

MATH 407 FALL 2009

QUIZ 1

1. a) How many hands of 5-card stud are possible? (Note that 5-card stud is a game in which 5 cards are dealt from a deck of 52 distinct cards, the order does not matter.)

$$\binom{52}{5} = \frac{52!}{47!5!} = \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

1. b) How many hands of bridge are possible? (Bridge is a game where 13 cards are dealt from a deck of 52 distinct cards, the order does not matter.)

$$\binom{52}{13} = \frac{52!}{39!13!}$$

2. There are 50 balls and 5 cells. In cell 1, there must be exactly 2 balls, in cell 2 at least 5, in cell 3 exactly 3, in cell 4 at least 10, and in cell 5 at least 1. How many possible ways are there to distribute the balls in this fashion?

One starts with 50 balls and 5 cells, and then one places 2 in the first, 5 in the second, 3, in the third, 10 in the fourth, and 1 in the last. Then one has 3 cells and 29 balls left to be distributed. This is equivalent to the number of nonnegative solutions to the equation $x_1 + x_2 + x_3 = 29$, whose solution is

$$\binom{n+r-1}{r} = \binom{3+29-1}{29} = \binom{31}{29} = \binom{31}{2}.$$

In other words, we have

$$50 \text{ balls such that } \boxed{= 2} \quad \boxed{\geq 5} \quad \boxed{= 3} \quad \boxed{\geq 10} \quad \boxed{\geq 1}$$

\Updownarrow

$$29 \text{ balls such that } \boxed{\geq 0} \quad \boxed{\geq 0} \quad \boxed{\geq 0}.$$