I. INTRODUCTION AND PROBLEM STATEMENT

In many practical wireless communication systems, every node (i.e., devices, user terminals, etc) has one-hop communication link to only a small number of other nodes due to the impact of pathloss and shadowing. Those nodes in direct radio link are called neighbors. As shown in Fig. 1, the neighbors are usually determined by transmission power, required receiver power, pathloss (i.e. distance), and shadowing (i.e., blocking objects). Each node in a network has its unique IDs, assigned from the ID space $\Omega = \{0, 1, \ldots, N\}$. The size of this set is in general very large. For instance, the 42 bits are allocated for ID space in IEEE 802.11, i.e., $N = 2^{42}$. Every node in a network should discovery and identify the IDs of its neighbors, which is referred as neighbor discovery. This problem has been studied in [2], which based on on-off signaling and group testing. In this report, we study a different strategy for compressed neighbor discovery. In formal, our problem is that each node $\ell$ wants to find the subset of the entire ID space $\Omega$, defined as $\mathcal{N}(\ell)$ that is the index set of neighbors of node $\ell$.

For neighbor discovery, each node $\ell$ in the network broadcasts its signature signal $s_\ell = [s_{\ell,1}, \ldots, s_{\ell,M}]$ during $M$ symbol intervals. These signature sequences are predetermined as a function of IDs and known to all nodes in the network. Then, the received signal at node $\ell$ is given by

$$y_\ell = \sum_{i \in \Omega \setminus \{\ell\}} h_{\ell,i} \gamma_\ell s_i + z_\ell \tag{1}$$

where

- $h_{\ell,i} \sim \mathcal{CN}(0, 1)$ denotes the Rayleigh fading
Fig. 1. The index set of neighbors of node 0 ($\mathcal{N}(0) = \{3, 15, 11\}$). The nodes 2 and 9 are not neighbor due to the blocking objects and others are due to the long distance.

Fig. 2. The simplified channel model to capture the impact of pathloss and shadowing.

- $\gamma_{\ell,i} \sim \text{Bern}(p)$ captures the impact of pathloss and shadowing (i.e., $\mathbb{P}(\gamma_{\ell,i} = 1) = p$)
- $z_\ell \sim \mathcal{CN}(0, \sigma^2 I)$ denotes the unit circularly symmetric complex Gaussian random variables

Specifically, the random variable $\gamma_{\ell,i}$ captures the following impacts (see Fig. 2):

- (Pathloss) For any two nodes, they can be closely located with probability $\alpha$, which makes it possible for one-hop communication
- (Shadowing) For any two nodes, some blocking objects can be located between them with
Based on this, we can get \( p = \alpha (1 - \beta) \). Without loss of generality, we focus on the node 0 and explain how to estimate the neighbors. All results derived in the report are straightforwardly applied to any node \( \ell \).

Our goal is to find the neighbor set of node 0 (i.e., \( \mathcal{N}(0) \)), for given observations \( y_0 \) and signature sequences \( S \), where

\[
S = [s_1, \ldots, s_N].
\]

The equation (1) can be rewritten in a matrix form such as

\[
y_0 = S\text{diag}(h_{0,1}, \ldots, h_{0,N})\gamma_0 + z_0
\]

where \( \gamma_0 = [\gamma_{0,1}, \ldots, \gamma_{0,N}]^T \) and \( \text{diag}() \) denotes the diagonal matrix. Since the \( \mathcal{N}(0) \) is completely determined by the binary vector \( \gamma_0 \), our problem is equivalent to estimate the \( \gamma_0 \). This can be obtained by maximum a posteriori (MAP) estimator:

\[
\hat{\gamma}_0 = \arg\max_{\gamma \in [0,1]^N} \mathbb{P}(\gamma | y_0, S)
\]

\[
= \arg\min_{\gamma \in [0,1]^N} \frac{1}{\sigma^2} \| y - S\text{diag}(h_{0,1}, \ldots, h_{0,N})\gamma_0 \|_2^2 - \sum_{i=1}^n \log \mathbb{P}(\gamma_{0,i})\mathbb{P}(h_{0,i}).
\]

Immediately, we can obtain the set of neighbors as \( \mathcal{N}(0) = \{ \ell \in [1 : N] : \hat{\gamma}_0(\ell) = 1 \} \). Since the objective function in (5) is non-convex and we want to find a binary vector, it is too difficult to solve the above optimization problem. Therefore, we are required to consider an approximation algorithm.

