Ex. Two rivers: Ra, Rb
A paper mill is allowed to dispose its waste into these rivers.

\[ E_a = \text{River Ra has pollutant greater than acceptable limit} \]
\[ E_b = \text{River Rb has pollutant} \]

\[ P(E_a) = 0.2 \]
\[ P(E_b) = 0.33 \]
\[ P(E_a \cap E_b) = 0.1 \]

On a given day what is the probability that both or at least one river has unacceptable level of pollutant?

\[ P(E_a \cup E_b) = P(E_a) + P(E_b) - P(E_a \cap E_b) = 0.43 \]
What is the probability that on a given day either one of the two rivers is polluted?

\[
P(A) = \frac{P(E_a E_b^c U E_a^c E_b)}{P(E_a E_b^c) + P(E_a^c E_b)}
\]

\[
= P(E_b^c | E_a) P(E_a) + P(E_a^c | E_b) P(E_b)
\]

\[
= 0.33
\]

Now,

\[
P(E_b^c | E_a) = 1 - P(E_b | E_a)
\]

\[
P(E_a^c | E_b) = 1 - P(E_a | E_b)
\]

\[
P(E_a | E_b) = \frac{P(E_a E_b)}{P(E_b)} = \frac{0.1}{0.3} = 0.33
\]

\[
P(E_b | E_a) = \frac{P(E_a E_b)}{P(E_a)} = \frac{0.1}{0.2} = 0.5
\]
Events:

- $R_1 = \text{Route 1 is open}$
- $R_2 = \text{Route 2 is open}$

During a wildfire:

- $P(R_1) = 0.8$
- $P(R_2) = 0.4$

$P(R_1 \cap R_2) = 0.25$

What is the probability that Route 1 is open given that Route 2 is open:

$$P(R_1 | R_2) = \frac{P(R_1 \cap R_2)}{P(R_2)} = \frac{0.25}{0.4} = 0.625$$
What is the probability that Route 1 is closed given that Route 2 is closed?

\[
P(R_1^c | R_2^c) = \frac{P(R_1^c \cap R_2^c)}{P(R_2^c)}
\]

\[
P(R_1^c) = 1 - P(R_1) = 0.2
\]

\[
P(R_2^c) = 1 - P(R_2) = 0.6
\]

\[
P(R_1^c \cap R_2^c) = 1 - P([R_1^c \cap R_2^c]^c)
\]

\[
= 1 - P(R_1 \cup R_2)
\]

\[
= 1 - \left[ P(R_1) + P(R_2) - P(R_1 \cap R_2) \right]
\]

\[
= 1 - [0.2 + 0.4 - 0.25]
\]

\[
= 1 - 0.95
\]

\[
= 0.05
\]

Hence, \( P(R_1^c | R_2^c) = \frac{0.05}{0.6} = 0.0833 \)
Monty Hall Problem

Gameshow:
Choice of three doors:
• behind one door is a car
• the other two = two goats

You pick door 1
Host opens door 3 and shows you a goat

Now, do you want to change your pick?

Classical Soln. $\frac{1}{3}$

[Diagram of three doors with门1, 2, 3, and the probability of $\frac{2}{3}$]
$c_1, c_2, c_3$ indicate that the car is behind door 1, 2, and 3 respectively.

\[ \therefore P(c_1) = P(c_2) = P(c_3) = \frac{1}{3} \]

The player chooses door 1 $\Rightarrow$ event $X_1$,

\[ P(c_1 | X_1) = P(c_2 | X_1) = P(c_3 | X_1) = \frac{1}{3} \]

Host opens door 3 $\Rightarrow$ event $H_3$

\[ P(H_3 | c_1, X_1) = \frac{1}{2} \]
\[ P(H_3 | c_2, X_1) = 1 \]
\[ P(H_3 | c_3, X_1) = 0 \]
\[ P(C_2 \mid H_3, X_1) = \frac{P(H_3 \mid C_2, X_1)P(C_2 \cap X_1)}{P(H_3 \cap X_1)} \]

\[ = \frac{P(H_3 \mid C_2, X_1)P(C_2 \cap X_1)}{P(H_3 \mid C_1, X_1)P(C_1 \cap X_1) + P(H_3 \mid C_2, X_1)P(C_2 \cap X_1) + P(H_3 \mid C_3, X_1)P(C_3 \cap X_1)} \]

\[ = \frac{P(H_3 \mid C_2, X_1)}{P(H_3 \mid C_1, X_1) + P(H_3 \mid C_2, X_1) + P(H_3 \mid C_3, X_1)} \]

\[ = \frac{1}{\frac{1}{2}+1+0} = \frac{2}{3}. \]