Playing 'Games' with Human Health The Role of Game Theory in Optimizing Reliability in Wireless Health Networks

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***Abstract*—Current advances in wireless sensor networks have contributed to the development of multi-hop wireless body sensor networks (WBSNs). These networks, which are primarily comprised of low-power sensor nodes, provide long term health and physiological monitoring of patients without obstructing their normal activities. However, the time-varying nature of network resources poses a significant challenge to the high reliability of operations required of a body sensor network. In this paper, we address the problem of reliability optimization in multi-hop WBSNs, where the nodes *co-operate* with each other in maximizing system performance.**

We model our problem as a co-operative game which inherently induces a distributed mechanism that *optimizes* the reliability of network operations in a multi-hop WBSN supporting concurrent applications. By the term 'reliability', we imply the system utility generated when each sensor node (user) services its specific application. We denote the system utility as an *instance* of the *vector* of utilities of individual users, where each user's utility is a function of its allocated bandwidth. Our mechanism *jointly* accounts for the dynamic nature of the wireless medium, application quality of service, user fairness in achieving the resources to service applications, and results in an optimal operating point (utility vector) at which the WBSN should operate. We arrive at the result that the *Nash bargaining solution* of the co-operative game gives us the optimal system operating point, i.e., the Nash bargaining solution optimizes the overall system reliability.

***Keywords:* Wireless Body Area Networks, Reliability, Co-operation, Bargaining, Nash Bargaining Solution**

I. Introduction

The recent advances in wireless networking, the Internet, and microelectronic technology have opened up opportunities to fundamentally modernize and revolutionize the way health care services are deployed and delivered. Traditional health care systems are mostly structured for reacting to medical crisis and managing illness rather than wellness. However, given the current economic, social, and demographic trends, it is well worth the effort to focus on prevention, early detection, and optimal maintenance of disease conditions. Recent studies show that 1) there will be about 150 million people in the world with chronic conditions by year 2015, 2) with the increase in life expectancy, the projected worldwide population over age 65 is expected t be around 800 million by 2025, and 3) the projected value of health care costs in the United States is expected to reach $12K per capita by 2015, which approximates to around 20% of the gross domestic product (GDP). Thus, these statistics emphasize the need for more scalable, efficient, and affordable health care solutions.

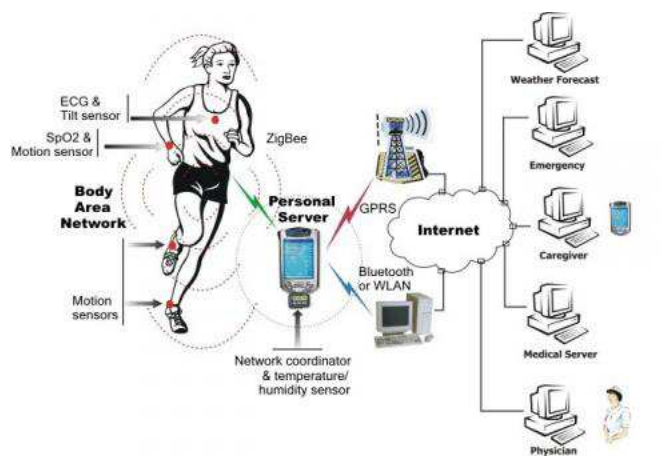

Fig. 1. A Wireless Body Sensor Network

Wearable/implanted devices for real-time and ubiquitous health monitoring are a key technology promoting proactive, non-obtrusive, and affordable next-generation health care solutions. A promising and revolutionary approach in building efficient health monitoring systems is to utilize the concept of emerging wireless body sensor networks (WBSNs), also commonly known as wireless health networks. A WBSN comprises of a set of mobile, low cost, low power intercommunicating sensors, either wearable or implanted into the human body, which continuously monitor vital body parameters in a wireless communication environment [7]. The strategically placed sensors operate on an unlicensed ISM band *(2.4 GHz, IEEE 802.15.4 standard)*[1], sample and process heterogenous body-centric information, and transmit it to a sink device like a portable digital assistant (PDA) or a home base station (Figure 1). Data is then transferred over the Internet from the sink to a hospital or clinic, for medical professionals to provide patients with real-time health

