Power Allocation in Linear and Tree WSN Topologies

(Invited Paper)

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***Abstract*— Estimation at a fusion center in a wireless sensor network is examined. The problem at hand is to perform power allocation subject to a total network power constraint while minimizing the mean-squared error of the estimate. In particular, amplify-and-forward and estimate-and-forward protocols are considered in linear and tree topologies. Analytical solutions for these cases appear to be intractable, and thus asymptotically optimal (for increasing measurement noise variance) solutions are derived. The optimal limiting power policy for the leaf nodes in branch and tree topologies is power equalization, whereas in linear networks, the optimal solution is weighted power equalization.**

I. Introduction

Wireless sensor networks (WSN) consist of small-scale nodes that cooperatively monitor physical and environmental conditions. In contrast to classical networks, WSNs are limited in energy, bandwidth and computational complexity, but are deployed in very high densities. Due to WSN applications, there has been much research in distributed estimation and detection. Scalar parameter estimation [1]-[5] and spatial field estimation [6] minimize the mean-squared error, for example.

We investigate optimal power allocation given a total network transmit power constraint to minimize the mean-squared error (MSE) for a parameter estimation problem. Optimizing the communication power is motivated by [4] which states that significantly more power (70%) is used by a node for transmission than for sensing and signal processing. A similar problem (focusing on gain allocation, instead of true power allocation) and its "dual" (power minimization to meet an MSE criterion) were considered in [2] and [3] for the simple star topology.

We focus on a WSN topology wherein a fusion center receives noisy measurements from geographically distributed sensors as in Figure 1. All sensors make measurements, and some also relay information to the fusion center, which determines the optimal power allocation strategy and conveys it, distortion-free, to the respective nodes. Multi-hop networks necessitate defining information passing protocols, so we consider the amplify-and-forward (AF) and estimate-and-forward (EF) protocols. Determining an analytical optimal power allocation scheme for an arbitrary topology appears intractable. Thus for more complex branch and linear topologies, asymptotically optimal solutions (for increasing measurement noise variance) are derived for both AF and EF protocols. We highlight the trade-offs between measurement noise and channel quality. The optimal power policy evolves as a function of both these factors; for example, for low measurement noise, sensor/branch selection is optimal, but all sensors must remain active as the measurement noise increases.

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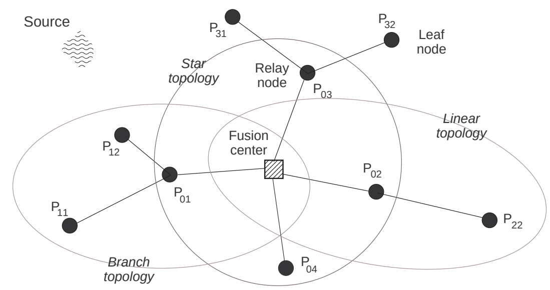

Fig. 1. WSN tree topology, and subset star, branch and linear topologies.

The remainder of this paper is organized as follows. Section II defines the signal models and transmission protocols used throughout. The optimal power allocation policy and the asymptotic analysis for the star topology are presented in Section III. Section IV develops the optimization and its asymptotic solution for the branch topology using AF and EF protocols. The branch topology is extended to the tree topology in Section V, and Section VI develops the linear topology case. Numerical results comparing different topologies are presented in Section VII and Section VIII draws conclusions.

II. System and Network Models

A. System Model

The simple star topology, with sensors directly connected to the fusion center, is shown in Figure 1. Nodes send a noisy measurement of the scalar parameter θ to the fusion center, and the received signal from the i^{th} node is given by

$$y_i = \sqrt{P_i} h_i \beta(z_i)(\theta + z_i) + n_i \ , \quad i = 1, \ldots, N, \qquad (1)$$

where P_i and h_i are the transmission power allocated and non-random channel attenuation coefficient for node i, respectively. The noise terms z_i and n_i represent the zero-mean measurement and channel noise terms with unknown PDFs and variances $\sigma^2_{z_i}$ and $\sigma^2_{n_i}$, respectively. We define the normalization factors

$$\beta(n_i) = \frac{1}{\sqrt{\sigma_\theta^2 + \sigma_{n_i}^2}} \quad \text{and} \quad \gamma(P_i) = \frac{1}{\sqrt{P_i h_i^2 + \sigma_{n_i}^2}}, \tag{2}$$

for generic noise and power terms n_i and P_i, respectively. σ_θ^2 is the variance of the random scalar parameter θ to be estimated at the fusion center.

