The Coskewness Premium: Where does it come from?

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ABSTRACT

This paper demonstrates that coskewness and volatility risk are closely related. In fact, coskewness can be decomposed into volatility risk and state-transition risk, where the latter is driven by stochastic volatility jumps. High exposure to volatility risk acts as hedge against unfavorable volatility movements. Exposure to state-transition risk provides hedge against volatility reversal following negative volatility jumps. However, this exposure implies higher risk following positive volatility jumps as it lowers returns when volatility fails to revert to its original level. The coskewness premium is affected by the premia given to both of its components. The sign of the time-series average of the premium is determined by the volatility risk component while the time-varying property resembles that of the state-transition risk. The coskewness, volatility risk and state-transition risk premia are negative following negative volatility jumps, but only the volatility risk premium is positive following positive jumps.

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The three-moment Capital Asset Pricing Model (3M-CAPM) driven by the second order stochastic discount factor (SDF) helps explain the cross-sectional variation of asset returns. When investors have preference for positive skewness in their wealth returns, Kraus and Litzenberger (1976) and Harvey and Siddique (2000) among others, show that a lower premium is required for assets of positive coskewness as they contribute positively to the skewness of the portfolio. In related literature, it is known that the market volatility process is cross-sectionally priced. Ang, Hodrick, Xing, and Zhang (2006), for example, argue that the option implied volatility as a factor is priced in the cross-section of stock returns. As Merton (1973) shows, the market volatility process may affect risk-averse investors when investors are concerned about inter-temporal decisions. Risk averse investors care about their exposure to the process if high volatility worsens their investment opportunity set.

While the coskewness premium is driven by investors’ preference for skewness, volatility risk premium at the individual stock level stems from variation in the investment opportunity set. Although the sources of these premia are theoretically unrelated, they may be empirically related as the volatility process is strongly affected by the square of the market returns. This paper studies the relation between coskewness and volatility risk. In fact, this paper suggests that coskewness of an asset can be decomposed into volatility risk and state-transition risk, which is driven by stochastic jumps. Furthermore, this paper introduces a framework that evaluates the dynamic dependencies among the coskewness, volatility and state-transition risk premia.

In the framework the market portfolio follows a double jump-diffusion process where jumps occur both in the price and the volatility process. The validity of jumps in both the market prices and volatility level has been well-documented. Moreover, these jumps are known to be correlated\[1\]. When bad news arrives that leads to a higher diffusion, the market, requiring a higher risk premium, will face a negative jump. Similarly, if there is

news that decreases variation of the diffusive process, it will be associated with a positive jump in the market prices. As a result, stochastic jumps driven by economic shocks appear both in the level and the variation in the diffusion process.

Correlated stochastic jumps are regarded as state-transitions, for the reason that the expected returns, volatility and other higher moments are simultaneously affected. While conditional moments remain constant within the same state, a jump leads to a different state where the conditional market moments vary. This paper assumes that the economy is governed by multiple states determined by the volatility of the diffusion process. When the volatility of the diffusion increases, investors perceive a higher risk. As a higher premium is required for taking additional risk, a negative price jump is observed when news arrives that increases the variation. A transition from a lower state to a higher state, therefore, leads to a negative jump in the price, and similarly, a transition from a higher to lower leads to a positive jump in the price. As Pindyck (1984) describes, positive stochastic jumps in volatility derive volatility feedback leading to lower prices.

When the market price and the volatility follows the double-jump process, coskewness is the sum of two components. The first component, the Volatility Risk (VR) component, is related to the exposure to variations in the diffusion process and measures how the returns co-vary with the volatility innovations. The second component, the State-Transition Risk (TR) component, is driven by the exposure to shocks caused by stochastic jumps. A high value of this term implies outperformance whenever there is a jump, or equivalently, when the volatility moves abruptly regardless of the direction. As a result, a risk-averse investor’s portfolio decision is affected by coskewness in two manner. While a high VR component is always preferred by the investor, a high TR is preferred only when possible future jumps are more likely to be positive. When there is mean-reversion in volatility, a high TR component provides hedge against volatility increases following negative jumps, but following positive jumps, this implies a relatively higher return when volatility fails to revert to its original low level.
A two-state economy helps understanding the roles of each of the components. Following a low state, the state could either persist (low) or transition (high) can occur. When the states are persistent, the current state is more likely to persist. As long as the investor acknowledges that a state-transition implies a negative shock in her wealth, she is willing to pay a premium for a group of assets that perform better when a negative shock appears. Since the negative price shock occurs at the same time as the positive volatility jump, both assets with high VR and with high TR have relatively higher returns when state transitions. In low states, VR and TR play an interchangeable role, providing hedge against future possible negative price jumps. Therefore, when coskewness can be decomposed as suggested in this paper, the premium given to positive coskewness as well as to both of the components must be negative.

Following a high state, the components, however, play countervailing roles. Since the investor expect a positive price shock when the state transitions, she is willing to hold assets that perform relatively well when state persists. While assets with a high VR component performs relatively well when the state persists, those with a high TR do not. Therefore, while the premium given to VR is negative, the TR premium must be positive as a high TR lowers the returns when state persists. As a result, the sign of the coskewness premium, being a miscellany of the two premia, is indeterminable.

When there are more than two states in the economy, a high VR component implies higher returns when volatility increases but lower returns when volatility decreases. Therefore, even in the case of more than two states, a high VR component provides hedge against unfavorable volatility movements. The role of the TR component depends on whether the volatility jump that follows is more likely to be positive or negative. When more likely to be positive, a state-transition is more likely to increase the volatility and decrease the price. Therefore, a high TR will provide hedge against negative price shocks when state transitions. On the other hand, when the jump is more likely to be negative, the volatility will drop and the price will increase. In this case, a high TR will relatively decrease the returns when state persists exacerbating the bad state returns. Hence, when there is mean-reversion in volatility,
the risk-averse investor is more likely to hold assets with high TR components as volatility decreases or following negative volatility jumps. As volatility increases or following positive jumps, a TR component adds risk to the market portfolio, hence is not likely to be held by the investor unless an additional premium is given.

This paper further investigates the dynamic relation between the premia given to coskewness and the components. According to the framework, the VR premium must be negative regardless of past volatility movements. Both the TR premium must be state-dependent. The premium is negative only following negative volatility jumps when a mean-reversion to a higher volatility level is expected in the future. The components are estimated using a stochastic volatility model with double jumps (SVJJ) as introduced by Duffie, Pan, and Singleton (2000), where jumps occur both in the price and the level of the volatility. The risk-adjusted returns of the top minus bottom quintiles of VR, TR components and coskewness-sorted portfolios are compared for the subsamples classified based on past realizations of volatility and stochastic jumps.

First, the VR component is estimated as the covariance of asset returns and the unexpected portion of the volatility. As the framework of this paper suggests, assets that positively covary with the volatility process tend to have lower subsequent returns. Moreover, the size of the premium does not depend on past realizations of the volatility process. This is because a high VR component acts as insurance against negative market shocks. Second, the TR component is estimated as the covariance with the absolute value of the jump process of the model. The risk-adjusted returns of the top minus bottom portfolio show that the TR premium is negative in the low states, followed by negative volatility jumps, but is positive premium following positive volatility jumps or in high states. This is because a high TR component reduces risk when volatility is low, but adds risk to the market portfolio when volatility is high.
Therefore, the coskewness premium, which is known to be negative if evaluated over the whole period\(^2\), may actually depend on the sign of past volatility movements. In low states, following negative volatility jumps, the sign of the premium is negative because both VR and TR premia are negative. However, in high states or following positive volatility jumps, the sign of the premium is ambiguous as the TR premium counteracts the VR premium. The sign of the premium evaluated over the whole period stems from the volatility risk part, but the state-dependency resembles that of the jump part. This paper further tests whether the state-dependency is driven by the TR premium. After controlling for the premium given to the two components, the coskewness premium no longer depends on past stochastic jumps.

Both the coskewness premium and the volatility risk premium has been extensively studied. Harvey and Siddique (2000) and Smith (2007) study time-varying conditional coskewness. Harvey and Siddique (2000) argue that the premium on conditional coskewness depends on the conditional moments of the market. Smith (2007) shows that size of the coskewness premium is higher when the historical market skewness is positive. It is also well-known that the volatility risk premium is negative overall. Ang, Hodrick, Xing, and Zhang (2006) show that innovation in option-implied volatility is priced in the stock market\(^3\). Jones (2006) studies volatility and jump risk premia in option prices. Adrian and Rosenberg (2008) show that both the short-term and long-term volatility is priced.

