MATH 407 FALL 2009

QUIZ 3

1. You have two coins. Coin 1 is unbiased (P(heads) = P(tails) = 1/2) and Coin 2 is biased, with P(heads) = 1/4, P(tails) = 3/4. With probability 1/3 you choose Coin 1 and with probability 2/3 you choose Coin 2, and you will flip whichever coin you choose. Compute the probability of heads.

   We must split up our space into two events, getting a heads with coin 1 and getting a heads with coin 2. Then we can apply Bayes rule and plug in numbers. Let C_i = we flip coin i, $i = 1, 2$. Then we have

$$\begin{aligned} P(heads) &= P(heads \cap C_1) + P(heads \cap C_2) \\ &= P(heads|C_1)P(C_1) + P(heads|C_2)P(C_2) \\ &= \frac{1}{2}\frac{1}{3} + \frac{1}{4}\frac{2}{3} \\ &= \frac{1}{6} + \frac{2}{12} = \frac{1}{3}. \end{aligned}$$

2. You wish to randomly select a number from the set $\{1, 2, 3, 4\}$ with equal probability (i.e. P(1)=P(2)=P(3)=P(4)=1/4). All you have is a standard six-sided die labeled $\{1, 2, 3, 4, 5, 6\}$, each number occuring with probability 1/6. You decide to roll the die, and if it is a 5 or a 6, roll again, otherwise record the value shown on the die. Prove that this method randomly generates a number from $\{1, 2, 3, 4\}$ with equal probability.

   This problem is similar to the duel problem. One stops rolling the die once a 1,2,3, or 4 is rolled, so we must sum over the possibility of a 5 or 6 occuring n times. For instance,

$$\begin{aligned} P(1) &= P(1 \text{ on first roll}) + P(5 \text{ or } 6 \text{ on first roll, } 1 \text{ on second roll}) + \dots \\ &= \frac{1}{6} + \frac{1}{6}\frac{2}{6} + \frac{1}{6}\left(\frac{2}{6}\right)^2 + \dots \\ &= \sum_{n=0}^{\infty} \left(\frac{2}{6}\right)^n \frac{1}{6} \\ &= \frac{1}{6}\frac{1}{1 - 2/6} \\ &= \frac{1}{4} \end{aligned}$$

   Finally, we note that $P(1) = P(2) = P(3) = P(4)$ since the 1/6 in the above expression is common to 1,2,3, and 4.