We further assume that the channel and measurement noises are independent, and that the channel gains are known at the fusion center which allocates the optimal powers. Given the model assumptions, the optimal linear estimate that minimizes the mean-squared error is the best linear unbiased estimate (BLUE) [7][1]. We also denote $r_i = |h_i|^2/\sigma_{n_i}^2$ to be the effective channel SNR for the i^{th} sensor. The optimization problem considered here is minimizing the BLUE MSE subject to a total network power constraint $\sum_{\forall i} P_i = P_T$. We assume the measurements of θ are orthogonally transmitted from the leaf nodes to the fusion center.

B. Relay Transmission Protocols

The more complex multi-hop branch, tree and linear topologies consist of both relay nodes and leaf nodes. The relay nodes make measurements and forward the leaf node measurements to the fusion center, thus the need for transmission protocols. We first define the AF scheme where the measurement from the leaf node is simply forwarded by the relay node. The second is the EF protocol which estimates the parameter at the relay using the received measurement and then transmits this new estimate. For both relay protocols, as in the star topology, the transmitted signal from any node is always normalized. Thus P_i represents the physical power used by the i^{th} node to forward information.

Consider a simple two-hop linear network (Figure 1) with measurements made at both the leaf and relay nodes. In the AF case, we assume that a relay node's power is split equally between all measurements that are forwarded through it.

Using the AF protocol, the fusion center receives two measurements

$$y_1 = \sqrt{\frac{P_0}{2}} h_0 \gamma(P_1) \left[\sqrt{P_1} h_1 \beta(z_1) (\theta + z_1) + n_1\right] + n_{01} \tag{3}$$

$$y_2 = \sqrt{\frac{P_0}{2}} h_0 \beta(z_0) (\theta + z_0) + n_{02} \tag{4}$$

from the leaf node and relay node, respectively. There is no estimation at the relay, and the received signal is simply re-transmitted. The two measurements received at the fusion center are used to compute the final BLU estimate.

However, the EF protocol involves computing an intermediate parameter estimate at the relay. The two measurements received are

$$y_1 = \sqrt{P_1} h_1 \beta(z_1) (\theta + z_1) + n_1 \tag{5}$$

$$y_2 = \theta + z_0 \tag{6}$$

[1]If the measurement and channel noises are Gaussian, the BLU estimate is equivalent to the optimal MMSE estimate.

and the BLU estimate computed at the relay $\widehat{\theta}_R = \theta + w_0$ is transmitted to the fusion center which receives

$$y = \sqrt{P_0} h_0 \beta(w_0) \widehat{\theta}_R + n_0 \tag{7}$$

and uses it to compute the final estimate.

The above models can be generalized for more complex, multi-hop networks. The communication power allocated to a relay node is divided equally between all measurements that must be transmitted through it. A key difference between the AF and EF protocols is that in the former, it is the noise in each link which is propagated through the system, while in the latter, it is the estimation error from each relay.

III. The Star Topology

A. Deriving the Optimal Solution

Recall the signal model for the star topology from Equation (1). The MSE corresponding to the BLU estimate formed at the fusion center is given by

$$\text{MSE}_{\text{star}} = \left(\sum_{i=1}^{N} \frac{1}{\sigma_{z_i}^2 + \frac{\sigma_\theta^2 + \sigma_{z_i}^2}{r_i P_i}} \right)^{-1}. \tag{8}$$

To minimize the MSE of the parameter estimate obtained at the fusion center subject to a total network power constraint, the following optimization problem is considered:

$$\min_{\mathbf{P}} \ \text{MSE}_{\text{star}} \quad \text{subject to} \quad \sum_{i=1}^{N} P_i = P_T \tag{9}$$

where $\mathbf{P} = (P_1, \ldots, P_N)$ are powers allocated to the N nodes.

We use Lagrange multiplier theory to solve the optimization problem since the MSE is a convex function in $\mathbf{P}$ (as done independently in [3] for gain optimization). The Lagrange multiplier ν is associated with the total network power equality constraint; both the objective function and the power constraint are convex, and the Karush-Kuhn-Tucker conditions are valid [8].