Transition risk has also been studied in the context of in the regime switching models. These studies include interest rates (Gray 1996), stock market overreaction (Veronesi 1999), asset allocations (Guidolin and Timmermann 2007, Cvitanic, Polimenis, and Zapatero 2008). Jump risk has been evaluated in the option pricing\(^4\) bond premium (Wright and Zhou 2009) and credit spreads (Tauchen and Zhou 2011) literature. Todorv (2010), Todorv and Bollerslev (2011), Bollerslev and Li (2013) disentangle diffusion risk from jump risk. Todorv (2010) among these papers studies the dynamics of jump premium, however, his paper

\(^2\)See, for example, Kraus and Litzenberger (1976), Harvey and Siddique (1999, 2000), Dittmar (2002) and Smith (2006)

\(^3\)See, also Chang, Christoffersen, and Jacobs (2013).

\(^4\)See, for example, Pan (2002) among others
considers jump risk to be uni-directional. None of the aforementioned papers discusses the role of jump risk as explaining the gap between volatility risk and coskewness. This is the first paper that studies the dynamic relation among volatility risk, coskewness and jump risk.

The idea that the coskewness premium is unrelated to preference for skewness is also not new. Post, van Vliet, and Levy (2008) argue that the coskewness premium is too large to represent the compensation that investors are willing to pay for assets that increase the portfolio skewness. Poti and Wang (2010) confirm that the coskewness premium should be substantially smaller if it represents investors’ preference for skewness. They argue that as the second order SDF is not sufficient, the premium must be driven by other factors. Their argument is closely related to the main idea of this paper showing that the missing part is driven by the state transition process not captured by the market factor. The framework of this paper supports the argument that the coskewness premium may not be driven by investors’ preference for skewness.

The rest of the paper is organized as follows: Section I discusses the setup of the framework. Section II summarizes a three-component decomposition of the coskewness. The stochastic volatility model is described in Section III. Section IV provides the main empirical results of this paper. Section V concludes.

I. The Framework

A. A Model with Two States

Traditional studies including the CAPM (Sharpe 1964, Lintner 1965) typically assume that the economy consists of a single state where the conditional moments of the returns remain constant over time. However, the fact that the moments vary over time are well-known. For example, we know that the market volatilities are clustered across time. This is an
assumption that any variants of the Auto-Regressive Conditional Heteroskedasticity (ARCH) (Engle 1982) model relies on. As suggested by Merton (1973) we also know that the expected market returns should be higher following times with high volatility. Regarding the relation between the first and second moment, we acknowledge that market returns tend to negatively co-move with the market volatility (See, for example, Nelson (1991), Glosten, Jagannathan, and Runkle (1993).) . The covariances or the correlations among assets are also time-varying. While the index returns, at least since the 1960’s, are negatively skewed while the returns an individual stock are on average positively skewed. The correlations among the asset pairs increase during bad times which creates negatively skewed index returns (Ang and Chen 2002, Krishnan, Petkova, and Ritchken 2009, Mueller, Stathopoulos, and Vedolin 2013). These evidences suggest that the economy may be governed by multiple states in which the conditional moments of asset returns vary over time.

Economic and asset pricing models that allow for multiple states incorporate dynamically and systematically varying market moments across time. Studies that rely on this type of framework often assume that even the past states are imperfectly known to the agents in the economy. These types of studies often assume that there is a latent variable and a transition process participants of the economy learn about the latent variable by observing other related variables. David (1997) uses firm productivity as a latent variable, which is difficult to observe. Other studies include using the investor’s perception about the growth rate of the economy (Veronesi 1999), expected dividend growth (Brennan and Xia 2001) or government policy (Pastor and Veronesi 2012). These studies often utilize a hidden-Markov Model (hMM) with Bayesian learning to describe the uncertainty about the regimes. A hMM, however, may not be appropriate when the states are ex-post known to the investors. For example, the level market volatility is relatively easy to infer given historical market returns are publicly known to the investors. Although investors might disagree about the exact level of the market volatility, it is not hard to infer whether the level of volatility has increased or decreased over a certain period with assistance of option-implied volatility or data from high-frequency trading.
When the market moments depend on the state realizations, a transition of the state implies jumps in the market moments. Jumps both in prices and the level of the volatility are correlated and appear simultaneously. This is often called as a double jump diffusion model, which has been studied extensively.\(^5\) I start with a simple two-state model and assume that the market price follow a jump-diffusion process. I assume that the two states differ mainly by the level of the volatility in the diffusion process, and a jump in the volatility occurs at the same time as a jump in prices when there is a state-transition. The first state, called the low state \(L\), has a relatively lower level of volatility \((h_L)\) of the diffusion process than the second state \((h_H)\), called the high state \(H\). A state-transition occurs upon an external random shock, which at the same time leads to a jump in both the price and volatility of the market portfolio. A transition from a low to a high state leads to a higher variation in the diffusion process, which will create a negative jump in prices due to a higher volatility premium. Similarly, a transition from a high to a low state decreases the variation in the diffusion process, which will be accompanied by a single positive jump in prices. I let the transition be governed by a Markov Process with a transition matrix with constant probabilities

\[
\Pi = \begin{pmatrix}
\pi_{LL} & \pi_{LH} \\
\pi_{HL} & \pi_{HH}
\end{pmatrix},
\]

where \(\pi_{ss'}\) is the probability of a state \(s' = L, H\) following a state \(s = L, H\). Therefore, \(\pi_{LL} > 0.5\) denotes the probability of state persistence given a low state, and \(\pi_{HL} < 0.5\) denotes the probability of state transition from a high to a low state. Since the random shock does not appear very often, investors believe that a high state is more likely to follow a high state than a low state and the same holds for the low state. I assume that the states are perfectly known to the investors after realization, as it is relatively easy to infer whether there is more short-term variation in the returns today compared to yesterday. The state transition process along with the exact timing when the states are known to the investors are summarized in Figure.\(^6\) The upper diagram illustrates possible state realizations in the low

\(^5\)See, for example, Duffie, Pan and Singleton (2000), Pan (2002), Eraker Johannes and Polsen (2003) and Todorv (2011) among others.
volatility state, and the lower diagram is for the high volatility state. For both diagrams the first (in the left) is a known state to the investors while the second (right) is yet unknown.

The variations in the diffusion process increases when the state transitions from a low to a high state. The market premium must increase at the same time since investors need to be compensated for higher risk. A transition from low to high must be accompanied by a negative jump in the market prices. Similarly, when state transitions from high to low, there must be a positive jump in the market. Thus, the market return process is assumed to be,

\[ r_{m,t+1} = \mu_s - \xi_s J_{s,s'} + h_{t+1} \epsilon_{m,t+1} \]  

(1)
where $\xi_s > 0$ for $s = L, H, J, J_{H,L} = -1, J_{L,H} = 1, J_{L,L} = J_{H,H} = 0$ and $\epsilon_{m,t+1}$ is standard normal that is independent of the state transition process. The volatility process described above, can be expressed as,

$$h_{t+1}^2 = h_t^2 + \xi_v J_{s,s'}$$

(2)

As there are only two possible states, both $h_t$ and $h_{t+1}$ either takes one of the two values: $h_H > h_L$ depending on the concurrent state.

There are two important implications of Equation (1) that are worth mentioning. First, the return of the market portfolio during a specific period depends both on the state at the beginning and at the end of the period. That is, the average market return between time $t$ and $t+1$ is determined by both the level of volatility at time $t$ and $t+1$. Therefore, when volatility increases there is a negative price jump even if the conditional returns of $t+1$ evaluated at time $t$ is positive. As there are two independent sources of variation in the market returns, the conditional distribution of the market returns is a mixture of two distributions as there are only two possible states. When the diffusion process is normally distributed given possible positive, negative or no jumps occurring, the conditional distribution of the market returns between time $t$ and $t+1$ given information about the realized state ($s_t$), is a mixture of two Gaussians. As a result, when assessing the market returns for an arbitrary interval, the empirical (unconditional) distribution would be a mixture of up to four Gaussian distributions since states spans up to four different pairs.

Second, this process assumes that jumps in the prices and volatility process occurs at the same time. A negative shock occurs when there is a transition from a low to a high state. Similarly, a transition from a high to a low is accompanied by a positive shock in the market. A number of studies show that the market returns move in the opposite direction from the instantaneous volatility (e.g. Nelson (1991), Engle and Ng (1993), Glosten, Jagannathan, and Runkle (1993) among others.). Several possible reasons have been explored. One of them proposed by Black (1976) is known as the ‘leverage effect’. A decrease in the return of an asset can increase the leverage of the firm, which will make assets more risky increasing
the variance. Alternatively, a shock to the market returns will increase the volatility. Since investors need to be compensated for higher volatility risk, a higher premium is required (Carr and Wu 2009, Bollerslev, Tauchen, and Zhou 2009). As a result, the market drops when the volatility increases, and since volatility is priced, the market has to devalue in order to bear a higher premium. This is known as the ‘volatility feedback effect’ (Pindyck 1984). In this framework, the asymmetries in the market returns and volatility comes from the correlated Jump process between the returns and the volatility.

These implications are summarized in Figure 1. Either a transition or persistence is possible following a low state. Although states are more likely to persist, there is a small probability of transition. Following a low state, a transition leads to a negative shock in the market prices. Following a high state, a transition leads to a positive shock in the market prices.

