MATH 407 FALL 2009

QUIZ 6

1. Let X be a binomial distribution with parameters $n = 100$, $p = .1$. Use the normal approximation with continuity correction to compute $P(10 < X \leq 19)$. (You may leave your answer in the form of $P(a \leq Z \leq b)$, where Z is the standard normal distribution.)
$\mathbb{E}X = np = 10$, $\sigma_X = \sqrt{np(1-p)} = 3$.

$$\begin{aligned} P(10 < X \leq 19) &= P(11 \leq X \leq 19) \\ &= P\left(\frac{10.5 - 10}{3} \leq \frac{X - \mathbb{E}X}{\sigma} \leq \frac{19.5 - 10}{3}\right) \\ &= P\left(\frac{1}{6} \leq Z \leq \frac{9.5}{3}\right) \\ &= P\left(\frac{1}{6} \leq Z \leq \frac{19}{6}\right) \end{aligned}$$

2. Let X be an exponential random variable with parameter λ. Find the density of (a) e^X; (b) $\ln X$. (Note: Be sure to include the *domain* of each function.)

$$F_X(x) = P(X \leq x) = \int_0^x \lambda e^{-\lambda y} dy = 1 - e^{-\lambda x}.$$

For (a), then, we have

$$F_{e^X}(x) = P(e^X \leq x) = P(X \leq \ln x) = 1 - e^{-\lambda \ln x} = 1 - x^{-\lambda},$$

and so the density is

$$f_{e^X}(x) = (F_{e^X}(x))' = \lambda x^{-\lambda - 1}, \qquad 1 < x < \infty.$$

For (b), we have similarly

$$F_{\ln X}(x) = P(\ln X \leq x) = P(X \leq e^x) = 1 - e^{-\lambda e^x},$$

and so the density is

$$f_{\ln X}(x) = (F_{\ln X}(x))' = \lambda e^x e^{-\lambda x} \qquad -\infty < x < \infty.$$