MATH 407 FALL 2009

QUIZ 5

1. In a collection of 70000 married couples find approximately the probability that in at least one of them, (a) both husband and wife were born on December 22; (b) both husband and wife were born on the same day.
   Let X = the number of couples both born on December 22. For (a), X is binomially distribution with $n = 70000$, $p = 1/365^2$. The approximate probability is then
   $$P(X \geq 1) = 1 - P(X = 0) \approx 1 - e^{-70000/365^2}.$$
   For (b), we have $n = 70000$, $p = 1/365$, so again
   $$P(X \geq 1) = 1 - P(X = 0) \approx 1 - e^{-70000/365}.$$

2. Suppose that a batch of 100 items contains 8 that are defective and 92 that are not defective. If X is the number of defective items in a randomly drawn sample of 10 items from the batch, find (a) $P(X = 0)$; (b) $P(X > 2)$; (c) Compute the distribution for the number of batches one has to inspect before one with at most 1 defective is found. (Hint: Let N = the number of batches inspected before one with at most 1 defective is found. Compute $P(N = k)$ for $k = 1, 2, 3, \ldots$.)

   a.
   $$P(X = 0) = \frac{\binom{8}{0}\binom{92}{10}}{\binom{100}{10}}.$$
   b.
   $$P(X > 2) = 1 - \frac{\binom{8}{0}\binom{92}{10}}{\binom{100}{10}} - \frac{\binom{8}{1}\binom{92}{9}}{\binom{100}{10}} - \frac{\binom{8}{2}\binom{92}{8}}{\binom{100}{10}}$$
   c. First
   $$p = P(X \leq 1) = \frac{\binom{8}{0}\binom{92}{10}}{\binom{100}{10}} - \frac{\binom{8}{1}\binom{92}{9}}{\binom{100}{10}},$$
   and then the distribution is geometric with $q = 1 - p$, so
   $$P(X = k) = q^{k-1}p, \quad k = 1, 2, 3, \ldots.$$