

MATH 126 SPRING 2011, QUIZ 7

- (1) Consider the region defined by  $x = 1, x = 2, y = e^{-x}$ .  
a. Find the total volume by rotating this region around the x-axis.

$$\int_1^2 \pi e^{-2x} dx = \frac{\pi}{2}(e^{-2} - e^{-4})$$

- b. Find the total volume by rotating this region around the y-axis.  
Using cylindrical shells we have

$$\int_1^2 2\pi x e^{-x} dx = 2\pi(2/e - 3/e^2).$$

Note that we could also use the washer method but that would involve inverting the equation (which is not hard) and then splitting up the region into two integrals (more complicated than the cylindrical shell method).

- (2) Consider the region between two upper semi-circles, the larger one with radius  $R$  and the smaller with radius  $r$ . Find the volume of the "hollow sphere" by rotating this region around the x-axis.

$$\int_{-R}^R \pi(R^2 - x^2) dx = \pi 2R^3 - 2R^3/3 = \frac{4}{3}\pi R^3$$
$$\int_{-r}^r \pi(r^2 - x^2) dx = \pi 2r^3 - 2r^3/3 = \frac{4}{3}\pi r^3$$

So the answer is

$$\frac{4}{3}\pi(R^3 - r^3).$$

- (3) Rotate the region defined by  $y = e^{-x^2}, x = 0, x = \sqrt{\ln 2}$  around the y-axis.  
Using cylindrical shells,

$$\int_0^{\sqrt{\ln 2}} 2\pi x e^{-x^2} dx = 2\pi \frac{1}{2} \int_0^{\ln 2} e^{-u} du = -\pi(e^{-\ln 2} - e^0) = \pi/2$$