[1]Presently, there are efforts to move into a different IEEE standard for wireless body area networks

care[2]. Typical applications of WBSNs include monitoring body parameters like heart rate, blood pressure, blood sugar, toxin levels, and oxygen saturation. Several academic and industrial organizations are presently investing in considerable research efforts to provide anytime, anywhere health care services. Some well known examples include *Project MIThril* (MIT), *Project Human++* (IMEC), *Project Codeblue* (Harvard), *Project AIDS-N* (Johns Hopkins), and projects in operation at companies like Qualcomm, Nokia, and Philips.

A. Research Motivation

Reliability of transmitted information is a big challenge in wireless networks and more so in low power body sensor networks since *critical* and *sensitive* medical data might be involved in the form of data /audio/video signals[3], and that they often need to be transmitted over channels of *low* bandwidth. For example, medical professionals may often have to deal with organ images or heart beat patterns for disease diagnosis. Highly reliable and good quality of audio/video output circulation at processing nodes helps a lot in doctors making proper judgements. Here,'reliability' implies the quantitative extent to which a system performs well with respect to certain performance parameters. However, reliability of a system could be defined in many ways. The proper and appropriate characterization of reliability in body sensor networks is an important open problem that needs to be addressed as it drives sound and practically implementable WBSN system design.

Heterogenous applications supported by WBSN systems demand high audio/video quality and data accuracy, low probability of retransmissions, and low delay. However, the transmission medium in a wireless network is time varying due to channel fading effects,[4] and as a result the network resources available at particular time instants may not be sufficient to meet potentially concurrent application demands. In addition, sensor nodes interfere in communication due to omnidirectional nature of wireless transmissions. This interference poses a barrier to achieving required application data rates, hence lowering reliability. There is also the phenomenon of congestion at various nodes and especially at the sink node, which increases the loss rate, decreases energy efficiency of the network, and worsens network fairness. Apart from the above mentioned traditional wireless communication challenges, WBSNs impose additional communication reliability challenges in terms of human mobility (and hence environment changes) and sensor node positions not remaining static relative to other node positions. Thus, in light of the above mentioned factors, resolving reliability issues *(improving and optimizing reliability)*, in WBSNs is of paramount importance to safe and efficient health care.

B. Related Work and Their Drawbacks

Reliability characterization and optimization in body sensor networks is a recent and important research topic. Till date, there have been no works focusing on the *proper* characterization of reliability metrics in wireless sensor networks, let alone body sensor networks. Most research efforts in general wireless and body sensor networks have not explicitly considered the dynamic/time-varying nature of wireless networks, and have modeled reliability naively and separately as either the probability of achieving data rate demands of heterogenous applications, or as the *mean* value of parameters like network throughput, network delay, number of retransmissions, and network life-time. However, modeling reliability this way does not give a complete picture of system performance, as the latter is jointly dependent on multiple parameters. Thus, improving on the values of existing reliability metrics have resulted in wireless sensor systems that do work in practice but those from which we cannot derive a clear conclusion[5]on overall system improvement. The existing works also do not account for an application's *quality of output*, as a factor in reliability calculation. Here, the term 'quality' refers mainly to audio/video applications, which are quite common to body sensor networks. Thus, one major challenge specific to WBSNs is the formation of a *unified* and *sophisticated* reliability metric that explicitly considers the time-varying nature of wireless networks and *jointly* accounts for factors like network delay, satisfaction of data rates, quality of data/audio/video signals, etc.

