complexities inherent in real-world economic behavior.

Computational techniques are not just about quantifying the qualitative results of theoretical models. In many problems we have no alternative but to rely on the computer for any solution. For such problems existing economic theory only provides the building blocks and one must generate the corresponding virtual economies to understand the complete implications of the theory. In these cases numerical solutions may not lead to the mere calibration of analytical results but instead actually suggest analytical theorems to researchers.

In the first part we will learn the basic numerical methods, including linear and nonlinear equation methods, complementarity methods, numerical integration and differentiation, and function approximation. In these chapters we will develop appreciation for basic numerical techniques by illustrating their application to equilibrium and optimization models familiar to most economists.

In recent years, a number of numerical methods have been proposed in the economic literature. The second part of the course is devoted to these methods for solving dynamic general equilibrium models.

EVALUATION

Your final grade in the class will be determined as the following weighted average of your work throughout the semester

Problem Sets	60%
Final Exam	40%

Steps in an Economic Computational Experiment (Kydland-Prescott)

Step 1: Pose a Question

Computational experiments are designed for questions of a quantitative nature. Thus the first step in conducting such an experiment is to have a well-designed quantitative question.

- What is/are the quantitative question(s) that the study seeks to answer?

Step 2: Use Well-Tested Theory

The next step is to choose a theory to guide in the development of a model. The theory can be thought of as "an explicit set of instructions for building...a mechanical imitation system."

- What theory does the study use to develop its computational experiment?
- Is the theory well-tested?
- Is this theory well-suited to address the question(s) at hand?

Step 3: Construct a Model Economy

Keeping the question in mind, the next step is to use the economic theory to guide in the construction of a model economy. There are two issues to focus on when writing down the model economy. First one has to determine how much detail to include. The second major issue to consider is the feasibility of conducting the computational experiment given the particular setup.

- What are the key features of the model economy?
- Why is each feature present?
- Does the inclusion of the feature help the researcher in addressing his question and, if yes, how?
- Are there any important or potentially important features that the researcher abstracts from?
- Why are these features not included? If they were included would the model still be computationally tractable?
- Is the model well-suited for addressing the question(s) at hand?

Step 4: Calibrate the Model Economy

Calibrating means choosing a set of parameter values so that the model is consistent with the real world along a limited, but clearly specified, number of dimensions.

- What is the researcher's calibration strategy? Along what dimensions do they make the model consistent with the real world?
- How do they obtain real world measurements? Do they use the right measurements?

Step 5: Run the Experiment

The final step in conducting computational experiments consists in using the instrument (a computer program that generates equilibrium realizations of our calibrated model economy) that we have created to answer the quantitative question(s) posed

- What algorithm is used to compute the model? Why was it chosen?
- How do the researchers use the calibrated model economy to answer their question?
- What are the results?

USEFUL TEXTBOOKS

Acemoglu, Daron. *Introduction to modern economic growth*. Cambridge, Princeton University Press, 2009.

Heer, Burkhard and Maussner, Alfred. *Dynamic general equilibrium modeling: computational methods and applications*. Berlin; New York, NY: Springer, 2005.

Judd, Kenneth L. *Numerical methods in economics.* Cambridge, MA: MIT Press, 1998.

Ljungqvist, Lars and Sargent, Thomas J. *Recursive Macroeconomic Theory*. Cambridge, MA: MIT Press, 2004.

Stokey, Nancy L.; Lucas, Robert E., Jr. and Prescott, Edward C. *Recursive Methods in Economic Dynamics*. Cambridge, MA: Harvard University Press, 1989.

Some additional books that will prove useful are:

Adda, Jérôme and Cooper, Russell. *Dynamic economics: Quantitative methods and applications*. Cambridge, Massachusetts: The MIT Press, 2003

Cooley, Thomas F. (Eds.). "*Frontiers of Business Cycle Research*." Princeton University Press, 1995.

