MATH 126 SPRING 2011, QUIZ 9

(1) Solve the differential equation $\dfrac{dy}{dt} = \dfrac{y}{t^2}$

$$\frac{dy}{y} = \frac{dt}{t^2}$$

$$\ln|y| = -t^{-1} + C$$
$$|y| = e^{-t^{-1}} e^C$$
$$y = Ce^{-1/t}$$

(2) Determine whether the following limits are convergent or divergent. If they are convergent, find the limit.

a. $\lim_{n\to\infty} \sin(2n)e^{-n}$

$$0 \lim_{x\to\infty} e^{-x} \leq \lim_{x\to\infty} \sin(2x)e^{-x} \leq \lim_{x\to\infty} e^{-x} = 0$$

Therefore by Squeeze Theorem $\lim_{x\to\infty} \sin(2x)e^{-x} = 0$, hence by a theorem in the book $\lim_{n\to\infty} \sin(2n)e^{-n} = 0$.

b. $\lim_{n\to\infty} \ln(2n^2 + 1) - \ln(n^2 + 1)$

$$\lim_{x\to\infty} \ln\left(\frac{2x^2+1}{x^2+1}\right) = \ln\left(\lim_{x\to\infty} \frac{2x^2+1}{x^2+1}\right) = \ln 2.$$

Hence, by a theorem in the book $\lim_{n\to\infty} \ln(2n^2+1) - \ln(n^2+1) = \ln 2$.

c. $\lim_{n\to\infty} \left(1 + \dfrac{e}{n}\right)^n$

$$\lim_{x\to\infty} \left(1 + \frac{e}{x}\right)^x = e^e$$

(you would need to show more details on an exam), hence by a theorem in the book $\lim_{n\to\infty} \left(1 + \frac{e}{n}\right)^n = e^e$.

(3) Newton's Method for finding square roots starts with an initial guess, say $x_1 = a$, then applies the formula

$$x_{n+1} = \frac{1}{2}\left(x_n - \frac{a}{x_n}\right).$$

Show that this sequence will converge to one of the two square roots of a.

Take the limits of both sides, assuming the limit is L, and we get

$$L = \lim_{n\to\infty} x_{n+1} = \frac{1}{2} \lim_{n\to\infty} \left(x_n - \frac{a}{x_n}\right) = \frac{1}{2}\left(L - \frac{a}{L}\right).$$

Solving for L we obtain the equation

$$L^2 = a$$

or in other words $L = \pm\sqrt{a}$.