

MATH 407 FALL 2009

QUIZ 8

1. A fair die is rolled n times. Compute the expected value of the sum.

The expected value of a single roll is 3.5, so we have

$$\sum_{i=1}^n \mathbb{E} X_i = \sum_{i=1}^n 3.5 = 3.5n$$

2. n students are given the numbers $\{1, \dots, n\}$ and take an exam where they place their number rather than their name on the exam. The exams are graded and handed back randomly. Compute the expected number of students that get their exam back.

Let 1_{A_k} be the indicator that the k^{th} student receives his/her exam back correctly. Then the number of students that get their exam back is

$$T = \sum_{k=1}^n 1_{A_k},$$

hence

$$\begin{aligned} \mathbb{E} T &= \sum_{k=1}^n \mathbb{E} 1_{A_k} \\ &= \sum_{k=1}^n P(A_k) \\ &= \sum_{k=1}^n 1/n \\ &= n(1/n) \\ &= 1. \end{aligned}$$