The rest of the analysis is devoted in analyzing the portfolio decision from a representative mean-variance investor’s perspective. There is no additional utility gained from a positive third moment or a negative forth moment in the portfolio returns. The only information that is important for the investor is whether it is feasible to reduce the conditional variance of the market portfolio for a fixed level of expected returns. When an asset has a relatively higher return during states that the market price shows a negative jump, this asset will provide hedge against the negative jump. When an asset has a relatively higher return when the market has a positive price jump, this asset will marginally increase the conditional variance of the market portfolio, hence should be given a positive premium. In this paper, I analyze whether the coskewness premium can be explained by a premium on a state variable that is correlated with the square of the market returns. The following section discusses the conditional moments of the market portfolio and the role of the jump process as a state variable when the market return follows a jump-diffusion process.
B. Conditional Moments and the Jump Process as a State-Variable

The mean-variance analysis implies that any strategy that marginally reduces the variance of the market portfolio must have lower expected returns. Thus, the sign of the premium given to the exposure with respect to a state-variable depends on whether a high exposure provides a hedge against bad state realizations. This section discusses the characteristics of the conditional distribution of the market portfolio when there are two states which differ by the level of volatility in the diffusion process. A distinction needs to be made between the volatility of the diffusion process and the conditional variance of the market portfolio since the latter is also affected by stochastic jumps. This section also provides a formula for the conditional variance of an arbitrary portfolio given a two factor market structure as in Equation (1). This will later help understand the marginal effect of the covariance with respect to a state variables on the conditional variance when the coskewness decomposition is introduced in the next section.

The market price process given as in Equation (1) implies that there are two independent sources of variation that affects the returns. In this case, distribution of the returns between time t and t+1 depend on whether there is a jump present between time t and t+1. While the hypothetical market returns given the information there is or is not a jump at time t+1 are normally distributed \( (r_{m,t+1}|J_{t+1}) \), the conditional distribution of the t+1 returns evaluated at time t \( (r_{m,t+1}|F_t) \) is a Normal-mixture of former, where the normal distributions are weighted each by the probability of jumps. The variance of the former distribution is the variation of the diffusion process, while the variance of the market returns is the sum of the conditional variation in the diffusion process and the conditional variation in the jump process. For convenience, I call the former distribution conditional on jumps as the \textit{ex-post distribution}, and save the word \textit{conditional} to refer the conditional distribution given investors information set. Following a low state, for example, either L will persist with probability \( \pi_{LL} \), or a transition to H will occur with \( \pi_{LH} \). The conditional distribution is a Gaussian-mixture of the ex-post distributions \( [N(\mu_L, h_L^2) \text{ and } N(\mu_L - \xi, h_H^2)] \) with probability
weights \((\pi_{LL}, \pi_L H)\) respectively. (Hamilton 1994). The conditional variance of the market portfolio is given as,

\[
\sigma^2_t = E[h^2_s|s] + \xi^2_s \text{Var}[J_{s,s'}|s]
\]

where \(s_t = s\) and \(s_{t+1} = s'\).

When the market returns depend on more than a single independent source of variation as assumed in the previous section, the traditional CAPM does not hold because the conditional market returns are not normally distributed. A single factor beta is insufficient to explain the cross-sectional variation of asset returns because it only able to capture a single dimension. Therefore, at least two factors, the jump factor and the diffusion factor, are necessary when there are two sources of variations. In fact, Todorv and Bollerslev (2011) and Bollerslev and Li (2013) study the two factor model driven by the jump and the diffusion process and show that the jump component is important in explaining the cross-sectional asset returns. Although the marginal contribution of the additional unit of diffusion risk to the conditional variance is dissimilar to that of the jump risk, a single market factor restricts a fixed relative ratio loadings given to the factors.

To observe this as an equation, consider the returns of an arbitrary portfolio \(p\) which is represented by the loadings on the two factors the diffusion and jump process.

\[
r_{p,t+1} = \alpha_{p,s} + \beta_{p,J} \xi_{s,J_{s,s'}} + \beta_{p,e}(h_{s'}) \epsilon_{p,t+1}
\]

where \(\alpha_s\) is a constant given a known state \(s\). \(\xi_{s,J_{s,s'}}\) and \(\epsilon_{m,t+1}\) are defined as in Equation (1). Note that for the market portfolio \(m\), \((\beta_{m,J}, b_{m,e}) = (1, 1)\) and hence, \(\alpha_{m,s} = \mu_s\).
For the mean-variance investor, a higher non-diversifiable conditional variance of a portfolio must be compensated by a higher premium. The conditional variance of a portfolio $p$ with loadings $\beta_J$ and $\beta_e$ on the jump and the diffusion process can be calculated as,

$$\begin{align*}
\operatorname{Var}[r_{p,t+1}|s] &= E[\operatorname{Var}(r_{p,t+1}|ss')|s] + \operatorname{Var}[E[r_{p,t+1}|ss']|s] \\
&= \beta_{p,e}^2 (\pi_s L^2 + \pi_s H^2) + \beta_{p,j}\pi_s L \pi_s H \xi_s^2
\end{align*}$$

When the diffusion and the jump process are independent, the conditional variance of the portfolio is the sum of the conditional variance that comes from the exposure to the diffusive process and that from the jump process. Both lower loadings on the diffusive process ($\beta_e$) and the jump process ($\beta_J$) implies a lower conditional variance. Therefore, a higher premium must be given if the investor takes more risk either on the diffusive or the jump part. Therefore, fitting a single market factor is insufficient because a single market beta restricts the loadings on diffusive and jump process to be identical.

The role of the loadings with respect to the diffusion and jump process can be observed by analyzing the conditional distribution of the market portfolio. The Figure 2 illustrates the conditional distribution of the market portfolio. The distribution of the low state is...
described in the left panel and the one for the high state is in the right panel. The ex-post
distribution, the hypothetical distribution conditional on future jumps weighted by the jump
probabilites are depicted as solids, and the sum of these densities are depicted as dotted
curves. In the low state, since the low state is more likely to persist the density with a
higher first moment is given a higher weight. The first moment when state persist will be
the sum of $\mu_L$ and the diffusive part. When there is a state-transition, which is less likely,
a negative jump appears in prices leading to lower returns. Thus, the market return will be
the sum of $\mu_L - \xi_L$ and the diffusive part. A lower weight is given to the distribution that
has a lower first moment. Two factor loadings, a lower beta on the diffusion process ($\beta_e$) has
a different effect to the market portfolio from a lower beta on the jump process ($\beta_J$). The
loadings on the diffusion process implies more variation in returns that is independent of the
jump process. Therefore, a lower exposure to the diffusion process shrinks the spread of the
densities depicted in solids. On the other hand, the loadings on the jump process affects the
spread between the solid densities. A lower exposure to the jump process implies a narrower
spread between the solid densities.

The case following a high state is depicted in the right panel. Similar to the case of the
low state, it is the high state that is more likely to persist. Since state transition implies
a positive shock in the prices, the density with a higher first moment is given a higher
weight. The first moment of the distribution when state persists is $\mu_h$ while the case of state
transition it is $\mu_h + \xi_h$. Therefore, the conditional distribution of the market portfolio is
the sum of the densities conditional on future jumps, weighted by the probability of jump
realizations. In short, when the market return depends on more than a single independent
source of variation, fitting the CAPM may be misleading since the model imposes restrictions
on the coefficients that needs to be considered separately.
II. Coskewness Decompositions

A. Volatility Risk and State-transition Risk

Coskewness is defined as the covariance between the return of an asset and the square of the market returns. For the reason that the coskewness adds up to be the market skewness if value-weighted, coskewness is regarded as the contribution to the market skewness. An asset that has a positive coskewness tends to perform well when the squared market returns are high. Therefore, holding an asset with positive coskewness increases the skewness of the portfolio. Thus, if investors hate the negative skewness in their portfolio returns, they need to be compensated additionally in order to hold assets that have negative coskewness. Thus, the coskewness premium must be negative. However, at least in the single state economy, when investors do not care about the third moment, coskewness should not affect asset prices.

However, when the economy consists of multiple states and the market returns vary with the state transition process, a single market factor is insufficient to explain the variation of the returns. Therefore, additional factors correlated with the jump process may affect asset returns. This section proposes a decomposition of coskewness and evaluates how the risk-averse investor’s portfolio decision is affected by each of the components. In particular, this section analyzes whether any of the components marginally decrease the conditional market variance by providing insurance against unfavorable state realizations.

