MATH 126 SPRING 2011, QUIZ 2

1. Find the derivative of $f(t) = \ln(\sinh(e^t))$. (Do not worry about domains.)

$$f'(t) = \frac{1}{\sinh(e^t)} \cosh(e^t)e^t = e^t \coth(t).$$

2. Find the limit.

a. $\displaystyle\lim_{x\to 1} \frac{\ln x}{\sin \pi x}$

$$= \lim_{x\to 1} \frac{1/x}{\pi \cos \pi x} = -\frac{1}{\pi}$$

b. $\displaystyle\lim_{x\to -\infty} x^2 e^x$

$$= \lim_{x\to -\infty} \frac{x^2}{e^{-x}} = \lim_{x\to -\infty} \frac{2x}{-e^{-x}} = \lim_{x\to -\infty} \frac{2}{e^{-x}} = \lim_{x\to -\infty} 2e^x = 0.$$

c. $\displaystyle\lim_{x\to 0} \frac{x}{\tan^{-1}(4x)}$

$$= \lim_{x\to 0} \frac{1}{\frac{4}{1+(4x)^2}} = \lim_{x\to 0} \frac{1+16x^2}{4} = \frac{1}{4}.$$

3. a. Prove the identity $\cosh x + \sinh x = e^x$.

$$\cosh x + \sinh x = \frac{e^x + e^{-x}}{2} + \frac{e^x - e^{-x}}{2} = 2e^x/2 = e^x$$

b. Use part a to prove the identity $(\cosh x + \sinh x)^n = \cosh nx + \sinh nx$. (Hint: Show that each side of the equality equals e^{nx}.)

$$(\cosh x + \sinh x)^n = (e^x)^n = e^{nx}.$$

$$\cosh nx + \sinh nx = \frac{e^{nx} + e^{-nx}}{2} + \frac{e^{nx} - e^{-nx}}{2} = 2e^{nx}/2 = e^{nx}$$