COMPOSITIONS

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This document is designed to clarify the concept and domain of a composition. A composition is simply a way to run an input through two or more functions to get an output. For instance, a function is like a water purifyer. You input any kind of water, and it returns also water but with slightly different properties. Once you have this filtered water, you can then run it through a coffee filter, and the output is a brown liquid with a peculiar taste.

The example above illustrates a composition of two functions

$$water \longrightarrow water\ filter \longrightarrow coffee\ filter \longrightarrow coffee$$

For a concrete math example, let's have $f(x) = \sqrt{2x+3}$ and $g(x) = x^2 + 1$. If we take the composition of g with itself, we get

$$(g \circ g)(x) = (x^2+1)^2 + 1 = x^4 + 2x^2 + 2,$$

which has domain $\mathbb{R}$ since it is just a polynomial. However, the composition above is different from the function $k(x) = x^4 + 2x^2 + 2$, which takes an input x and outputs according to the single formula given in $k(x)$.

To see the distinction, let us consider

$$(g \circ f)(x) = g(f(x)) = g(\sqrt{2x+3}) = (\sqrt{2x+3})^2 + 1.$$

It is temping to simplify this to $2x + 4$ and write that the domain is all real numbers. But we must be careful! Let us think more closely about the difference between $(g \circ f)(x)$ and $h(x) = 2x + 4$. $h(x)$ is a single input output function that works for any real number because it is a polynomial. The composition, on the other hand, first must run through $f(x)$ and *then* go through $g(x)$, so the domain of $(g \circ f)(x)$ cannot include points that are not in the domain of $f(x)$.

With this in mind, we have

$$\begin{array}{lcl} \text{the domain of} & h(x) & \text{is all real numbers, } \mathbb{R}\text{, and} \\ \text{the domain of} & (g \circ f)(x) & \text{is } x \geq -3/2, \end{array}$$

since the domain of $f(x)$ is $x \geq -3/2$. Of course, one must also make sure that the range of $f(x)$ is in the domain of $g(x)$. This concept should already be familiar with the reader.

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