MATH 126 SPRING 2011, QUIZ 11

(1) Find the interval and radius of convergence of $\sum_{n=1}^{\infty} \frac{(x+3)^n}{n(-9)^n}$.

First we apply the RATIO test,

$$\left|\frac{(x+3)^{n+1}}{(n+1)(-9)^{n+1}} \frac{n(-9)^n}{(x+3)^n}\right| = \left|\frac{(x+3)}{-9} \frac{n}{n+1}\right|$$
$$\to \left|\frac{x+3}{-9}\right| = \left|\frac{x+3}{9}\right|.$$

The series converges when

$$\begin{aligned} \left|\tfrac{x+3}{9}\right| &< 1 \\ |x+3| &< 9 \\ -9 < \quad x+3 &< 9 \\ -12 < \quad x &< 6, \end{aligned}$$

and diverges otherwise. We now need only check the points $x = -12$ and $x = 6$. When $x = 6$, we have

$$\sum_{n=1}^{\infty} \frac{9^n}{n(-9)^n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}.$$

This is an alternating series with $b_n = 1/n \to 0$, and $1/(n+1) < 1/n$, hence b_n is decreasing and the limit is zero, so by the Alternating Series test this series is convergent. When $x = -12$, we have

$$\sum_{n=1}^{\infty} \frac{(-9)^n}{n(-9)^n} = \sum_{n=1}^{\infty} \frac{1}{n}.$$

this is the Harmonic Series, which is a p-series with $p = 1 \leq 1$, hence divergent. Therefore the Interval of Convergence is

$$(-12, 6]$$

and the Radius of Convergence is

$$R = 9.$$

(2) Determine whether the series is convergent or divergent. You should justify your answer and indicate the test you are using

(a) $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{\sqrt{n+9}}$ (b) $\sum_{n=1}^{\infty} (-1)^n \left(1 - \frac{1}{n}\right)$ (c) $\sum_{n=2}^{\infty} (-1)^n \frac{n2^n}{(n-1)!}$.

(a) We must notice that $\cos(n\pi) = (-1)^n$ (just think about it!). Then we can write this series as

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+9}}.$$

This series is alternating, with $b_n = 1/\sqrt{n+9} \to 0$. It is obvious that $\sqrt{(n+1)+9} = \sqrt{n+10} > \sqrt{n+9}$, so $b_{n+1} < b_n$, hence b_n is decreasing and the limit is zero, hence by the Alternating series test the series is convergent.

(b) Apply the divergence test. First look at the limit of the *sequence* without the $(-1)^n$. This limit is $(1 - 1/n) \to 1$, which is not zero. This means that the limit of $(-1)^n(1 - 1/n)$ does not exist since it is bouncing back and forther from numbers close to -1 and +1. Hence by the DIVERGENCE TEST this series is divergent.

(c) Apply the Ratio test,

$$\left| \frac{(n+1)2^{n+1}}{n!} \frac{(n-1)!}{n2^n} \right| = \left| \frac{2(n+1)}{n^2} \right| \to 0 < 1,$$

therefore convergent.