The number of active sensors $N' \le N$ is the greatest integer that satisfies the following inequality

$$\sqrt{\nu^\star} = \frac{\displaystyle\sum_{i=1}^{N'} \frac{1}{\sigma_{z_i}^2} \sqrt{\frac{\sigma_\theta^2 + \sigma_{z_i}^2}{r_i}}}{\displaystyle P_T + \sum_{i=1}^{N'} \frac{\sigma_\theta^2 + \sigma_{z_i}^2}{r_i \sigma_{z_i}^2}} < \sqrt{\frac{r_{N'}}{\sigma_\theta^2 + \sigma_{z_{N'}}^2}}, \tag{10}$$

and the optimal power allocated to each sensor is

$$P_i^\star = \frac{\sigma_\theta^2 + \sigma_{z_i}^2}{r_i \sigma_{z_i}^2} \left(\sqrt{\frac{r_i}{\nu^\star \left(\sigma_\theta^2 + \sigma_{z_i}^2\right)}} - 1 \right)^+, \ \forall\, i \tag{11}$$

where $(x)^+ = 0$ when $x < 0$, and is otherwise equal to x.

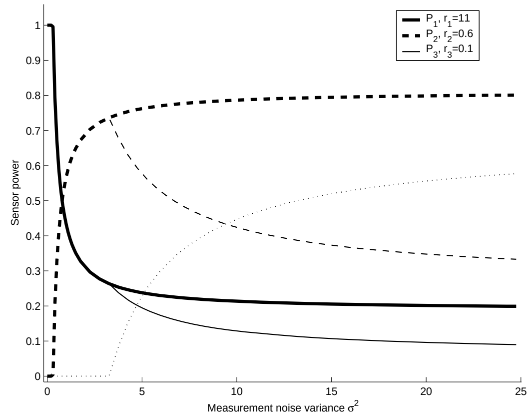

Fig. 2. Optimal power allocation for $N = 2, 3$ star topology.

B. Analysis and Asymptotic Results

First consider the case when $\sigma^2_{z_i} = 0\ \forall i$; we have noiseless measurements. In this case, the equivalent objective function is $\sigma^2_\theta \left(\sum_{i=1}^{N} r_i P_i\right)^{-1}$, and the optimal solution is allocating all the power to the sensor(s) with the highest (common) SNR, yielding a sensor selection solution[2].

Since only "perfect" measurements of the *same* scalar parameter are transmitted by all sensors, all the power is allocated to the sensor(s) with the best channel SNR. The optimal solution described in (11) is a type of waterfilling solution [8] for low measurement noise variance. With $\sigma^2_{z_i} = 0\ \forall i$, an example of extreme waterfilling, all power is allocated to the sensor with the maximum SNR. As the measurement noise variance increases, waterfilling is *not* optimal.

Now let the measurement noise variance of all sensors be equal, $\sigma^2_{z_i} = \sigma^2_z\ \forall i$, and consider $\sigma^2_z \to \infty$; *i.e.*, all measurements get noisier. Figure 2 shows the evolution of the optimal powers as a function of σ^2_z.

As measurement noise variance increases, other sensors in addition to that with the best SNR become active. For large σ^2_z, all sensors are active, and the optimal limiting power allocation is given by

$$\overline{P}^\star_i = \frac{\alpha}{\sqrt{r_i}}\left(P_T + \sum_{i=1}^{N}\frac{1}{r_i}\right) - \frac{1}{r_i} \quad \text{with} \quad \alpha = \left(\sum_{j=1}^{N}\frac{1}{\sqrt{r_j}}\right)^{-1}. \tag{12}$$

The form of the optimal limiting solution $\overline{\mathbf{P}}^\star$ is termed *power equalization*[3]. Since $\overline{P}^\star_i$ is proportional to $r_i^{-\gamma}$ (for some $\gamma > 0$), the sensor with the highest SNR is allocated the smallest fraction of the total power.

[2]If more than one sensor has the same maximum SNR, any arbitrary power allocation amongst that subset of sensors is optimal.

[3]The optimal limiting power allocation policy $\overline{\mathbf{P}}^\star$ is termed power equalization and is defined as allocating the greatest fraction of the total power to the node with the weakest channel SNR.

IV. The Branch Topology

A. Deriving the MSE Expressions

The branch topology, a generalization of the star topology, consists of N leaf nodes connected to the fusion center via a single relay node as in Figure 1. Measurements are taken at all nodes and transmitted to the fusion center using either the AF or EF protocol. The time resources used to form the estimate at the fusion center in the branch topology are the same for both AF and EF protocols.

