MATH 126 SPRING 2011, QUIZ 1

1. Find the derivative of the function. Find the domains of the function and its derivative.

$$f(x) = \ln \cos^{-1} e^{2x}$$

$$f'(x) = \left(\frac{1}{\cos^{-1}(e^{2x})}\right)\left(\frac{-1}{\sqrt{1-e^{4x}}}\right)2e^{2x}$$

The domain of f is found by looking at the set of x for which

$$-1 \leq e^{2x} \leq 1$$

and

$$0 < \cos^{-1} e^{2x}.$$

For the first inequality, we have $x \leq 0$. For the second, we need $e^{2x} < \pi/2$, which will always happen for $x \leq 0$ so our domain for $f(x)$ is $x \leq 0$,

$$domain(f) = \{x : x \leq 0\}.$$

For $f'(x)$, we need $\cos^{-1}(e^{2x}) \neq 0$ and $1 - e^{4x} > 0$. So we need $x < 0$ and $x \neq \pi/2$, which is included in $x < 0$. Thus the domain of f' is $x < 0$,

$$domain(f') = \{x : x < 0\}.$$

2. Find the limit.

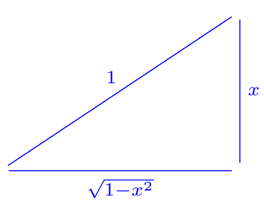
$$\lim_{x\to\infty} \tan^{-1}\left(\frac{5+x^2}{2\pi+x^3}\right)$$

$$\begin{aligned}\lim_{x\to\infty} \tan^{-1}\left(\frac{5+x^2}{2\pi+x^3}\right) &= \tan^{-1}\left(\lim_{x\to\infty}\frac{5+x^2}{2\pi+x^3}\right) \\ &= \tan^{-1}(0) = 0.\end{aligned}$$

3. Simplify the expression, and include the domain in your final answer.

$$\tan \sin^{-1} x$$

Let $\theta = \sin^{-1}(x)$, so that $\sin\theta = x$ for $-1 \leq x \leq 1$. Then the triangle looks like



so the answer we are looking for is $\tan\theta = \frac{x}{\sqrt{1-x^2}}$. The domain is $-1 < x < 1$, so our final answer is

$$\tan \sin^{-1} x = \frac{x}{\sqrt{1-x^2}}, \quad -1 < x < 1.$$