Once we have characterized reliability properly in WBSNs, the main challenge is to optimize it. The critical nature of medical applications in WBSNs demands very high reliability, which is not the case when it comes to applications in general wireless networks. To cite a simple example, inaccurate diagnosis based on slightly unreliable WBSN data may lead to patient death, whereas the same amount of unreliability may be perfectly acceptable (tolerable) to applications in general wireless networks. Thus, even though reliability improvement may be good enough for some applications in general wireless networks, reliability optimization is nearly imperative in body area networks. The state of the art solutions in WBSNs involve simple distributed heuristics to improve on the so called improper reliability metrics like network throughput, number of retransmissions, network life-time, and network delay. However, the proposed solutions are not provably optimal (do not guarantee high reliability), do not account for application fairness, and fail to take proper advantage of interactions amongst network nodes to obtain optimal/near optimal performance. By the term 'fairness', we imply the availability of enough resources to each application such that they meet their quality of service (QoS) requirements. The readers are referred to [3][4][5] for details on some recent research work related to reliability in WBSNs.

C. Overview of Research Contribution

In this paper, we investigate the important problem of *unified reliability characterization* and *optimization* in multi-hop[6] wireless body sensor networks. The goal of this paper is two fold: 1) to derive a unified reliability metric for individual WBSN nodes with bandwidth as the limiting resource. The metric will be able to jointly reflect performance parameters like throughput, network delay, quality of audio/video, network life-time, and application fairness, for a network with time-varying channel conditions, and 2) to design an efficient distributed mechanism to optimize our proposed reliability metric. As a solution approach to the optimization problem, we use an appropriate *co-operative game-theoretic mechanism* called bargaining and apply *optimization theory* tools to maximize reliability in WBSNs. Our solution technique is novel, formal, concrete, efficiently implementable, and best suits the problem at hand. By solving the reliability problem, we find *optimal* network operating point/s, which in turn provides valuable insights to WBSN designers for efficiently designing practical routing and scheduling algorithms/protocols to achieve optimal/near optimal system performance. Our proposed methodology would also provide theoretical insights to modeling reliability, and solving related optimization problems in general co-operative networks.

II. Role of Co-operation and Bargaining

It is a well known fact in network information theory that *co-operation* leads to improved system performance, i.e., enhancement of network capacity. Existing research works have proved this fact

[2]Examples of health care provided might include monitoring effects of drug therapy, optimal maintenance of chronic conditions, monitoring in ambulatory settings, and supervising recovery from a surgical operation

[3]A multimedia sensor [9] is involved in the processing and communication of audio/beat patterns and video images of body organs.

[4]The extent of fading varies depending on whether signals are transmitted within the body or around the body [1]. There is high attenuation through the human body of radio waves at particular frequencies.

[5]A vague and unclear perspective of overall system improvement in WBSNs might prove to be costly when it comes to medical decision making.

[6]In a recent work [2], the authors suggest that multi-hop is a better WBSN architecture in many respects.

using sophisticated information-theoretic and co-operative communication techniques. It has also been shown in the light of information theory that there is a significant breakdown in system performance without co-operation. In multi-hop wireless networks, co-operation is necessary as intermediate nodes *must* relay packets for source nodes for communication to be successful. Co-operation here is mainly in the form of message passing and information exchange regarding various system parameters. However, in energy constrained multi-hop systems like WBSNs, co-operation *cannot be taken for granted* as relaying foreign packets cost power. On the other hand, without co-operation, critical WBSN applications will fail to achieve their required quality of service (QoS).

In this paper, we employ *bargaining techniques* from co-operative game theory and mathematical economics to *incentivize* co-operation amongst the WBSN nodes, thereby improving system performance. We model our problem as a bargaining task such that the optimal bargaining solution/s correspond to optimal values of our reliability metric. The main strength of bargaining mechanisms is that they can be efficiently implemented in a distributed manner in dynamic environments, and generate Pareto-optimal outcomes, which are inherently fair w.r.t each player utility (see Section IV). We note here that although our idea of using co-operation in wireless networks is inspired by results from information theory, we analyze the effect of co-operation from a totally different angle, i.e., from a purely game-theoretic and network economics approach. We also emphasize the fact that information-theoretic techniques prove capacity improvement using co-operation, but provide no incentive mechanisms to achieve co-operation.

III. Reliability Metric

In this section, we propose our unified reliability metric that jointly captures relevant information regarding the dynamicity of wireless networks as well as multiple user performance parameters.