The following proposition proposes a decomposition of coskewness.
Proposition 1. In a two-state economy, when the market returns follow a jump diffusion process as defined in Equation (1), coskewness can be decomposed into market beta, exposure to volatility risk and exposure to state-transitions. That is,

\[
\text{Cov}_t(r_{i,t+1}, r_{m,t+1}^2) = \text{Cov}_t(r_{i,t+1}, h^2_s \epsilon_{m,t+1}^2) + \xi_s^2 (1 + 2|\bar{\pi}_s|) \text{Cov}_t(r_{i,t+1}, 1_{s \neq s'}) \\
+ 2(\mu_s - \xi_s \bar{\pi}_s) \text{Cov}_t(r_{i,t+1}, r_{m,t+1})
\]

(5)

where \(\bar{\pi}_L = \pi_{LH}\) and \(\bar{\pi}_H = -\pi_{HL}\), \(r_{i,t+1}\) and \(r_{m,t+1}\) are the excess individual and market returns respectively, \(h^2_{s'}\) is the variance of the diffusion process at time \(t+1\) when the state is \(s'\) at \(t+1\) and \(\xi_s\) is the degree of the jump in the market price given a state \(s_t = s\).

Proof. See Appendix. ∎

The first component, referred to as the Volatility Risk (VR) component, measures an asset’s exposure to the volatility movements. This component measures whether and how much an asset outperforms when there are positive innovations in the variation of the diffusion process. The investor acknowledges that a higher variation leads to lower wealth returns. Therefore, the risk-averse investor is more willing to hold assets that have higher returns when there is more variation in the diffusion process. When volatility is low, a high VR component provides hedge against volatility increases. When volatility is high, a high VR provides hedge against volatility remaining high. Thus, a premium given to the VR component must be negative and state independent when the investor is risk averse.

The second component, referred to as the State Transition Risk (TR) component, measures whether and how much an asset outperforms when there is a state-transition or a jump shock. The jump-diffusion process defined in Equation (1) implies that a transition from a low to a high state is accompanied by a negative jump and a transition from high to low by a positive jump. Therefore, the role of the TR component depends on which state the investor is situated on. When volatility is low, a transition must lead to a high state in which there is more variation in the diffusion process. Since a transition implies a negative jump in prices,
a high TR component alleviates the negative shock driven by state-transitions. In contrast, when volatility is high, a transition would lead to a low state in which there is less variation in the diffusion process. A high TR component reinforces the positive jump, but this also aggravates the returns when state remains high. Therefore, the premium given to a high TR is negative when the volatility is low but positive when it is high.

The decomposition of coskewness shows that it is an intermingled measure of the two components. An asset that has high coskewness can be a member of two completely different types. Both assets with a high VR component or a high component may have high coskewness. First, an asset that has a high positive VR component are assets that have higher returns when volatility increases but lower when it decreases. For example, a replicating portfolio of the long straddle or strangle position, or betting on a delta-hedged VIX is essentially a strategy of the first type. The covariance with respect to the state transition indicator should be close to zero for these portfolios. The second type consists of assets that have a high TR but a neutral VR component. These assets have higher returns when there is a state-transition but does not necessarily comove with the volatility. Since state-transitions occur when there are volatility jumps, a bet on the vega-neutral volatility-of-volatility or a replicating portfolio of VVIX index are examples of the this type. This decomposition suggests that coskewness is an intermingled measure of the two components in which the role of these two types should be discussed separately as they play different roles depending on the state.

As coskewness can be decomposed into VR and TR, the premium given to coskewness must also be related to the premium given to VR or TR. First, suppose that current variations in the diffusion process is low. There is mean reversion in the volatility, thus investors expect the variations to increase eventually. In addition, the investor knows that when a news arrives that increase the variations the market will face a negative jump in the price. Therefore, she is willing to pay a premium for assets that alleviates the negative shock when such news arrives. Since both positive VR or TR components have higher returns when news arrives, any asset with positive coskewness will perform well upon a transition shock. Therefore, in
low states, coskewness always provides hedge against volatility shocks, requiring a negative premium.

The roles of the VR and the TR components are counterveiling when volatility is high. Although there are high variations in the diffusion process, the investor acknowledges that the volatility level will eventually decrease. When the state transitions to a low state, there is also a positive jump in prices. The returns are lower when states persists. A high VR component provides hedge against state persistence, because high VR implies a higher return when state remains high. However, a high TR component increases the returns when there is a transition, but exacerbates the bad state. As a result, the premium on coskewness, being an intermingled measure of the components, may either be positive or negative. In short, the sign of the coskewness premium is negative in low states. In high states, the sign is ambiguous as it depends on the relative magnitude of these two components and the relative size of the premia.

In conclusion, there are two components that affect the coskewness of an asset in a different manner. If coskewness is simply a hodge-podge of the components, the negative time-varying coskewness premium may not necessarily be driven by investors’ preference for skewness. The first component always decreases the risk of the market portfolio, and thus, contributes to the negative coskewness premium. The second component is state-dependent. While the TR premium also contributes to the negativeness of the the coskewness premium in the low states, there is a counterveiling effect in the high states. Therefore, the coskewness premium is negative when volatility is low but the sign is indeterminable.

B. Extension to \( S \geq 3 \) States

The previous section introduces a decomposition of coskewness in the case of a simplification to the two-state economy. The analysis of the two-state economy is simple because conditional on a known state, state-transitions are uni-directional. A generalization of the decomposition to the case when there are more two states is simple and similar, although a
more careful interpretation may be necessary. Assume that there are $S$ states where state 1 refers to the state when the variation in the diffusion process is the lowest and $S$ as the state where the variation is the highest. Suppose that the transition matrix is given as,

$$
\Pi = [\pi_{s,s'}]
$$

where $\Pi$ is a $S \times S$ matrix, $\pi_{s,s'} = 0$ if $|s - s'| > 1$. That is, a two step transition is not feasible. Similar to the previous section, assume that a increase in the variation in the diffusion process is associated with a negative jump in the market prices. Also, assume that a transition from $s$ to $s-1$ leads to a positive jump in prices. There is also mean-reversion in volatility, so the probability of a down-jump is decreasing as $s$ decreases, and the probability of up-jump is increasing as $s$ decreases. The following proposition proposes a decomposition of coskewness for the $S$-state case, when the state $s$ is known.

**Proposition 2.** When the market returns follow a jump diffusion process as defined in Equation (1), coskewness can be decomposed into market beta, exposure to volatility risk and exposure to state-transitions. That is,

$$
\text{Cov}_t(r_{i,t+1}, r_{m,t+1}^2) = \text{Cov}_t(r_{i,t+1}, h_{s,t+1} e_{m,t+1}^2) + \xi_s^2(1 + 2(|\pi_{s,s-1}| + |\pi_{s,s+1}|))\text{Cov}_t(r_{i,t+1}, 1_{s \neq s'}) + 2(\mu_s - \xi_s (\pi_{s,s+1} - \pi_{s,s-1}))\text{Cov}_t(r_{i,t+1}, r_{m,t+1})
$$

(6)

**Proof.** See Appendix.

The decomposition shows that even when there are more than two states, the two-component decomposition of coskewness is valid. The role of the two components in the $S$ state economy is comparable to that of a two state economy. A high VR implies a higher return when the volatility increases and a lower return when it decreases. A high TR implies a higher return when there is variation in volatility but a lower return when the volatility remains stable. Since volatility increases are accompanied by negative price jumps, a high VR provides hedge against the negative jumps in the prices. Therefore, even when there
are more than two states, a high VR provides hedge to the investor by marginally reducing the conditional variance of the market portfolio. As in the two-state case the sign of the premium is negative and state independent.

The role of a high TR component depends on whether the probability of a transition to a higher state is more likely than a transition to a lower state. Detailed proofs are shown in the Appendix. I provide the intuition here only. When volatility jumps in the future are more likely to be positive, a state-transition is more likely lead to a negative jump. Thus, a high TR will provide a hedge against such negative jumps. When jumps are more likely to be negative, a low TR component will provide hedge against no positive jumps occurring in the future. Thus the sign of the premium depends on whether possible future jumps are more likely to be positive or negative. When volatility is persistent and mean-reverting, the role of a high TR component will depend on past observations of the volatility process. When positive jumps are observed today, mean-reversion implies that jumps are more likely to be negative rather than positive. Similarly, when negative jumps are observed today and in the past, investors believe that the level of volatility is low, will increase in the future. Thus, the sign of the TR premium, and also the coskewness premium may depend on past realizations of the stochastic jumps.

C. Empirical Predictions

Several testable hypothesis can driven from the framework. This section summarizes the main empirical predictions of the framework.

First, assets that covary positively with volatility innovations must have lower subsequent returns. Ang, Hodrick, Xing, and Zhang (2006) evaluate sensitivities to volatility innovations in the cross-section, and find that assets that positively covary with the innovations have a lower premium. The prediction of the framework of this paper is consistent with their findings. An additional prediction of this paper is that the premium should not vary over
time, depending on the level of volatility or depending on past realizations of stochastic jumps.

Second, assets that have higher returns when there are volatility jumps must have lower subsequent returns when volatility is expected to rise in the future. When there is mean-reversion, these are times following a decrease or a negative jump in volatility. Following positive volatility jumps or following an increase in volatility, the level of the volatility is expected to drop in the near future. During these times, a high TR component should be given a positive premium. A high TR component act as hedge in low states, but makes exacerbates the bad states in high states.