For the AF case, the fusion center receives $N+1$ measurements given by

$$y_0 = \sqrt{\frac{P_0}{N+1}} h_0 \beta(z_0)(\theta + z_0) + n_0 \tag{13}$$

$$y_i = \sqrt{\frac{P_0}{N+1}} h_0 \gamma(P_i)\Big[\sqrt{P_i} h_i \beta(z_i)(\theta + z_i) + n_i\Big] + n_{0i},\ i = 1, \ldots, N \tag{14}$$

from the relay node and the N leaf nodes, respectively. As before, we assume equal powers are used to forward each of the $N+1$ measurements from the relay to the fusion center. The fusion center forms the optimal BLU estimate which has the corresponding MSE in Equation (15).

In the case of EF, the relay node receives $N+1$ measurements given by

$$y_0 = \theta + z_0 \tag{17}$$

$$y_i = \sqrt{P_i} h_i \beta(z_i)(\theta + z_i) + n_i\ , \quad i = 1, \ldots, N. \tag{18}$$

The relay makes the BLU estimate $\widehat{\theta}_{\text{R}}$ and transmits this single measurement to the fusion center which receives

$$y = \sqrt{P_0} h_0 \beta(n_R)\,(\theta + n_R) + n_0 \tag{19}$$

and computes the final estimate $\widehat{\theta}_{\text{FC}}$ with an estimation error given by

$$\text{MSE}_{\textsf{branch,EF}} = \frac{1}{r_0 P_0 \beta(n_R)} + \sigma^2_{n_R} \tag{20}$$

where

$$\sigma^2_{n_R} = \left(\frac{1}{\sigma^2_{z_0}} + \sum_{i=1}^{N}\frac{1}{\sigma^2_{z_i} + \frac{1}{r_i P_i \beta^2(z_i)}}\right)^{-1}. \tag{21}$$

The minimization of the MSE expressions in (15) and (20) under a total network power constraint can only be solved numerically, so we now derive an upper bound which is asymptotically optimal for increasing measurement noise variance.

B. Asymptotic Analysis of the Branch Topology

Recall the arithmetic mean — harmonic mean (AM-HM) inequality [9] which can be expressed as

$$\frac{1}{N^2}(x_1 + \ldots + x_N) \quad \geq \quad \left(\frac{1}{x_1} + \ldots + \frac{1}{x_N}\right)^{-1}. \tag{22}$$

$$\text{MSE}_{\text{branch,AF}} = \left(\frac{1}{\sigma_{z_0}^2 + \frac{N+1}{r_0 P_0 \beta^2(z_0)}} + \sum_{i=1}^{N} \frac{1}{\sigma_{z_i}^2 + \frac{1}{r_i P_i \beta^2(z_i)}\left(1 + \frac{N+1}{r_0 P_0 \gamma^2(P_i)}\right)} \right)^{-1} \tag{15}$$

$$\overline{\text{MSE}}_{\text{branch,AF}} = \frac{\sigma_z^2}{N+1} + \frac{1}{(N+1)^2}\left[\frac{N+1}{r_0 P_0 \beta^2(z_0)} + \sum_{i=1}^{N} \frac{1}{r_i P_i \beta^2(z_i)}\left(1 + \frac{N+1}{r_0 P_0 \gamma^2(P_i)}\right)\right] \tag{16}$$

For the simple case with equal measurement noise variances for all nodes, applying (22) to the MSE expression in (15) yields[4] the upper bound in Equation (16).

Substituting Equation (2) for $\beta^2(P_i)$ into (16) and re-writing the upper bound yields

$$\overline{\text{MSE}}_{\text{branch,AF}} = S + \sum_{i=1}^{N} \frac{1}{t_i P_i} \tag{23}$$

for some constants S and $\{t_i\}_{i=1}^N$. This illustrates the dependence of $\overline{\text{MSE}}_{\text{branch,AF}}$ on the leaf node powers $\{P_i\}_{i=1}^N$.