Our main emphasis is to account for the fact that in addition to data signal transmission, video and audio transmissions in/around the human body are common w.r.t certain image/signal processing applications in body sensor networks. We first adopt the rate-distortion model proposed in the seminal paper by Stuhlmuller et.al. [12] to suit this purpose. We then formulate our reliability metric using the rate-distortion model. Stuhlmuller's rate-distortion model is widely used in the video coding community and accounts for the most important multimedia system parameters including transmission channel variations. The model generalizes the performance of data, video, and audio signal transmissions. It also inherently captures application performance parameters like signal quality, network delay and throughput, which in turn depend on the allocated bandwidth to an application. In this extended abstract, we deal with central modeling concepts in [12]. The readers are referred to [12] for more details regarding the Stuhlmuller's model.

Stuhlmuller's distortion model defines D_e to be the distortion of an encoded video sequence[7], measured as the mean square error (MSE) (averaged over all frames of a video sequence) and formulates it as follows.

$$D_e = \frac{\theta}{R_e - R_0} + D_0, \quad R_e \geq R_0, \quad D_0 \geq 0, \quad \theta > 0, \quad (1)$$

where R_e is the rate for the video sequence, θ, R_0, and D_0 are the parameters of the rate distortion model, which are dependent on video sequence characteristics, spatial and temporal resolutions, and delay. The corresponding peak-signal-to-noise ratio (PSNR) is given by

$$PSNR = 10\, log_{10} \frac{255^2}{D_e} \quad (2)$$

[7]We emphasize that the model generalizes all types of sequences, be it data, audio, or video.

We formulate our unified reliability metric as a *vector* of utilities for each user in the network, where each user is sensor/actuator capable of acting as a source to an application that needs to be serviced at a given rate demand. We define the utility for each user i as a function of the *bandwidth* allocated to the user, and it follows from the structure of the PSNR formula[8] for each user i, except that we do not involve the logarithm and the constant. Mathematically, we define the utility for each user i as

$$U_i(b_i) = \frac{k}{D_{ei}} = \frac{k \cdot (b_i - R_{0i})}{D_{0i}(b_i - R_{0i}) + \theta_i}, \quad b_i \geq R_{0i}, R_{0i} > 0, k \geq 0 \quad (3)$$

From the utility formulation follows the fact that a user i receives a utility of zero when allocated a bandwidth of R_{0i}. Our formulation is extremely realistic in the sense that a network user servicing bandwidth dependent applications is *positively* satisfied only when it gets an amount of bandwidth above a particular threshold. In our formulation, we assume the threshold $R_{0i} > 0$ for each user i. Thus, our *system reliability metric* is a vector of the form $(U_1(b_1), ..., U_n(b_n))$, where n is the number of users in the system. The utility derived by each user determines the system fairness.

IV. Bargaining Model

We have n body sensor nodes, which co-operate to divide the total available network bandwidth. Each user/player i, i.e., a sensor node, has its own utility function, $U_i(b_i)$ (as formulated in Section III) that it derives from the rate b_i allocated to it. Each user also desires a minimum level of utility $U_i(R_{0i})$ called the *disagreement point*. This is the minimum utility that each user expects by joining a game without co-operation. In this paper, we assume that the minimum utility $U_i(R_{0i}) = 0$ *is guaranteed* for each user in the co-operative game, which implies that no user gets a bandwidth less than $R_{0i} > 0$. We define the vector **d** as a disagreement vector, where **d** = $(d_1, ..., d_n)$ = $(U_1(R_{01}), ..., U_n(R_{0n})) \,\epsilon\, \Re^n$, the vector of individual disagreement points for each user in the network. From our utility formulation, **d** = (0,......,0).