Finally, the framework shows that coskewness can be decomposed into the VR and TR components. The coskewness premium must also be a mixture of the premia on the two components. The coskewness premium must be highly correlated with both the VR and the TR premia. Also, the coskewness premium is negative following negative volatility jumps or decreases. However, when volatility is high or following positive volatility jumps, the sign of the coskewness premium is ambiguous.

III. Estimation Methodology

When coskewness can be decomposed into two components, a high value of either one of the components can lead to a high positive coskewness. Vice versa, if coskewness is low (highly negative), at least one of the VR or the TR component must be low as well. In the following sections, I evaluate whether the same can be said for the premia on coskewness and the components. I evaluate the dynamic relation among the coskewness, VR and TR premia. When the coskewness premium is negative, it is highly likely that either the VR or the TR premium is negative. Daily S&P500 index is used in order to estimate the volatility and the jump process of the stochastic volatility model where jumps in prices and the level of volatility are correlated as in Duffie, Pan, and Singleton (2000).
A. The Stochastic Volatility Model with Double Jumps

In order to estimate the two components of coskewness VR and TR components, a stochastic volatility model (SVJJ) of Duffie, Pan, and Singleton (2000) and Eraker, Johannes, and Polson (2003) where the jumps occur both in the price and volatility process is used. The SVJJ model is appropriate for the purpose of testing the predictions of the framework in several aspects. First, both in the SVJJ and in the framework, stochastic jumps occur simultaneously in both the volatility and the price process, which are correlated. The leverage effect in both models appear through stochastic jumps. Second, the jumps increases or decreases the level of the volatility, in which in the framework is represented as state-transitions. Third, the process of the market prices are identical. There are two sources of variation that affect the market price, one of which is Gaussian and the other is a jump process. In the SVJJ model, the volatility process also has a Gaussian diffusive term which is a slight modification of the framework. However, the diffusive term in the volatility is independent of the process of the market prices which does not affect any of the analysis.

The returns of the market portfolio are assumed to follow a jump-diffusion process which is given as,

\[ r_{m,t+1} = r + \xi_{t+1} J_{t+1} + \exp(h_{t+1}/2) \epsilon_{m,t+1} \]  \hspace{1cm} (7)

The log of the volatility process is also a jump process with mean-reversion which is given as,

\[ h_{t+1} = \mu_h + \phi_h (h_t - \mu_h) + \xi^v_{t+1} J_{t+1} + \sigma_h \epsilon_{h,t+1} \]  \hspace{1cm} (8)

where \( J_{t+1} \) can either take values -1, 0 or 1, \( \exp(h_{t+1}/2) \) is the latent value of the market variance, \( \xi \) is the jump size in the market price upon stochastic jumps and \( \xi^v \) is the jump-size in the log-volatility process. By assuming that \( \xi_t \sim N(\rho_0 + \rho_1 \xi^v_t, \sigma^2_y) \), the leverage effect comes through asymmetric and simultaneous jumps in price and volatility processes. The only difference in the assumption of this model and the SVJJ model of Duffie, Pan, and
Singleton (2000) is that the model of this paper allows both positive and negative jumps in the process. Allowing for negative jumps does not cause an issue for a log-volatility process.

Monte Carlo Markov Chain (MCMC) method is used to estimate the latent log-volatility process ($h$), jump sizes ($\xi$ and $\xi^v$), the Jump process $J$ and the parameters $\Theta$ over the sample period (1957-2012). Following the convention, jumps are innovations in the volatility process that are not Gaussian nor linear. Also, the Jumps follows a Bernoulli process with jump probability $\lambda_p$ for positive jumps and $\lambda_n$ for negative jumps. In short, I assume $\xi \sim Exp(\lambda)$, $J \sim Multinomial(\lambda_n, 1 - \lambda_n, \lambda_p)$. The dimension of the parameter space $\Theta = (r, \mu_h, \phi_h, \sigma_h, \lambda_p, \lambda_n, \lambda, \rho_0, \rho_1, \sigma_y)$ is 10.

Eraker, Johannes, and Polson (2003) provides an algorithm for the MCMC simulation. Like any other volatility process, the estimates are robust to the priors. The priors are $r \sim N(0, 5), \mu_h \sim N(0, 5), \phi_h \sim Beta(20, 1.5), \sigma_h^2 \sim IG(10, 0.18), \sigma^2_x \sim IG(10, 0.18), \lambda \sim Gamma(3, 2), \rho_0 \sim N(0, 5), \rho_1 \sim N(0, 5), \lambda_p \sim Beta(5, 60)$ and $\lambda_n \sim Beta(5, 60)$. The MCMC sampling scheme is given as follows.

Step 1. Initialize $\Theta^{(0)}, h^{(0)}, \xi^{(0)}$ and $J^{(0)}$

Step 2. Sample $\Theta_j^{(1)}$ from $p(\Theta_j|\Theta_{-j}, h^{(0)}, \xi^{(0)}, J^{(0)})$ for $j = 1, 2, 10$ in sequence

Step 3. Sample $J^{(1)}$ from $p(J|\Theta^{(1)}, h^{(0)}, \xi^{(0)})$

Step 4. Sample $\xi^{(1)}$ from $p(\xi|\Theta^{(1)}, h^{(0)}, J^{(1)})$

Step 5. Sample $h^{(1)}$ from $p(h|\Theta^{(1)}, \xi^{(1)}, J^{(1)})$

Step 6. Repeat Step 1 - Step 5

The steps are straightforward except for step 5. It is known that there is a substantial improvement in efficiency when highly correlated parameters are sampled at the same time. Therefore, the volatility process needs to be sampled simultaneously. Although the volatility follows a Gaussian process in the second equation, the challenge comes from the first equation where the log volatility follows a Chi-square (1) distribution. Kim, Shephard, and Chib (1998) propose approximating the Chi-square distribution by seven normal-mixtures.
Daily S&P 500 index returns between 1956 and 2012 to estimate the whole volatility process. A total of 200,000 iterations with 5,000 burn-in samples. Table I summarizes the parameter estimates of the stochastic volatility model. The level of the average daily market volatility is estimated as 0.834% over the whole sample period. This is equivalent to 13.24% in annual terms. The level of volatility estimated through the SVJJ Model is lower than the average standard deviation of market returns because the latent variable precludes the Jump part in the market returns process. Notably, \( \phi_h \) is the degree of persistence in the level of the daily market volatility, which is high (\( \phi_h = 0.990 \)) even when there are stochastic jumps in the model. Also, \( \rho_1 < 0 \) shows that there is leverage effect that comes through the jump process. The latent volatility process is illustrated in Figure 3. The first peak is the ‘Black Monday’ of 1987 and the second peak appears during the financial crisis in 2008.

I propose a two-state classification based on two different measures. First, the framework suggests that the role of the TR component depends on whether future volatility jumps are more likely to be positive or negative. An ideal classification would be by comparing the conditional positive and negative future jump probabilities at a certain point. However,
future jump probabilities are hard to identify. Alternatively, when there is mean-reversion in volatility, negative jumps or movements in volatility are more likely following a series of positive jumps, and positive jumps are more likely to follow extreme negative movements in volatility. When positive jumps are observed in the previous month, investors expect a reversal in the near future. Thus, a month $T$ is classified as a high volatility month if the aggregated volatility jumps ($\xi J$) for month $T-1$ is greater than the historical third quartile (75%) of the all the historical jumps between month $T-61$ and $T-1$. Similarly, a month $T$ is classified as a low volatility month if volatility jumps at $T-1$ are smaller than the first quartile (25%) of the degree of historical jumps between $T-61$ and $T-1$. Second, I consider is a non-parametric measure that does not depend on the model specification. I compute the realized volatility for each month, and classify a month $T$ as a high volatility month if there was an increase in the level of volatility at time $T-1$. If the realized volatility decreases in month $T-1$, it is classified as a low volatility month. The volatility classification based on the first measure is provided in Figure 3. High volatility months are depicted in greyscale.

The first 7 years are dropped from the volatility estimates these are later used for VR and TR estimation. Hence, the sample consists of months between 1962 and 2012. There are a total of 612 months during the 51 years. According to the classification based on the stochastic volatility model, 111 months (18.1%) are classified as low volatility months and 194 months (31.7%) are classified as high. Among 111 low volatility months, 21.6% are classified as a high volatility month according to the second measure, while 84.2% of the high volatility months according to the first measure is also classified as a high volatility month. Therefore, two classifications are positively correlated. Table II provides the summary statistics of the economic variables for each of the volatility states. Out of the 90 NBER recession months in the sample, 39 of them (43.3%) are classified as high volatility months with respect to the first measure. Only 10% of the NBER months are classified as low volatility months. High volatility months tend to have a lower term premium, higher default premium, lower unemployment rate higher inflation rate and lower market returns.
The framework suggests that the state classifications should be based on whether volatility movements in the future are more likely to be positive or negative. Following a low state, volatility must be low but must increase. Following a high state, volatility may stay high but must decrease. The first moment of the market returns are higher following a high state as additional premium should be given to higher level of variation in the volatility. The last four rows of Table II summarizes the moments of ex-post market returns. For January classification, for example, the moments are computed using the daily market returns for February and March. Consistent with the predictions of the inter-temporal CAPM of Merton (1973), high volatility periods have a higher subsequent return for both classifications. The level volatility is also high during these periods, and the market tends to be less negatively skewed. Finally, the change in volatility level, calculated by the difference between time monthly standard deviation of returns of month t+2 and t, shows that there is reversal in volatility. Positive volatility movements are predicted following negative volatility jumps and negative movements are predicted following positive jumps. This shows that both of the classifications are consistent with the framework.

