MATH 407 FALL 2009

QUIZ 2

1. Four married couples are arranged in a row. Compute the probability that no wife is next to her husband.

   This was a homework problem. Let $E_i = i^{th}$ couple sitting next to each other, $i = 1, 2, 3, 4$. Then

$$P(E_i) = \frac{2(N-1)!}{N!} = 2/N = 2/8 = 1/4,$$

$$P(E_i \cap E_j) = \frac{6!2!2!}{8!} = \frac{1}{14},$$

$$P(E_i \cap E_j \cap E_k) = \frac{5!(2!)^3}{8!} = \frac{1}{42},$$

$$P(E_1 \cap E_2 \cap E_3 \cap E_4) = \frac{4!(2!)^4}{8!} = \frac{1}{105}.$$

   The answer we are looking for is

$$\begin{aligned}
P((E_1 \cup E_2 \cup E_3 \cup E_4)^C) &= 1 - P(E_1 \cup E_2 \cup E_3 \cup E_4). \\
&= 1 - (\sum_i P(E_i) - \sum_{i<j} P(E_i \cap E_j) + \sum_{i<j<k} P(E_i \cap E_j \cap E_k) \\
&\quad -P(E_1 \cap E_2 \cap E_3 \cap E_4)) \\
&= 1 - \binom{4}{1}\frac{1}{4} + \binom{4}{2}\frac{1}{14} - \binom{4}{3}\frac{1}{42} + \binom{4}{4}\frac{1}{105} \\
&= \frac{12}{35}
\end{aligned}$$

2. Two fair dice are rolled. For $i = 2, 3, \ldots, 12$, compute the probability that the first one shows 1 given that the sum is i. (Hint: There is a simple answer for $i = 8, 9, 10, 11, 12$).

   For $i = 8, 9, 10, 11, 12$ the answer is 0 since there is no way one of the dice could be 1 and still add up to a number greater than 7.

   For $i = 2, \ldots, 7$, consider the events $A =$ the first die shows 1, $B_i =$ the sum of the two dice is i. We want to know $P(A|B_i)$. Let $D_1 =$ the value of the first die and $D_2 =$ the value of the second die. Using Bayes' Theorem, we have

$$\begin{aligned} P(A|B_i) &= \frac{P(A \cap B_i)}{P(B_i)} \\ &= \frac{P(D_1 = 1 \cap D_2 = i-1)}{P(D_1 + D_2 = i)} \\ &= \frac{\frac{1}{6} \times \frac{1}{6}}{P(D_1 + D_2 = i)} \\ &= \frac{1}{i-1}. \end{aligned}$$