Marimon, Ramon and Scott, Andrew (Eds.). "*Computational Methods for the Study of Dynamic Economies*." Oxford: Oxford University Press, 1999.

Miranda, Mario J. and Fackler, Paul L. *Applied computational economics and finance*. Cambridge, Massachusetts: The MIT Press, 2002.

Press, William H.; Teukolsky, Saul A.; Vetterling, William T. and Flannery, Brian P. (Eds.) *Numerical Recipes in C.* New York, N.Y.: Press Syndicate of the University of Cambridge, 1992.

Ralston, Anthony and Rabinowitz, Philip. *A first course in numerical analysis*. Mineola, New York: Dover Publications, Inc. 1978 (2nd edition).

COURSE OUTLINE

I Numerical Analysis

Heer and Maussner, Chapter 8

Judd, Chapter 5

Nonlinear Equations

Bisection
Broyden's Method
Fixed-Point Iteration
Gauss-Jacobi Algorithm
Gauss-Seidel Algorithm
Newton's Method
Secant Method

Numerical Integration and Differentiation

Judd, Chapter 7

Newton-Cotes Formulas
Gaussian Formulas
Singular Integrals
Adaptive Quadrature
Numerical Differentiation

II Solving the Stochastic Growth Model

Markov Chains

Heer and Maussner, Chapter 9

Ljungqvist and Sargent, Chapter 2

Stokey and Lucas, Chapter 8

Flodén, Martin. "A Note on the Accuracy of Markov-Chain Approximations to Highly Persistent AR (1) Processes." *Economics Letters,* June 2008, 99(3), pp. 516-520.
Matlab code

Tauchen, George. "Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions." *Economics Letters,* 1986, 20(2), pp. 177-181.

Tauchen, George and Hussey, Robert. "Quadrature-Based Methods for Obtaining Approximate Solutions to Nonlinear Asset Pricing Models." *Econometrica,* March 1991, 59(2), pp. 371-396.

Value-Function Iteration

Ljungqvist and Sargent, Chapter 3, 4

Christiano, Lawrence J. "Solving the Stochastic Growth Model by Linear-Quadratic Approximation and by Value-Function Iteration." *Journal of Business & Economic Statistics,* January 1990, 8(1), pp. 23-26.

Coleman, John Wilbur. "Solving the Stochastic Growth Model by Policy-Function Iteration." *Journal of Business & Economic Statistics,* January 1990, 8(1), pp. 27-29.

Tauchen, George. "Solving the Stochastic Growth Model by Using Quadrature Methods and Value-Function Iterations." *Journal of Business & Economic Statistics,* January 1990, 8(1), pp. 49-51.

Projection Methods

Heer and Maussner, Chapter 4

Judd, Chapter 11

Judd, Kenneth L. "Projection Methods for Solving Aggregate Growth Models." *Journal of Economic Theory,* December 1992, 58(2), pp. 410-452.

Parameterized Expectations

Den Haan, Wouter J. and Marcet, Albert. "Solving the Stochastic Growth Model by Parameterizing Expectations." *Journal of Business & Economic Statistics,* January 1990, 8(1), pp. 31-34.

Marcet, Albert and Lorenzoni, Guido. "The Parameterized Expectations Approach: Some Practical Issues." In Ramon Marimon and Andrew Scott eds., *Computational Methods for the Study of Dynamic Economies,* Oxford University Press, 1999, pp. 143-171.

Comparing Solution Methods

Aruoba, S. Borağan; Fernández-Villaverde, Jesús and Rubio-Ramírez, Juan F. "Comparing Solution Methods for Dynamic Equilibrium Economies." *Journal of Economic Dynamics and Control,* December 2006, 30(12), pp. 2477-2508.

Taylor, John B. and Uhlig, Harald. "Solving Nonlinear Stochastic Growth Models: A Comparison of Alternative Solution Methods." *Journal of Business & Economic Statistics,* January 1990, 8(1), pp. 1-17.