II. PROPOSED TWO-STAGE ALGORITHM

In this section, we introduce the two-stage algorithm to approximately solve the optimization problem in (5). We let

\[
x_0 = \text{diag}(h_{0,1}, \ldots, h_{0,N})\gamma_0.
\]

Since the \( x_{0,i} = \gamma_{0,i}h_{0,i} \) (\( i \)-th component of \( x_0 \)), the sparsity of \( x_0 \) is equal to that of \( \gamma_0 \). Recall that \( y_0 = Sx_0 + z_0 \). Our approach can be summarized in the following way:

- Instead of using the full knowledge on a priori information \( \mathbb{P}(x_0) = \mathbb{P}(\gamma_0)\mathbb{P}(h_0) \), we first use the sparsity of \( x_0 \) to estimate the \( x_0 \) (i.e., compressed sensing problem)
• For estimated $x_0$, we perform the “Hard-Decision” with threshold where the full knowledge on a priori information is used to determine the threshold.

Using the sparsity of $x_0$, we consider the LASSO estimation, which is widely used for estimation of sparse vectors. The LASSO estimate [3] is given by

$$\hat{x} = \arg\min_{x \in \mathbb{C}^N} \frac{1}{2} \|y_0 - Sx_0\|_2^2 + \lambda \|x\|_1$$

(7)

where $\lambda > 0$ is an algorithm parameter. Intuitively, this estimator is a least-squares estimator with an additional regularization term $\|x_0\|_1$ to encourage the sparsity solution. The algorithm parameter $\lambda$ should be carefully chosen to tradeoff the sparsity of estimate with the prediction error. In [4], it was introduced to find an optimal parameter $\lambda$ to minimize the mean square error, by employing the result of asymptotic analysis based on statistical physics. However, the procedures are still complicated. In this report, the $\lambda$ is simply chosen as $1/\sqrt{\text{SNR}}$.

In addition, we need to estimate the $\gamma_0$ from the solution of LASSO estimator $\hat{x}_0$. Here, we use the “Hard-Decision” with Threshold ($\eta$) such as

$$\left\{ \begin{array}{ll} \hat{\gamma}_{0,i} = 1 & \text{if } |\hat{x}_{0,i}| > \eta \\ \hat{\gamma}_{0,i} = 0 & \text{otherwise.} \end{array} \right.$$ 

(8)

If $x_0$ is equal to $\hat{x}_0$ then we can easily obtain the optimal threshold value as $\eta = 0$, since $h_{\ell,i}$ is non-zero with probability 1. However, $\hat{x}_0$ is not exactly matched to $x_0$ with high probability, although the mean square error $\|x_0 - \hat{x}_0\|_2^2$ is small. Therefore, we should carefully choose the $\eta$, which will be explained in the next section.

III. Determining the Threshold

In this work, we consider two performance metrics called Miss Detection Probability (MDP) and False Alarm Probability (FAP). For example, let $\mathcal{N}(0) = \{11, 13, 73, 131\}$ denote the actual neighbors of node 0 and $\hat{\mathcal{N}}(0) = \{11, 15, 73, 131\}$ denote the estimated neighbors. In this example, the node 13 is not detected by the estimator. This error is called miss detection. In general, the difference set, $\mathcal{N}(0) - \hat{\mathcal{N}}(0)$, includes all miss detected nodes. Then, MDP is defined as

$$E \left[ \frac{|\mathcal{N}(0) - \hat{\mathcal{N}}(0)|}{N} \right]$$

(9)
Fig. 3. Relative Mean Square Error. SNR = 20dB, $N = 100$, and $p = 0.1$ (i.e., $|\mathcal{N}(0)| \approx 10$)

where $|S|$ denotes the number of elements in the set $S$ and the expectation is over $h_{\ell,i}$ and $\gamma_{\ell,i}$. Also, in the above example, the node 13 is not actual neighbor node. This error is called false alarm. The difference set, $\hat{\mathcal{N}}(0) - \mathcal{N}(0)$, contains all false alarm nodes. Then, FAP is defined as

$$E\left[\frac{|\hat{\mathcal{N}}(0) - \mathcal{N}(0)|}{N}\right]$$

(10)

where $|S|$ denotes the number of elements in the set $S$ and the expectation is over $h_{\ell,i}$ and $\gamma_{\ell,i}$.

As shown in Fig. 3, the mean square error $\|x_0 - \hat{x}_0\|_2^2$ is not exactly zero, which means that we have some residual error, although their magnitudes are small. In this case, the hard-decision with $\eta = 0$ definitely produces false alarm, since all non-zero components are set by 1. This impact is shown in Fig. 5 where the estimator with $\eta = 0$ results in high FAP. Natural step is to introduce a positive threshold (i.e., $\eta > 0$), in order to reduce the FAP, as shown in Fig. 4. However, it is non-trivial to determine the threshold due to the performance tradeoff between FAP and MDP. For example, if we increase the threshold, it can increase the MDP. Also, if decreasing the threshold, it will increase the FAP. The main contribution of this report is to
Fig. 4. Miss Detection Probability (MDP) and False Alarm Probability (FAP): SNR = 20dB, $N = 100$, $p = 0.1$ (i.e., $|N(0)| \approx 10$), and Threshold is chosen with MDP = 0.96.