The *feasible* values for our system reliability metric can be considered as a joint feasible utility set, **U** = $\subset\ \Re^n$, where **U** = $\{(U_1(b_1), ..., U_n(b_n))\}$, the set of feasible vectors that form an *instance* of our reliability metric. We *define* the bargaining problem in our abstract as *finding the vector* in **U** that is both, *Pareto optimal*, as well as the best for the WBSN in question. A vector $(U_1(b_1), ..., U_n(b_n)) \,\epsilon$ **U** is Pareto optimal if for each $(U'_1(b_1), ..., U'_n(b_n)) \,\epsilon$ **U**, $(U'_1(b_1), ..., U'_n(b_n)) \geq (U_1(b_1), ..., U_n(b_n))$ implies the equality of the utility vectors $(U'_1(b_1), ..., U'_n(b_n))$ and $(U_1(b_1), ..., U_n(b_n))$, where the inequality among the vectors denote component-wise inequality. In practice, for a game of multiple players/users, there may exist an *infinite* number of Pareto optimal solutions [8]. Thus, we need a selection criteria (decide on the type of bargaining solution) to decide which Pareto optimal point is the best for the system. In this abstract, we consider the well known *Nash bargaining solution* (NBS) to be the *preferred (best)* solution of our bargaining problem, which is both, Pareto optimal and possess certain other properties [10][6] [11] that we expect our network system to have.

A. Practical Significance of Optimizing Reliability

In the process of finding the best Pareto optimal vector of user utilities, where each vector (either Pareto-optimal or non Pareto-optimal) in **U** is an instance of our proposed reliability metric, we implicitly optimize the following network parameters in practice by *fairly* allocating bandwidth to all applications : 1) network delay for each application, 2) quality of application output (content characteristics), and 3) power consumption at nodes, which in turn

[8]PSNR is a good indicator of the satisfaction of a user with regard to signal quality.

influences network life-time. We also achieve data rate demands of concurrently running heterogenous applications.

B. Mathematical Properties of the Feasible Utility Set

In Section IV, we mentioned that all feasible instances of our reliability metric can be considered as the set of joint utility vectors for the n system users. The Nash bargaining solution (our preferred solution for the co-operative game) requires the feasible utility set to be *convex* and *compact*. In this section, we mathematically prove that our joint feasible utility set $\mathbf{U}$ is both, convex and compact in the *metric space* $\Re^n$. We first review some basic definitions on convexity and compactness of sets in metric spaces.

Definition 1. A set $\mathbf{U}$ in $\Re^n$ is said to be convex if for any two points $u_1, u_2 \,\epsilon\, \mathbf{U}$, the linear combination $\alpha u_1 + (1-\alpha)u_2, \epsilon\, \mathbf{U}$, where $0 \leq \alpha \leq 1$. (See [13] for more details.)

Definition 2. A set $\mathbf{U}$ is compact in $\Re^n$ if and only if its *closed* and *bounded*[9]. A set $\mathbf{U}$ is closed in a metric space[10] if $\mathbf{U}$ contains all its limit points, i.e., every converging *Cauchy sequence* of points in $\mathbf{U}$ converges to a point in $\mathbf{U}$. A set $\mathbf{U}$ in $\Re^n$ is bounded if any two points in the set are within a finite distance of each other, where the distance is defined by the *Euclidean distance*. The reader is referred to [14] for more details on topological properties in metric spaces.

Theorem 1. *The set* $\mathbf{U}$ *of feasible instances of the reliability metric is convex and compact.*

Proof. By our assumptions in Section III, for each user i, its allocated bandwidth $b_i > R_{0i}$. Let the maximum total available network bandwidth be B_{max}. Thus, $\sum_{i=1}^{n} b_i \leq B_{max}$. Let $S_i = U_i(b_i)$, for each user i, for a particular feasible instance $(U_1(b_1), ..., U_n(b_n))$. Therefore, the utility of each user i, $U_i(b_i)$, is bounded above by $U_i(B_{max})$. This proves that the set $\mathbf{U}$=$\{(U_1(b_i), ..., U_n(b_n))\}$ of feasible instances is *bounded*. From equation 3, b_i = $\frac{\theta_i S_i}{k - D_{0i} S_i}$ + R_{0i}. Thus, we have $\sum_{i=1}^{n} \frac{\theta_i S_i}{k-D_{0i}S_i} \leq B_{max} - \sum_{i=1}^{n} R_{0i}$. The feasible utility set $\mathbf{U}$ can be expressed as

$$\mathbf{U} = \{\mathbf{S} | \sum_{i=1}^{n} \frac{\theta_i S_i}{k - D_{0i} S_i} \leq B_{max} - \sum_{i=1}^{n} R_{0i},\, S_i \geq 0,\, \forall i\} \quad (4)$$