As an aside, we can show that the optimization problem

$$\text{minimize} \quad \sum_{i=1}^{N} \frac{1}{a_i x_i} \quad \text{subject to} \quad \sum_{i=1}^{N} x_i = X_T \tag{24}$$

is optimized by $x_i^\star = \frac{1/\sqrt{a_i}}{(\sum_{j=1}^N) 1/\sqrt{a_j}} X_T$. Applying this result to the AF upper bound (23), we obtain

$$\overline{P}_i^\star = \frac{\alpha_t}{\sqrt{t_i}}\left(P_T - \overline{P}_0^\star\right) \quad \text{with} \quad \alpha_t = \left(\sum_{j=1}^{N} \frac{1}{\sqrt{t_j}}\right)^{-1}, \tag{25}$$

which minimizes Equation (16); the optimal power allocated to the relay node can be computed as the result of a one-variable numerical optimization.

Similarly, the corresponding upper bound for the branch EF case can be computed, and is minimized by Equation (25) with $t_i = r_i$, and whose relay node optimal power is also computed numerically. In both the AF and EF cases, the optimal limiting power allocated to the leaf nodes corresponds to power equalization.

This limiting power scheme is intuitive. Since the relay node is used by all leaf nodes to forward measurements to the fusion center, $\overline{P}_0^\star$ is not a function of any one of the individual leaf node SNRs. On the other hand, the optimal limiting leaf node powers are allocated using power equalization; the leaf nodes resemble a star topology with respect to the relay node. Sensors with weaker channel SNRs are allocated a greater fraction of the remaining power $P_T - \overline{P}_0^\star$.

V. The Tree Topology

The single branch topology can be replicated to form a 'one-layer' tree topology. Assume K relaying nodes connected to the fusion center, each with N_k leaf nodes attached to the k-th relay node. The MSE for the tree topology is

$$\text{MSE}_{\text{tree,AF/EF}} = \left(\sum_{k=1}^{K} \frac{1}{\text{MSE}^{\text{k}}_{\text{branch,AF/EF}}}\right)^{-1}, \tag{26}$$

which suggests that the MSE for the tree can be minimized by independently minimizing the estimation errors for each individual branch. The asymptotically optimal power policy for the AF/EF branch case can be used to obtain the limiting power solution for the one-layer tree topology.

We find that the optimal powers assigned to the branches of a tree are simply scaled versions of the single branch optimal powers. In particular, power equalization is used to allocate leaf node powers to ensure that all measurements are useful at the fusion center.

VI. Power Allocation in Linear Topologies

The linear topology is shown in Figure 1. For both AF and EF protocols, measurements are made at all nodes.

A. Problem Formulation for the AF Protocol

We have assumed that each relay node equally divides its allocated power amongst the measurements forwarded through that relay. Thus the N signals received at the fusion center are given by

$$y_1 = \sqrt{\frac{P_1}{N}} h_1 \beta(z_1)(\theta + z_1) + n_1 \tag{27}$$

$$\vdots$$

$$y_N = \sqrt{\frac{P_1}{N}} h_1 \gamma\left(\frac{P_2}{N-1}\right)\Big(\cdots\gamma(P_N)\Big(\sqrt{P_N} h_N \times\beta(z_N)(\theta + z_N) + n_N\Big)\cdots\Big) + n_1 \tag{28}$$

and these are used to compute the BLUE. The optimization problem in this case can only be solved numerically as power variables are non-separable.

B. Problem Formulation and Solution for the EF Protocol

Using EF, each relay combines two measurements (its own and the one received) to compute $\widehat{\theta}_R$. Finally the fusion center computes the BLUE with the MSE given recursively as:

$$\text{MSE}_{\text{linear,EF}} = \frac{1}{r_1 P_1 \beta^2(\overline{z}_1)} + \sigma_{\overline{z}_1}^2 \tag{29}$$

$$\text{where} \quad \sigma_{\overline{z}_i}^2 = \left(\frac{1}{\sigma_{z_i}^2} + \frac{1}{\sigma_{\overline{z}_{i+1}}^2 + \frac{1}{r_{i+1} P_{i+1} \beta^2(\overline{z}_{i+1})}}\right)^{-1} \tag{30}$$

for $i = 1, \ldots, N-1$, and $\sigma_{\overline{z}_N}^2 = \sigma_{z_N}^2$.

[4]The MSE upper bound is denoted by $\overline{\text{MSE}}$, and thus $\text{MSE} \leq \overline{\text{MSE}}$.