(B. Estimation of Coskewness and its Components)

The framework suggests that coskewness is an intermingled measure of its two components. The VR component, measures the covariance between the asset returns and the innovations in the market volatility. A portfolio with a high VR component is a hedge to the market regardless of the current or past realizations of the volatility. The TR component measures the relative performance upon a state transition, which occurs when there is a substantial movement in volatility. A high TR component is a hedge in low states as it increases the relative returns of the bad state. In high states, this component adds risk to the market portfolio so that a positive premium is required. As a result, both components marginally reduce the risk of the market portfolio during the low states and the premia on both components

(Insert Table II here)
should be negative in the low states. In contrasts, the two components play an opposite role in the high states. The premium given to the VR component should be negative while the premium given to the TR component should be positive. This section discusses how the VR and the TR components are estimated.

Ang, Hodrick, Xing, and Zhang (2006) define innovations in volatility as the difference between the current level of the option-implied volatility index (VXO) and the lagged value of VXO. They use one-month horizon of daily data to estimate the VR component and find that assets with high volatility sensitivity have lower subsequent returns. Adrian and Rosenberg (2008) use a component GARCH model aggregating daily data to define monthly innovations in volatility and show that a negative premium is given to both the short-run and the long-run innovations in volatility. The stochastic volatility model directly assumes volatility innovations (VI) to be

$$VI_{t+1} = h_{t+1} - \mu_h - \phi_h(h_t - \mu_h)$$

where $\mu_h, \phi_h, h$ are the estimates of the MCMC simulation.

The TR component measures an asset’s exposure to jump risk. Therefore, the TR component can be computed as the covariance with the absolute value of the jump variable.

The daily series of the volatility innovation and the absolute value of the jumps are then aggregated to the monthly level by taking the sum of the variables over a particular month. Based on these monthly variables, I run a three-factor regression including the market factor, volatility innovations and stochastic jumps. Then, the VR and the TR components of an asset is the rolling beta coefficient of the following time-series regression.

$$r_{i,t+1} = \alpha_i + \beta_{VR,i}VI_{t+1} + \beta_{m,t+1} + \beta_{TR,i}|J_{t+1}| + \epsilon_{i,t+1}$$ (9)
Finally, coskewness is defined as the covariance between the square of the market returns and the return of a particular asset. The coskewness ($\hat{\gamma}_{i,t}$) in this paper is estimated using the following rolling 3M-CAPM time-series regression.

$$r_{i,t+1} = \alpha_i + \beta_i r_{m,t+1} + \gamma_{it} r^2_{m,t+1} + \epsilon_{i,t+1}$$

(10)

where $r_{i,t+1}$ is the excess return of an asset and $r_{m,t+1}$ is the market excess return. Frisch-Waugh-Lovell theorem implies that coskewness estimated using this regression equivalent to a single factor $\gamma_{i,t}$ when both the asset returns and the squared market returns are orthogonalized by the market returns. Accurate estimates that rely on higher moments are more susceptible to misspecification error as these estimates rely on observations with extreme values. Therefore, these values need to be estimated using a higher number of observations. Therefore, I use 100-month to estimate the components on rolling basis.

C. Data and Portfolio Strategy

The volatility model is estimated using daily S&P 500 returns and are aggregated to the monthly level. VR is the covariance of the return of an asset and the sum of the volatility innovation in that particular month. Similarly, TR is the covariance of an asset with respect to the sum of the jumps in that particular month. Given these components, the dynamic relation among the coskewness, VR and TR can be analyzed either (1) by directly comparing the relation (2) or by comparing the time-variation of the premia given to coskewness, VR and TR.

Data of monthly stock returns acquired from the Center for Research in Security Prices (CRSP) is used to compute the portfolio returns. The sample consists of individual common stocks traded on NYSE, AMEX and NASDAQ between 1962 and 2012. I exclude ADRs, REITs, closed-end funds and foreign stocks. I also require individual stocks to be traded at least for twelve months. Finally, the market value of the individual stock has to be non-
missing at each time point which is necessary to compute the value-weighted return at the portfolio level. Therefore, any stock that has a missing price is naturally omitted.

At the end of each quarter, I sort the stocks with respect to the level of coskewness, VR and TR components. Equally-weighted quintiles are formed based on the rankings. Coskewness-quintile 1 consists of assets that the estimated coskewness is most negative. Coskewness-quintile 5 consists of assets that have the highest covariance. The quintile portfolios are then held for twelve months. For example, in January of 2010, Quintile 1 consists of four value-weighted portfolios that have the lowest coskewness measured in March, June, September and December of 2009 each of which is weighted by 25%. The VR-quintiles and the TR-quintiles are formed in the same manner.

(Insert Table III here)

Table III provides the summary statistics of the sorted portfolios. Panel A., Panel B. and C. provides the summary statistics of some key variables for VR, TR component and coskewness-sorted quintiles, respectively. The last three columns of Panel A., B., and C. shows the relation between the three covariances. The table shows that assets with high VR tend to have higher coskewness but lower TR covariances. Assets with higher TR tend to have lower VR components but higher coskewness. Finally, Assets with positive coskewness tend to have both higher VR and TR components. This inter-relation suggests that VR and TR may be components of coskewness that are negatively correlated to each other.

Other statistics such as average size, idiosyncratic volatility, skewness of the excess returns, skewness of the idiosyncratic risk are also summarized in the table. Size is measured as the time-series average of the total market capitalization of each portfolio. Adesi, Gagliardini and Urga (2004) argue that the coskewness premium is driven by small firms. Panel C. shows that firms that have equity with positive coskewness tend to be larger than negative coskewness firms. This is consistent with their finding that coskewness premium is related to the size of the firm. Notably, both VR and TR premium also tend to be small stock phenomena. Idiosyncratic volatility is measured using the $R^2$ from a market model. A high $R^2$
implies that a higher proportion of the variation in returns is explained by the market factor. Thus, a high $R^2$ means lower idiosyncratic volatility. Ang, Hodrick, Xing, and Zhang (2006) provide evidence that the premium on high idiosyncratic risk stocks is related to coskewness but coskewness alone is not enough to explain the premium. Panel C. confirms that they are related since negative coskewness assets have higher idiosyncratic volatility. The reason stems from the TR rather than the VR part of coskewness. The third and the forth column summarizes the skewness of the excess returns and the residuals from a single factor model. Although coskewness measures the marginal contribution to the market skewness, returns of positive coskewness assets are not necessarily positively skewed. This is because higher VR assets tend to be more positively skewed, but higher TR assets tend to be more negatively skewed.

The summary statistics supports the idea that coskewness is the sum of two components which depends on the exposure to variability in the diffusion process and stochastic jumps. The following section presents the inter-relation among the VR, TR and the coskewness premium.

**IV. Empirical Results**

This section provides the main empirical result of this paper. The framework of this paper predicts a negative VR premium that is state independent and a time-varying, state-dependent TR premium. In particular, when there is mean-reversion in volatility, the TR premium is negative following negative volatility jumps but positive following positive volatility jumps. Therefore, both the VR and the TR premia are negative in the low states, but only the VR premium is negative in the high states. The sign and size of the coskewness premium, a miscellany of VR and TR, may depend on the relative size and the importance of the two components. In low volatility periods, the coskewness premium may be negative, but there may be a reversal in high volatility periods.
Table IV summarizes the risk-adjusted returns (Jensen’s alpha) of the VR (Panel A.), TR (Panel B.) and coskewness Panel C.) sorted portfolios evaluated over the whole sample period as well as over a subsample based on the classification provided in the previous section. The first column of each panel summarizes the risk-adjusted returns of the portfolios evaluated over the whole period. The second and the third columns of each panel summarize the risk-adjusted returns evaluated over the sub-periods where the classification is based on the realized volatility. The final two columns summarize the returns when the classification is based on the stochastic volatility model. Low periods are times following a negative jump in the market (SVJJ) or when there is a decrease in the level of realized volatility.