To prove the convexity of $\mathbf{U}$, we need to show the following

$$\sum_{i=1}^{n} \frac{\theta_i(\alpha X_i + (1-\alpha)Y_i)}{k - D_{0i}(\alpha X_i + (1-\alpha)Y_i)} \leq B_{max} - \sum_{i=1}^{n} R_{0i} \;\forall \mathbf{X}, \mathbf{Y} \,\epsilon\, \mathbf{U}, \quad (5)$$

where $\mathbf{X}$ = $(X_1, ..., X_n)$ = $(U_1(x_1), ..., U_n(x_n)) \,\epsilon\, \mathbf{U}$, and $\mathbf{Y}$ = $(Y_1, ..., Y_n)$ = $(U_1(y_1), ..., U_n(y_n)) \,\epsilon\, \mathbf{U}$, and $0 \leq \alpha \leq 1$. (5) in turn gives reduces/decomposes to the following set of three inequalities as shown.

$$CE = \begin{cases} \sum_{i=1}^{n} \frac{\theta_i X_i}{k - D_{0i} X_i} \leq B_{max} - \sum_{i=1}^{n} R_{0i} & \text{if } \alpha = 1 \\ \sum_{i=1}^{n} \frac{\theta_i Y_i}{k - D_{0i} Y_i} \leq B_{max} - \sum_{i=1}^{n} R_{0i} & \text{if } \alpha = 0 \\ \leq max\{\sum_{i=1}^{n} \frac{\theta_i X_i}{k - D_{0i} X_i}, \sum_{i=1}^{n} \frac{\theta_i Y_i}{k - D_{0i} Y_i}\} & \text{otherwise} \end{cases}$$

where $CE = \sum_{i=1}^{n} \frac{\theta_i(\alpha X_i + (1-\alpha)Y_i)}{k - D_{0i}(\alpha X_i + (1-\alpha)Y_i)}$. The first two inequalities follow directly from (4). The last of the three inequalities follows from the fact that the function $f(\alpha) = \sum_{i=1}^{n} \frac{\theta_i(\alpha X_i + (1-\alpha)Y_i)}{k - D_{0i}(\alpha X_i + (1-\alpha)Y_i)}$ is *convex* for $0 < \alpha < 1$. *The three inequalities combined prove the convexity of* $\mathbf{U}$. To prove the convexity of $f(\alpha)$, we need to explore the *second derivative* of f and show that it is non-negative for all $\alpha \,\epsilon\, (0, 1)$ [13]. The second derivative of each component of f is given by

$$f_i''(\alpha) = \frac{2k\theta_i D_{0i}(X_i - Y_i)^2}{(k - D_{0i}(\alpha X_i + (1-\alpha)Y_i))^3} \quad (6)$$

The numerator of $f_i''(\alpha)$ is obviously non-negative. Since $x_i > R_{0i}$, we have $\frac{\alpha_i X_i}{k - D_{0i} X_i} > 0$. We know that all X_i, D_{0i} are non-negative. Thus, for each component i, $k - D_{0i} X_i > 0$. By a similar argument, $k - D_{0i} Y_i > 0$. Therefore, $k - D_{0i}(\alpha X_i + (1-\alpha)Y_i) > 0$, implying the fact that the denominator of $f_i''(\alpha) > 0$. Hence we get $f_i''(\alpha) > 0$. Since $f(\alpha) = \sum_{i=1}^{n} f_i(\alpha)$, and that the sum of convex function is convex [13], $f(\alpha)$ is *convex*.