III Heterogeneous Agent Models

There are many questions in economics for which heterogeneous-agent dynamic models have to be used to provide answers. Examples of these questions where the desired answer is quantitative are as follows:

- What changes in the distribution of wealth will occur if the tax system is changed from progressive to proportional?
- What increases in taxation are needed to maintain the current level of US social security benefits under current population patterns?
- What type of policy changes can be expected from changes in constitutions?

All these questions require models where the households that populate are not identical. With respect to the first question, note that the key property of progressivity of the tax system is that different households face different tax rates. For the second question, the age distribution of the population determines the amounts collected and paid by the administrators of social security. Finally, the determinants of policy should be affected by the relations between different groups of households that do not have the same preferences over policies.

Computation of equilibria in these models is usually substantially more difficult than in standard representative agent models, as equilibrium laws of motion become functions not only of aggregate variables, but also of the distribution of these variables across different types of agents. Solving for the laws of motion of such distributions is a nontrivial task.

Computation of Stationary Distributions

Heer and Maussner, Chapter 5

Aiyagari, S. Rao. "Uninsured Idiosyncratic Risk and Aggregate Saving." *Quarterly Journal of Economics,* August 1994, 109(3), pp. 659-84.

Huggett, Mark. "The Risk-Free Rate in Heterogeneous-Agent Incomplete-Insurance Economies." *Journal of Economic Dynamics and Control,* September-November 1993, 17(5-6), pp. 953-69.

Dynamics of the Distribution Function

Heer and Maussner, Chapter 6

Krusell, Per and Smith, Anthony A., Jr. "Income and Wealth Heterogeneity in the Macroeconomy." *Journal of Political Economy,* October 1998, 106(5), pp. 867-96.

Ríos-Rull, José-Víctor. "Computation of Equilibria in Heterogeneous-Agent Models." In Ramon Marimon and Andrew Scott eds., *Computational Methods for the Study of Dynamic Economies*, Oxford University Press, 1999, pp. 238-264.

IV Computational Experiments in Macroeconomics

Methodology

The researcher constructs the model economy. A quantitative answer is found for the proposed question to the model economy. The results are compared to the real world data. When the results aren't accurate, measurements must be redone and further testing follows. The theories used in the setup of the model may not match the proposed question. The model then needs to be reconstructed. The instructions of the theory used to build the model are tested to see if they are realistic to the actual data. The predictions of the model for future results are tested to the actual data. If the predictions match, further testing is done by including realistic scenarios for the data that have not been observed before. These new conditions can induce further development of the theory.

Hansen, Lars Peter and Heckman, James C. "The Empirical Foundations of Calibration." *Journal of Economic Perspectives,* Winter 1996, 10(1), pp. 87-104.

Kydland, Finn E. and Prescott, Edward C. "The Econometrics of the General Equilibrium Approach to Business Cycles." *Scandinavian Journal of Economics,* June 1991, 93(2), pp. 161-178.

Kydland, Finn E. and Prescott, Edward C. "The Computational Experiment: An Econometric Tool." *Journal of Economic Perspectives,* Winter 1996, 10(1), pp. 69-85.

Sims, Christopher A. "Macroeconomics and Methodology." *Journal of Economic Perspectives,* Winter 1996, 10(1), pp. 105-120.

Calibration

Calibration is a strategy for finding numerical values for the parameters of artificial economic worlds. Some of the parameter values are chosen based on observed features of actual economies. The others are determined based on the theory.

"Economic Growth and Business Cycles", **Cooley and Prescott**

Gomme, Paul and Rupert, Peter. "Theory, Measurement and Calibration of Macroeconomic Models." *Journal of Monetary Economics,* March 2007, 54(2), pp. 460-497.

Prescott, Edward C. "Theory Ahead of Business-Cycle Measurement." *Carnegie-Rochester Conference Series on Public Policy,* Autumn 1986, 25, pp. 11-44.