The first column of Panel A. shows that the premium given on the volatility risk component is negative and statistically significant. Investors perceive high VR assets as hedge to the market portfolio, thus, demand a lower premium. Moreover, the premium does not depend on past realizations of volatility. The difference between low and high periods is not statistically significant, suggesting that the role of VR is state-independent. Panel B. shows that the premium given to the TR component is state-dependent. The sign of the TR premium is indeterminable evaluated if evaluated over the whole period. However, the TR premium can be predicted by past volatility jumps as shown in the table. In the high states, the TR premium is positive and is higher than the low states. This table shows that high TR assets are hedge assets in a sub-period following negative jumps in the market, but when state transitions, the role of the TR component switches.

Panel C. summarizes the risk-adjusted returns for the coskewness-sorted portfolios. The table suggests that coskewness might be a hodge-podge of the two components. There are at least two things to note from this table. First, evaluated over the whole sample period, the premium is negative but statistically insignificant. However, Harvey and Siddique (2000) and Dittmar (2002) report a negative coskewness premium. Smith (2006) finds a negative coskewness premium conditional on positively skewed historical market returns.
These papers use a generalized methods of moments to estimate the sign and the size of the premium. The coskewness premium may be negative, but may not be statistically significant because the TR premium crowds-out the VR premium. As the sign of the TR premium is indeterminable, the sign of the premium must be affected by the VR premium. Second, the premium is time-varying and depends on past realizations of the stochastic jumps. The coskewness premium is negative in the low states following negative stochastic jumps, but positive in the high states following positive jumps. The time-variation resembles that of the TR premium. Therefore, the state-dependency must be driven by the transition risk component. To see if this is the case, consider a regression model where the coskewness premium is a dependent variable, and where the lagged jump process is an independent variable.

(Insert Table V here)

Table V summarizes the coefficients of the regression along with the Newey-West adjusted t-statistics. CSK is the coskewness premium defined as the top minus bottom quintile risk-adjusted returns of the coskewness-sorted portfolios. The VR and TR premia are constructed similarly. HML, SMB, UMD are the Fama and French (1993) three factors obtained from French’s website. High Dummy variables are based on the state-classifications proposed in the previous section. $\xi_{t-1}$ is the lagged stochastic jump aggregated to the monthly level and $RV_{t-1}$ is the level of the realized volatility over the past month. (1) and (3) show that the coskewness premium is related to the size premium but not to the value premium or momentum. This is consistent with the findings of Adesi, Gagliardini and Urga (2004). In any specification, the coskewness premium is highly correlated with both the VR premium and TR premium, even after controlling for the Fama-French three factors. (4), (6), (8), (10) show that the coskewness premium is higher following positive volatility jumps or when volatility has recently increased. However, after controlling for both the VR and TR premium, the coskewness premium is no longer increasing in lagged volatility jumps. The regressions suggests that the time-variation of the coskewness premium is driven by the transition risk premium.
V. Conclusion

In summary, this paper shows that coskewness is an intermingled measured of two components: the volatility risk (VR) component and the state-transition risk (TR) component. A high VR component provides a hedge against volatility increases while the role of the TR component depends on the level of the volatility. When volatility is low, a high TR provides hedge against volatility increases. When it is high, a high TR exacerbates the bad state when volatility remains high.

The Coskewness of an asset can be high when the sensitivity of its returns to the innovations in market volatility is high. This provides a hedge because increase in volatility typically is related to negative shocks in the market. Coskewness can also be high when an asset has higher returns when volatility moves abruptly. These exponential jumps can be either positive or negative, and the relative probability of a positive jump determines whether this second component provides a hedge. If there is mean reversion in volatility, a high value provides a hedge during low volatility times. During other times, a low value provides a hedge.

Evaluating the two components of coskewness helps explain the time-variation of the coskewness premium. The premium given to coskewness is positively correlated with the market volatility. It is also known that the premium is on average negative. The negativity can be explained by the VR component as the premium is negative and statistically significant. The time-varying property can be explained by the TR component since the TR premium is also positively correlated with market volatility. In subsequent research, I evaluate whether and how much of the cross-sectional variation of asset returns can be explained by the two components.
References


A. Coskewness Decompositions

The coskewness decomposition in the case of two states is a special case of the decomposition when there are S states. After taking the squares on the market returns process in Equation (1), the squared market returns can be expressed as,

\[
r_{m,t+1}^2 = \mu_s^2 + \xi_s^2 \mathbb{1}_{s \neq s'} + 2\{\mu_s \xi_s J_{s,s'} + (\mu_s - \xi_s J_{s,s'}) \sigma_s \epsilon_{m,t+1}\} + \sigma_{s'}^2 \epsilon_{m,t+1}^2
\]

\[
= \mu_s^2 + \xi_s^2 \mathbb{1}_{s \neq s'} + \sigma_{s'}^2 \epsilon_{m,t+1}
\]

\[
+ 2\mu_s r_{m,t+1} - 2\xi_s J_{s,s'} \sigma_s \epsilon_{m,t+1}
\]

where \( s_t = s \) and \( s_{t+1} = s' \). Note that \( J_{s,s'} \) and \( \sigma_s \epsilon_{m,t+1} \) are independent. Using this relation, coskewness of an asset i can be decomposed into,

\[
Cov_t(r_{i,t+1}, r_{m,t+1}^2) = Cov_t(r_{i,t+1}, \sigma_{s'}^2 \epsilon_{m,t+1}^2) + \xi_s^2 Cov_t(r_{i,t+1}, \mathbb{1}_{s \neq s'})
\]

\[
+ 2\mu_s Cov_t(r_{i,t+1}, r_{m,t+1}) - 2\xi_s \bar{\pi}_s Cov_t(r_{i,t+1}, \epsilon_{m,t+1} \sigma_{t+1}^2)
\]

(11)

\[
= Cov_t(r_{i,t+1}, \sigma_{s'}^2 \epsilon_{m,t+1}^2) + \xi_s^2 (1 + |\pi_{s,s+1}| + |\pi_{s,s-1}|) Cov_t(r_{i,t+1}, \mathbb{1}_{s \neq s'})
\]

\[
+ 2(\mu_s - \xi_s (|\pi_{s,s+1} - |\pi_{s,s-1}|)) Cov_t(r_{i,t+1}, r_{m,t+1})
\]

(12)

. In the last equation, the fact that \( J_{s,s+1} = -1 \) is used to derive the decomposition.

B. The Role of the TR component

Suppose \( r_{p,t+1} \) follows a two factor structure determined by the market returns process. That is,
\[ r_{p,t+1} = \alpha_p + \beta_J \xi_s J_{s,s'} + \beta_e \epsilon_{m,t+1} + \epsilon_{p,t+1}. \]

where \( \epsilon_{m,t+1} = \sigma_{t+1} \epsilon_{t+1} \) and others as defined in the framework.

\[
Cov(r_{p,t+1}, 1_{s \neq s'}) = \beta_J \xi_s Cov(J_{s,s'}, J_{s,s'}^2)
\]

\[
= \beta_J \xi_s \{ E[J_{s,s'}^3] - E[J_{s,s'}] E[J_{s,s'}^2] \}
\]

\[
= \beta_J \xi_s \{ E[J_{s,s'}](1 - E[J_{s,s'}]) \}
\]

\[
= \beta_J \xi_s \{ (\pi_{s,s+1} - \pi_{s,s-1})(1 - \pi_{s,s+1} - \pi_{s,s-1}) \}
\]

When \( \pi_{s,s+1} - \pi_{s,s-1} > 0 \), TR is linearly increasing in \( \beta_J \).

\rightarrow \text{High TR is bad (increases the conditional variance of the portfolio) if a transition to a higher state is more likely.}
### Table I
Parameter Estimates of the SVJJ Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>95% HPD Credible Interval</th>
<th>95% HPD Credible Interval</th>
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</thead>
<tbody>
<tr>
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<td>µ&lt;sub&gt;h&lt;/sub&gt; -1.178 -1.576 -0.660</td>
</tr>
<tr>
<td>φ&lt;sub&gt;h&lt;/sub&gt;</td>
<td>0.990 0.988 0.993</td>
<td>σ&lt;sup&gt;2&lt;/sup&gt;&lt;sub&gt;h&lt;/sub&gt; 0.009 0.007 0.011</td>
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<tr>
<td>λ&lt;sub&gt;p&lt;/sub&gt;</td>
<td>0.022 0.013 0.035</td>
<td>σ&lt;sup&gt;2&lt;/sup&gt;&lt;sub&gt;ξ&lt;/sub&gt; 0.693 0.521 0.948</td>
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<tr>
<td>λ&lt;sub&gt;m&lt;/sub&gt;</td>
<td>0.011 0.004 0.020</td>
<td>ρ&lt;sub&gt;0&lt;/sub&gt; 0.002 -0.136 0.144</td>
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<td>λ</td>
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<td>ξ</td>
<td>0.015 e^ξ 0.834</td>
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<td>Volatility States</td>
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<tr>
<td></td>
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<td>Low</td>
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<td># of Months</td>
<td>111</td>
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<td>% of High (Other Classification)</td>
<td>21.62%</td>
<td>84.02%</td>
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<td>Default Premium</td>
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<td>Inflation</td>
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<td>Skewness</td>
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<td>Δ Volatility</td>
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### Table III
Characteristics and Summary Statistics of VR, TR and Coskewness-Sorted Portfolios

