Sample Quiz

TBA

Machine Learning

CSCI 567 Spring 2016

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Name: ____________________

USC ID: ____________________

It is completely closed-book (and consulting Internet or other electronic resources is not permitted). Must be done individually, no discussion is allowed!

This exam contains **ten multiple choice questions** (ONLY one correct choice for each question) and **four short questions**.

MULTIPLE CHOICE QUESTIONS: ONLY one correct choice for each question. Please write down your answer in front of each question ID.

Q1 Matrix Algebra Which identity is NOT correct for two real-valued matrices A and B? Assume that inverses exist and multiplications are legal.

(a) $(AB)^{-1} = B^{-1}A^{-1}$

(b) $(A + B)^\top = A^\top + B^\top$

(c) $(A^\top)^{-1} = (A^{-1})^\top$

(d) $(I + A)^{-1} = I - A^{-1}$

Q2 Probability Rules Suppose A and B are two events, which of the following identity is in general NOT true?

(a) $P(A \cap B) = P(A)P(B)$

(b) $P(A \cup B) \leq P(A) + P(B)$

(c) $P(A \cup B) \geq P(A \cap B)$

(d) $P(A) = 1 - P(\overline{A})$, where $\overline{A}$ denotes the complement of event A.

Q3 Conditional Probability A family has two children with equal probability of being a girl or a boy. Suppose that we know at least one of them is a girl. What is the probability that both of them are girls? Suppose that the first child is a girl. What is the probability that the second child is also a girl?

(a) $\frac{1}{3}$ and $\frac{1}{2}$

(b) $\frac{1}{2}$ and $\frac{1}{2}$

(c) $\frac{1}{3}$ and $\frac{2}{3}$

(d) $\frac{1}{2}$ and $\frac{2}{3}$

Q4 Distributions Suppose you want to simulate the number of students who are taking this exam. What is the most suitable distribution for characterizing the number of students taking this exam?

(a) Gaussian

(b) Exponential

(c) Bernoulli

(d) Poisson

Q5 Mean and Variance Suppose x is a random variable which is distributed uniformly between 0 and 2. What are the mean and variance of x?

(a) $1, \frac{1}{3}$

(b) $0, 1$

(c) $1, \frac{\sqrt{3}}{3}$

(d) $1, 1$

Q6 Convexity Suppose $\mathbf{x}, \mathbf{a} \in \mathbb{R}^{n\times 1}$ are two arbitrary vectors . Which one of the following functions is NOT convex:

(a) $f(\mathbf{x}) = \|\mathbf{x}\|_1 = \sum_{i=1}^n |x_i|$

(b) $f(\mathbf{x}) = \sum_{i=1}^n a_i x_i$

(c) $f(\mathbf{x}) = \min_{i=1}^n a_i x_i$

(d) $f(\mathbf{x}) = \sum_{i=1}^n \exp(x_i)$

Q7 Calculus Suppose $f(\boldsymbol{\beta}, \{y_i, \mathbf{x}_i\}_{i=1}^n) = \sum_{i=1}^n (\boldsymbol{\beta}^\top \mathbf{x}_i - y_i)^2$ where $\mathbf{x}_i, \boldsymbol{\beta} \in \mathbb{R}^{p\times 1}$ and y_i is a scalar. What is the derivative $\frac{\partial f(\boldsymbol{\beta}, \{y_i, \mathbf{x}_i\}_{i=1}^n)}{\partial \boldsymbol{\beta}}$?

(a) $2\sum_{i=1}^n |\boldsymbol{\beta}^\top \mathbf{x}_i - y_i|\boldsymbol{\beta}$

(b) $2\sum_{i=1}^n (\boldsymbol{\beta}^\top \mathbf{x}_i - y_i)\mathbf{x}_i$

(c) $2\sum_{i=1}^n |\boldsymbol{\beta}^\top \mathbf{x}_i - y_i|\mathbf{x}_i$

(d) $2\sum_{i=1}^n (\boldsymbol{\beta}^\top \mathbf{x}_i - y_i)\boldsymbol{\beta}$

Q8 Gaussian Distribution Suppose x and y are two independent Gaussian random variables. Which of the following variable is Gaussian?

(a) $4x + 3y$

(b) x^2

(c) $x \times y$

(d) $z = \begin{cases} x & \text{with probability } \frac{1}{2} \\ y & \text{with probability } \frac{1}{2} \end{cases}$

Q9 Eigenvalues Suppose $A \in \mathbb{R}^{n \times n}$ and $\mathbf{v} \in \mathbb{R}^{n \times 1}$ are given such that $\text{rank}(A) = n$. Choose the matrix whose eigenvalues are ALL real and positive.

(a) $A + A^\top$

(b) $AA^\top$

(c) $\mathbf{v}\mathbf{v}^\top$

(d) $A.^2$ (Every element of A is squared)

Q10 Euclidean Distance In a d-dimensional Euclidean space, what is the shortest distance from a point $\mathbf{x}_0$ to a hyperplane $\mathcal{H} : \mathbf{w}^\top \mathbf{x} + b = 0$? (Notation: $\|\mathbf{w}\|_2 = \sqrt{\sum_i w_i^2}$)

(a) $|\mathbf{w}^\top \mathbf{x}_0 + b|$

(b) $|\mathbf{w}^\top \mathbf{x}_0 + b| / \|\mathbf{w}\|_2$

(c) $|(\mathbf{w}/\|\mathbf{w}\|_2)^\top \mathbf{x}_0 + b|$

(d) There is no closed form solution.

SHORT QUESTIONS: Please write down your answers on the answer sheet.

P1 Bayes Formula A gambler has two coins in his pocket: a fair coin (with equal chance of showing head and tail) and a 2-headed coin (heads on both sides). These two coins are indistinguishable and thus, has equal probability of $\frac{1}{2}$ to be selected. He draws one of the coins and flips it. The coin shows head. What is the probability that he had chosen the fair coin?

P2 Transformation of Random Variables Suppose two random variables A and B are independent and identically distributed according to the Exponential distribution (The cumulative distribution function is given by $F_A(a) = P(A \leq a) = 1 - e^{-a}, a \geq 0$). Find the distribution of random variable $X = \max\{A, B\}$. *Hint*: Start with calculating the cumulative distribution function $F_X(x) = P(X \leq x)$.

P3 Optimization Find the minimum of $f(x) = x - 1 - \ln x$, $\forall x > 0$.

P4 Information Theory Using the result in P3, show that, for any probability vectors $\boldsymbol{p} = (p_1, \cdots p_K)$ and $\boldsymbol{q} = (q_1, \cdots q_K)$,

$$KL(\boldsymbol{p} \| \boldsymbol{q}) = \sum_{k=1}^{K} p_k \ln \frac{p_k}{q_k} \geq 0,$$

where $KL(\boldsymbol{p} \| \boldsymbol{q})$ is called the Kullback-Leibler divergence between $\boldsymbol{p}$ and $\boldsymbol{q}$. Note that for probability vectors, $p_k \geq 0$ and $q_k \geq 0$ and additionally, $\sum_k p_k = 1$ and $\sum_k q_k = 1$.