To prove the closed property of $\mathbf{U}$, we just need to show that every Cauchy sequence in $\mathbf{U}$ converges to a point in $\mathbf{U}$. Let $\{t_p\}$ = $\{(U_1(b_1), ..., U_n(b_1))\}$ be one such arbitrary Cauchy sequence. Since b_i for each user i is bounded by B_{max}, so is $U_i(b_i)$ by $U_i(B_{max})$, and thus we have the fact that the limit of t_p, as $p \to \infty$, lies in $\mathbf{U}$. Since $\{t_p\}$ was arbitrary, $\mathbf{U}$ is *closed* for all Cauchy sequences $\{t_p\}$. Since, $\mathbf{U}$ is closed as well as bounded in $\Re^n$, it is *compact* by the Heine-Borel theorem [14]. ■

C. The Nash Bargaining Solution

In this section, we provide a formulation of a Nash bargaining solution for our problem to provide the readers with a basic algebraic-geometric intuition.

The Nash bargaining solution for the n-user game is a function $F(\mathbf{U}, \mathbf{d})$ defined as

$$F(\mathbf{U}, \mathbf{d}) = \{\mathbf{l} \,\epsilon\, \mathbf{BS} | \mathbf{l} = \sum_{i=1}^{n} \beta_i \mathbf{v}_i;\, \sum_{i=1}^{n} \beta_i = 1; \beta_i \geq 0, \forall i, \} \quad (7)$$

where $\mathbf{BS}$ is the bargaining set, the $\beta_i's$ are the *bargaining powers*[11] of each user/player, and the $\mathbf{v}_i's$ are the intersection of the *supporting hyperplane* to $\mathbf{U}$, and the axes passing through the disagreement point $\mathbf{d}$ = (0,....,0). The supporting hyperplane at $\mathbf{l}$ is a $(n-1)$ dimensional plane through points $\mathbf{v}_1$., , , $\mathbf{v}_n$, and $\mathbf{l}$. We note that each $\mathbf{v}_i$ is of the form $v_i \mathbf{e}_i$, where $\mathbf{e}_i$ is the unit vector in the direction of utility axis S_i. The bargaining set $\mathbf{BS}$ of $\mathbf{U}$ can be formulated as

$$\mathbf{BS} = \{\mathbf{S} = (S_1, ..., S_n) | \sum_{i=1}^{n} \frac{\theta_i S_i}{k - D_{0i} S_i} = B_{max} - \sum_{i=1}^{n} R_{0i},\, S_i > 0,\, \forall i\} \quad (8)$$

In fact the Nash bargaining solution (NBS) is the vector $(S_1^*, ..., S_n^*)$ that lies in the bargaining set $\mathbf{BS}$ and must satisfy

$$\sum_{i=1}^{n} \frac{\theta_i S_i^*}{k - D_{0i} S_i^*} = B_{max} - \sum_{i=1}^{n} R_{0i},\, S_i^* > 0,\, \forall i \quad (9)$$

or, equivalently

$$\sum_{i=1}^{n} b_i^* \leq B_{max},\, b_i^* > R_{0i},\, \forall i \quad (10)$$

The $(n-1)$ dimensional supporting hyperplane at point $\mathbf{l}$ is also perpendicular to the *gradient* at $\mathbf{l}$, which is evaluated as

$$\nabla \mathbf{BS}|_{\mathbf{l}} = [\frac{k\theta_1}{(k - D_{01}\beta_1 v_1)^2} .... \frac{k\theta_n}{(k - D_{0n}\beta_n v_n)^2}]^T \quad (11)$$

The supporting hyperplane includes all points $\mathbf{v}_i$, and as a result the gradient at $\mathbf{l}$ is perpendicular to the vector $\mathbf{v}_i - \mathbf{v}_j$ for all i, $j \,\epsilon\, \{1, ..., n\}$, $i \neq j$. Thus, we have the following relationship regarding the orthogonality of the hyperplane at $\mathbf{l}$ and vectors of the form $\mathbf{v}_i - \mathbf{v}_j$.

$$\{\nabla \mathbf{BS}|_{\mathbf{l}}\}^T \cdot (\mathbf{v}_i - \mathbf{v}_j) = 0, \quad (12)$$

[9] This is the celebrated *Heine-Borel* theorem in mathematical analysis.