**Panel A. VR-sorted Quintiles**

<table>
<thead>
<tr>
<th>Quintile</th>
<th>Idiosyncratic Size (1MM USD)</th>
<th>$R^2$</th>
<th>Skewness</th>
<th>Skewness</th>
<th>Coskewness</th>
<th>VR</th>
<th>TR</th>
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<tr>
<td>Quintile 1</td>
<td>521.21</td>
<td>0.256</td>
<td>0.912</td>
<td>0.978</td>
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<td>-2.143</td>
<td>0.097</td>
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<td>0.572</td>
<td>0.647</td>
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<td>-0.643</td>
<td>0.012</td>
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<td>Quintile 3</td>
<td>1,839.40</td>
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<td>0.548</td>
<td>0.612</td>
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<td>-0.093</td>
<td>-0.016</td>
</tr>
<tr>
<td>Quintile 4</td>
<td>1,706.81</td>
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<td>0.634</td>
<td>0.680</td>
<td>-2.457</td>
<td>0.429</td>
<td>-0.043</td>
</tr>
<tr>
<td>Quintile 5</td>
<td>737.92</td>
<td>0.266</td>
<td>1.081</td>
<td>1.093</td>
<td>-3.006</td>
<td>1.797</td>
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<td>5-1</td>
<td>216.71</td>
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<td>0.115</td>
<td>3.170</td>
<td>3.940</td>
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<td>(5.39)</td>
<td>(3.26)</td>
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**Panel B. TR-sorted Quintiles**

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<th>Quintile</th>
<th>Idiosyncratic Size (1MM USD)</th>
<th>$R^2$</th>
<th>Skewness</th>
<th>Skewness</th>
<th>Coskewness</th>
<th>VR</th>
<th>TR</th>
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<tr>
<td>Quintile 1</td>
<td>466.68</td>
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<td>1.099</td>
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<tr>
<td>Quintile 2</td>
<td>1,298.87</td>
<td>0.285</td>
<td>0.633</td>
<td>0.708</td>
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<td>0.241</td>
<td>-0.058</td>
</tr>
<tr>
<td>Quintile 3</td>
<td>1,882.88</td>
<td>0.296</td>
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<tr>
<td>Quintile 4</td>
<td>1,875.85</td>
<td>0.296</td>
<td>0.632</td>
<td>0.586</td>
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<td>Quintile 5</td>
<td>710.24</td>
<td>0.273</td>
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<td>-1.551</td>
<td>0.145</td>
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<td>5-1</td>
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<td>(4.80)</td>
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**Panel C. Coskewness-sorted Quintiles**

<table>
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<tr>
<th>Quintile</th>
<th>Idiosyncratic Size (1MM USD)</th>
<th>$R^2$</th>
<th>Skewness</th>
<th>Skewness</th>
<th>Coskewness</th>
<th>VR</th>
<th>TR</th>
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<tr>
<td>Quintile 1</td>
<td>295.21</td>
<td>0.239</td>
<td>0.959</td>
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<td>-0.041</td>
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<tr>
<td>Quintile 2</td>
<td>756.86</td>
<td>0.268</td>
<td>0.690</td>
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<td>-0.022</td>
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<tr>
<td>Quintile 3</td>
<td>1,436.11</td>
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<td>0.600</td>
<td>0.669</td>
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<td>Quintile 4</td>
<td>2,109.94</td>
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<td>0.604</td>
<td>0.637</td>
<td>0.875</td>
<td>-0.040</td>
<td>-0.006</td>
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<tr>
<td>Quintile 5</td>
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<td>0.298</td>
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<td>0.872</td>
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<td>(-1.01)</td>
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# Table IV

Performance of VR, TR and Coskewness-sorted Portfolios

## Panel A. VR-sorted Quintiles

| Quintile | Excess Returns | CAPM alpha |  |
|-----------|----------------|-------------|
|           | Realized Vol   | SVJJ        | Realized Vol | SVJJ |
|           | Whole Low High | Low High    | Whole Low High | Low High |
| Quntile 1 | 0.74% 1.82% -0.48% | 1.85% -0.87% | 0.21% 0.23% 0.17% | 0.06% 0.13% |
| Quntile 2 | 0.55% 1.46% -0.45% | 1.56% -0.73% | 0.08% 0.13% 0.05% | 0.05% 0.11% |
| Quntile 3 | 0.50% 1.27% -0.36% | 1.51% -0.58% | 0.06% 0.01% 0.11% | 0.03% 0.23% |
| Quntile 4 | 0.47% 1.34% -0.51% | 1.56% -0.78% | 0.00% 0.00% 0.01% | -0.04% 0.10% |
| Quntile 5 | 0.50% 1.59% -0.72% | 1.86% -1.25% | -0.13% -0.13% -0.06% | -0.15% -0.14% |
| 5-1       | -0.24% -0.23% -0.23% | 0.01% -0.38% | -0.34% -0.35% -0.23% | -0.21% -0.27% |
| High - Low| -0.01% -0.39% 0.12% | -0.05% | (-0.03) (-1.22) (0.58) | (-0.18) |

## Panel B. TR-sorted Quintiles

| Quintile | Excess Returns | CAPM alpha |  |
|-----------|----------------|-------------|
|           | Realized Vol   | SVJJ        | Realized Vol | SVJJ |
|           | Whole Low High | Low High    | Whole Low High | Low High |
| Quntile 1 | 0.59% 2.05% -1.15% | 1.48% -0.17% | -0.01% 0.26% -0.29% | 0.26% -0.21% |
| Quntile 2 | 0.55% 1.50% -0.70% | 1.13% 0.04% | 0.06% 0.11% 0.00% | 0.15% -0.01% |
| Quntile 3 | 0.50% 1.38% -0.56% | 0.89% 0.14% | 0.06% -0.01% 0.13% | -0.02% 0.11% |
| Quntile 4 | 0.47% 1.30% -0.56% | 0.84% 0.11% | 0.01% -0.11% 0.13% | -0.11% 0.09% |
| Quntile 5 | 0.57% 1.71% -0.81% | 1.03% 0.13% | 0.07% 0.07% 0.07% | -0.05% 0.16% |
| 5-1       | -0.02% -0.34% 0.34% | -0.44% 0.30% | 0.08% -0.18% 0.36% | -0.31% 0.37% |
| High - Low| 0.69% 0.74% 0.97% | 0.55% 0.67% | (3.18) (3.44) (2.67) | (3.36) |

## Panel C. Coskewness-sorted Quintiles

| Quintile | Excess Returns | CAPM alpha |  |
|-----------|----------------|-------------|
|           | Realized Vol   | SVJJ        | Realized Vol | SVJJ |
|           | Whole Low High | Low High    | Whole Low High | Low High |
| Quntile 1 | 0.52% 2.18% -1.34% | 1.84% -1.57% | 0.03% 0.31% -0.28% | 0.04% -0.32% |
| Quntile 2 | 0.52% 1.79% -0.91% | 1.66% -0.96% | 0.07% 0.19% -0.05% | 0.08% 0.09% |
| Quntile 3 | 0.54% 1.60% -0.65% | 1.59% -0.65% | 0.09% 0.08% 0.09% | 0.10% 0.10% |
| Quntile 4 | 0.43% 1.36% -0.61% | 1.46% -0.71% | 0.08% 0.00% 0.17% | 0.07% 0.21% |
| Quntile 5 | 0.47% 1.43% -0.62% | 1.75% -0.74% | -0.06% -0.22% 0.16% | -0.25% 0.26% |
| 5-1       | -0.05% -0.75% 0.72% | -0.09% 0.83% | -0.09% -0.53% 0.44% | -0.29% 0.58% |
| High - Low| 1.47% 0.92% 0.97% | 0.87% | (4.55) (1.84) (3.96) | (2.40) |
Table V  
Dynamic Relation among the VR, TR and the Coskewness Premium

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<th>CSK</th>
<th>CSK</th>
<th>CSK</th>
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<td>(2.24)</td>
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<td>(\xi_{J_{t-1}})</td>
<td>0.28</td>
<td>0.08</td>
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<td>(2.61)</td>
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<td>High Dummy(RV)</td>
<td>5E-05</td>
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<td>(2.02)</td>
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<td>(RV_{t-1} - RV_{t-2})</td>
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<td>0.93</td>
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<td>(2.62)</td>
<td>(-0.47)</td>
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<tr>
<td>Adj (R^2)</td>
<td>17.23%</td>
<td>25.00%</td>
<td>28.56%</td>
<td>0.76%</td>
<td>25.10%</td>
<td>0.97%</td>
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<td>25.18%</td>
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<td>0.04%</td>
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