MATH 126 SPRING 2011, QUIZ 5

a. Show that $\int_0^\pi x\sin(x)dx = \pi$.

$$\begin{aligned} \int_0^\pi x\sin(x)dx &= -x\cos(x)|_0^\pi + \int_0^\pi \cos(x)dx \\ u = x,\ dv = \sin(x)dx & \qquad du = dx,\ v = -\cos(x) \\ &= \pi + 0 - 0 = \pi. \end{aligned}$$

b. Use Simpson's Rule to compute the approximation of the integral for $n = 6$.

We use the partition $x_0 = 0, x_1 = \pi/6, x_2 = 2\pi/6, x_3 = 3\pi/6, x_4 = 4\pi/6, x_5 = 5\pi/6, x_6 = \pi$ with $\Delta x = \pi/6$. The integral is then approximately equal to

$$\begin{aligned} \int_0^\pi x\sin(x)dx &\approx \frac{\Delta x}{3}\left(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + 4f(x_5) + f(x_6)\right) \\ &= \frac{\pi}{18}\left(0 + 4\frac{\pi}{6}\frac{1}{2} + 2\frac{\pi}{3}\frac{\sqrt{3}}{2} + 4\frac{\pi}{2} + 2\frac{2\pi}{3}\frac{\sqrt{3}}{2} + 4\frac{5\pi}{6}\frac{1}{2} + 0\right) \\ &= \frac{\pi^2}{18}\left(\frac{1}{3} + \frac{\sqrt{3}}{3} + 2 + \frac{2\sqrt{3}}{3} + \frac{5}{3}\right) \\ &= \frac{\pi^2}{18}\left(\frac{12 + 3\sqrt{3}}{3}\right) \\ &= \frac{\pi^2(4+\sqrt{3})}{18} \end{aligned}$$

c. Set your answer in b equal to π and solve for π.

$$\begin{aligned} \frac{\pi^2(4+\sqrt{3})}{18} &\approx \pi \\ \pi &\approx \frac{18}{4+\sqrt{3}} \end{aligned}$$

d. What is the maximum error of this estimate for π?

We have $|E_S| \leq K(b-a)^5/180n^4$, where $|f^{(4)}(x)| \leq K$ for all $0 \leq x \leq \pi$. By taking derivatives, we have

$$f^{(4)}(x) = -4\cos(x) + x\sin(x)$$

$$|f^{(4)}(x)| \leq 4 + |x| \leq 4 + \pi < 8$$

So taking $a = 0, b = \pi, K = 8, n = 6$, we obtain

$$|E_S| \leq \frac{8\pi^5}{180 \times 6^4}.$$

Note that you could take $K = 4 + \pi$ to obtain a more accurate answer, but this is less important than understanding how to find a reasonable value of K.