MATH 126 SPRING 2011, QUIZ 10

(1) Determine whether the series is convergent or divergent. (Note you do NOT need to determine the actual value for convergent series.)

a. $\sum_{n=2}^{\infty} \frac{\sqrt{n}}{n-1} > \sum_{n=2}^{\infty} \frac{\sqrt{n}}{n} = \sum_{n=2}^{\infty} \frac{1}{n^{1/2}}$,

This series on the right is divergent by the $p-$series test with $p = 1/2 \leq 1$, hence by the Comparison Theorem the series is DIVERGENT.

b. $\sum_{n=2}^{\infty} \frac{n^2-1}{n^2}$

Since

$$\lim_{n\to\infty} \frac{n^2-1}{n^2} = 1,$$

this series is DIVERGENT by the divergence test.

c. $\sum_{n=1}^{\infty} \frac{1+\sin n}{10^n}$

Since $-1 \leq \sin n \leq 1$ for all n, we have

$$\sum_{n=1}^{\infty} \frac{1+\sin n}{10^n} \leq \sum_{n=1}^{\infty} \frac{2}{10^n} < \infty$$

by the geometric series with $-1 < r = 1/10 < 1$. Therefore by the Comparison Theorem the series is CONVERGENT.

d. $\sum_{n=0}^{\infty} \frac{(-\pi)^n}{3^{n+1}} = \sum_{n=0}^{\infty} \frac{1}{3}\left(\frac{-\pi}{3}\right)^n$,

but this series is divergent since it is geometric with $r = -\pi/3 < -1$.

e. $\sum_{n=1}^{\infty} \arctan n$.

$\lim_{n\to\infty} \arctan n = \lim_{x\to\infty} \arctan x = \pi/2 > 0$, so by the divergence test the series is DIVERGENT.

(2) What is the value of c if $\sum_{n=2}^{\infty}(1+c)^{-n} = 2$?

$$\begin{aligned}
\sum_{n=2}^{\infty}(1+c)^{-n} &= \sum_{n=2}^{\infty}\left(\frac{1}{1+c}\right)^2\left(\frac{1}{1+c}\right)^{n-2} \\
&= \left(\frac{1}{1+c}\right)^2 \frac{1}{1-\frac{1}{1+c}} \\
&= \frac{1}{(1+c)^2-(1+c)},
\end{aligned}$$

so we wish to solve $(1+c)^2 - (1+c) = 1/2$. You can expand out for c or let $y = c+1$ and solve the quadratic $y^2 - y - 1/2 = 0$, in which case we get

$$y = \frac{1 \pm \sqrt{1+2}}{2} = \frac{1 \pm \sqrt{3}}{2}.$$

We need to have $-1 < y < 1$, so the only viable solution is $y = \frac{1-\sqrt{3}}{2}$, hence the answer is

$$c = \frac{\sqrt{3}-1}{2}.$$