Fig. 5. The impact of Threshold.

provide a way to determine the threshold, by exploiting a priori information $P(x) = P(\gamma)P(h)$, where $P(\gamma)$ is a bernoulli distribution with $p$ and $P(h)$ is a Gaussian distribution with mean zero and variance 1. Further, we use the fact that the system provides the performance criterion on MDP and FAP as a system parameter. For example, both MDP and FAP should be less than
Our approach is to derive the MDP as a function of threshold, using the \( P(x) \). Then, for given MDP as system parameter, we can immediately compute the corresponding threshold by using the derived formula. Intuitively, our approach is to find the largest threshold to minimize the FAP, while satisfying the required performance on MDP.

For given \( \eta \), we can compute the MDP that is the impact of unknown Rayleigh fading such as

\[
\text{MDP}(\eta) = E \left[ \frac{1}{N} \sum_{i=1}^{N} \mathbb{1}_{\{x_{0,i} \neq 0, |x_{0,i}| \leq \eta\}} \right]
\]

\[
= P(\{x_{0,i} \neq 0, |x_{0,i}| \leq \eta\})
\]

\[
= P(x_{0,i} \neq 0)P(\{|x_{0,i}| \leq \eta\})
\]

\[
= P(\gamma_{0,i} = 1)P(\{|h_{0,i}| \leq \eta\})
\]

\[
= p(1 - \text{erfc}(\eta/\sqrt{2}))
\]

where

\[
\text{erfc}(x) = \frac{2}{\pi} \int_{x}^{\infty} e^{-t^2} dt.
\]

Notice that when MSE is low (i.e., \( \hat{x}_0 \rightarrow x_0 \)), the MDP will converge to the MDP(\( \eta \)). From the (15), we can obtain the \( \eta \) with MDP(\( \eta \)) = 0.01 when the required performance is 1%.

IV. NUMERICAL RESULTS

In this section we show some numerical results obtained based on the proposed algorithm in Section II and Threshold in Section III. We consider the wireless network with \( N \) valid IDs and \( p = 0.1 \) (i.e., about 0.1\( N \) neighbors). Threshold was selected such that the MDP is less than 1%.

For given SNR, we plotted the MDP and FAP as a function of the number of observations. Fig. 6 shows that the proposed scheme provides a substantial gain compared with Fig. 4, where the gain comes from the threshold effect. As a consequence of this result, it can definitely decrease the required observations from > 150 symbols to 50 symbols. In other words, we only require the 50 symbols to estimate the length-100 sparse vector with sparsity 10. Also, the MDP converges to about 1%, which is well matched to the analytical result.

We also considered a low-complexity algorithm to approximately solve the LASSO estimate, named Iterative-Reweighted-Least-Squares (IRLS) algorithm [5]. This algorithm can be briefly described as follows. In this algorithm, there are two algorithm parameters to be determined. As
Algorithm 1 IRLS Algorithm

- Initialization: $x_0 = 1$ and $X_0 = I$
- Iteration with $k$:
  1) Regularized Least-Squares:
     \[
     (2\lambda X_{k-1}^{-1} + S^T S) x_k = S^T y 
     \]  
     \(17\)
  2) Weight Update:
     \[
     X_k(j, j) = |x_k(j)| + \epsilon. 
     \]  
     \(18\)
  3) Stopping Rule: if $\|x_k - x_{k-1}\|_2$ is smaller than some threshold, stop

suggested in [5], we choose the $\epsilon = \sqrt{N}\sigma$ and $\lambda = \sigma/2$. In Fig. 7, the IRLS algorithm shows the worse performance, requiring more observations (i.e., 80 symbols). However, Fig. 8 shows that IRLS algorithm almost achieves the performance of LASSO estimate at high SNR (i.e., SNR = 50dB), having much lower complexity. For lower SNR (i.e., SNR = 20dB), we need to optimize the algorithm parameter $\lambda$, which is not considered in the report and left for future work.

V. CONCLUSIONS

In this report, we considered the neighbor discovery problem for wireless networks. Since it is too complicated to solve this problem optimally, we approximately solved this problem by introducing two-stage algorithm consisting of compressed sensing and hard-decision with threshold. Instead of using full knowledge on a priori information, we first used the sparsity and estimated the complex vector using LASSO estimate. Then, we obtained the binary vector using hard-decision with threshold from the estimated vector. Here, threshold has been determined by using the a priori distribution. Finally, we provided some numerical results to confirm that the proposed schemes work well. In future works, we need to consider the half-duplex system and compare with the previous schemes proposed in [2].

REFERENCES

Fig. 6. SNR = 20dB. The MDP and FAP as a function of the number of observations.


Fig. 7. SNR = 20dB. The MDP and FAP as a function of the number of observations.
Fig. 8. SNR = 50 dB. The MDP and FAP as a function of the number of observations.