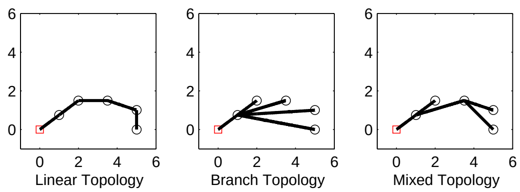

Fig. 3. Different topologies imposed on the same set of nodes.

Even in the linear EF case, $\{\overline{P}_i^\star\}_{i=1}^N$ is difficult to characterize due to non-separable variables. Thus we further assume that total power P_T is high[5]. For increasing P_T, the upper bound on the MSE expression can be written as

$$\lim_{\sigma_z^2\to\infty} \lim_{P_T\uparrow} \ \text{MSE}_{\text{linear,EF}} = \sum_{i=1}^{N} \frac{1}{c_i r_i P_i}, \qquad (31)$$

where $c_i = N/(N-i+1)$. The asymptotically optimal power allocation strategy (for increasing P_T) is given by

$$\overline{P}_i^\star = \frac{\alpha}{\sqrt{c_i^2 r_i}} P_T \quad \text{where} \quad \alpha = \left(\sum_{k=1}^{N} \frac{1}{\sqrt{c_k^2 r_k}} \right)^{-1}, \qquad (32)$$

which is a weighted power equalization solution; c_i is a "weighting" for the position of a sensor relative to the fusion center. The c_i values show that nodes closer to the fusion center are given more weight. Furthermore, for low σ_z^2, sensors further away from the fusion center remain inactive.

VII. Numerical Results

The optimal power solutions for different topologies, obtained using AMPL [10], are globally optimal and unique since the MSEs can be shown to be convex functions.

Consider different topologies imposed on the same set of nodes in Figure 3. The EF protocol is used to compare the linear, branch, and "mixed" topologies for increasing σ_z^2. Note we assume σ_z^2 is equal for all the nodes, and that the channel SNR is $r_i = Cd_i^{-\alpha}$, where d_i is the distance between adjacent communicating nodes, and C is an arbitrary constant. Assume $C = 20$ and path-loss exponent $\alpha = 2$. We find

$$\text{MSE}_{\text{linear}} > \text{MSE}_{\text{branch}} > \text{MSE}_{\text{“mixed”}}, \qquad (33)$$

which suggests that MSE decreases as the depth of branching increases. Figure 4 compares the MSE for these topologies as we vary $\alpha \in [2.0, 4.0]$. We see that the linear topology, with shorter hops, yields a lower MSE than the branch topology as the distance increases. The mixed topology yields the lowest MSE $\forall\ \alpha$, so more hierarchical networks seem preferable for the constant σ_z^2 case.

VIII. Conclusions

A power allocation problem for parameter estimation with distributed observations under a total network power constraint was considered for various topologies. An analysis of the star topology was presented which noted that the measurement noise variance changes the nature of the optimal solution. For $\sigma_z^2 \approx 0$, sensor selection is optimal, but as σ_z^2 increases, the solution evolves from waterfilling to power equalization: sensors with lower SNRs are allocated a greater fraction of the total power.

A similar solution space is derived for the leaf nodes of the branch topology, but only when an alternate optimization problem is solved. An upper bound is obtained for the MSE that is asymptotically equal to the true MSE. The single branch topology is easily extended to the generic one-layered tree topology, which exhibits the same limiting power allocation. The linear topology has a limiting optimal weighted power equalization solution.

For common values of the path-loss exponent, hierarchical networks yield the lowest MSE for the equal measurement noise variance case. Future work includes investigating the suboptimality of decentralized power allocation schemes that do not require perfect knowledge of all the channel gains at the fusion center.

[5]The relative value of P_T depends on the channel SNRs $\{r_i\}_{i=1}^N$

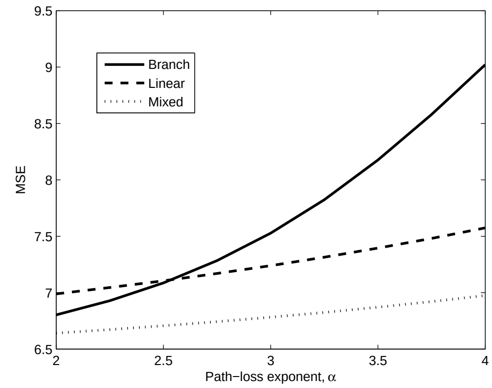

Fig. 4. MSE of different topologies for varying path-loss exponent α.

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