MATH 126 SPRING 2011, QUIZ 4

1. Write out the form of the partial fraction decomposition of the function. Do NOT determine the numerical values of the coefficients.

$$\frac{2x+17}{(x^3+x^2)(x+1)(x^2+\pi)^3}$$

First we MUST simplify the expression to

$$\frac{2x+17}{x^2(x+1)(x+1)(x^2+\pi)^3} = \frac{2x+17}{x^2(x+1)^2(x^2+\pi)^3}.$$

Now the solution is

$$\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1} + \frac{D}{(x+1)^2} + \frac{Ex+F}{x^2+\pi} + \frac{Gx+H}{(x^2+\pi)^2} + \frac{Ix+J}{(x^2+\pi)^3}$$

2. Evaluate the integral.

$$\int \frac{5x^2+3x-2}{x^3+2x^2}dx$$

We can rewrite the integral as

$$\int \frac{5x^2+3x-2}{x^2(x+2)}dx$$

and use partial fraction decomposition,

$$\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+2}$$

$$Ax(x+2) + B(x+2) + Cx^2 = 5x^2 + 3x - 2$$

Set $x = 0$, we get $2B = -2$, so $B = -1$.

Set $x = -2$, we get $4C = 20 - 6 - 2 = 12$, so $C = 3$.

Finally, we can compare the coefficients of x^2, which means $A + C = 5$, so $A = 2$.

Putting this all together, we have

$$\begin{aligned} \int \frac{5x^2+3x-2}{x^3+2x^2}dx &= \int \frac{2}{x} - \frac{1}{x^2} + \frac{3}{x+2}dx \\ &= 2\ln|x| + \frac{1}{x} + 3\ln|x+2| + C. \end{aligned}$$