[10] In this paper we only consider the finite dimensional metric space $\Re^n$. This is the commonly known *Euclidean space*.

[11] In co-operative game theory, bargaining power is the ability of a player to influence the system to achieve its desired goal, i.e., in this problem a player's desired goal is to achieve a required QoS. Each player may be a source to a particular application that demands service at given data rates.

which implies

$$\frac{k\theta_i v_i}{(k - D_{0i}\beta_i v_i)^2} = \frac{k\theta_j v_j}{(k - D_{0j}\beta_i v_j)^2} \tag{13}$$

Replacing S_i^* by $\beta_i v_i$ in (11), we have

$$\frac{\beta_i(k - D_{0i}S_i^*)^2}{\theta_i S_i^*} = \frac{\beta_j(k - D_{0j}S_j^*)^2}{\theta_j S_j^*},\ \forall i, j\,\epsilon\,\{1, ..., n\},\ i \neq j, \tag{14}$$

or, equivalently

$$\frac{\beta_i\theta_i}{(b_i^* - R_{0i})(D_{0i}(b_i^* - R_{0i}) + \theta_i)} = \frac{\beta_j\theta_j}{(b_j^* - R_{0ij})(D_{0j}(b_j^* - R_{0j}) + \theta_j)} \tag{15}$$

Solving the set of equations in (14) and (15) gives us the optimal values of S_i^* and b_i^*. However, there exists no general solution to the system of equations in (14) and (15) for arbitrary dimensions n. In this paper, we use a bisection search method *(http://en.wikipedia.org/wiki/Bisection-method)* to solve the system of equations. A bisection search procedure is a simple and robust numerical root finding procedure that can be easily adopted when the lower and upper bounds of the root variable are known. To use a bisection search procedure (bisection method), we transform our root finding problem into a problem of a single scalar variable. We focus on expressing each b_i in terms of b_1 and use our bisection method to compute b_1^*. Each S_i^* can be evaluated by plugging in the corresponding values of b_i^* in the utility function $U_i(b_i^*)$. We derive the relationship between each $b_i | i \neq 1$ and b_1 as

$$b_i = \begin{cases} \frac{-\beta_1\theta_1\theta_i + \sqrt{(\beta_1\theta_1\theta_2)^2 + 4\beta_1\beta_i\theta_1\theta_i D_{0i} z_1}}{2\beta_1\theta_1 D_{0i}} + R_{0i} & \text{if } D_{0i} \neq 0 \\ \frac{\beta_i z_1}{\beta_1\theta_1} + R_{0i} & \text{if } D_{0i} = 0 \end{cases} \tag{16}$$

where $z_1 = (b_1 - R_{01})(D_{01}(b_1 - R_{01}) + \theta_1$. Our bisection search algorithm is adopted from [13] and is as follows.

Algorithm: *Bisection Search*
Begin
Input: lower bound, $lb = min\{R_{01}, ..., R_{0n}\}$; upper bound, $ub = B_{max}$; tolerance, $\epsilon > 0$
do
1. let $b_1 = \frac{lb+ub}{2}$
2. compute all b_i; $i \neq 1$ using equation 16
3. check for the feasibility condition that $\sum_{i=1}^{n} b_i \leq B_{max}$
4. **if** feasibility condition satisfied, $lb = b_1$, **else** $ub = b_1$

until $ub - lb \leq \epsilon$
Output all optimal b_i^*; $i = 1, ..., n$
end *Bisection Search*

Once we get the final values of the b_i^*'s, we can compute and output the vector $\mathbf{S}^* = (U_1(b_1^*), ..., U_n(b_n^*))$ as the NBS. The Nash bargaining solution for our problem gives the optimal bandwidth allocated to each application to achieve their required QoS. We emphasize that for different bargaining power vectors we get different NBSs. The bargaining power vector plays a pivotal role in the allocation of bandwidth to applications being serviced in a WBSN.

D. Results

We derive the following theorems regarding the solution to our bargaining problem. We omit the proofs of the theorems 2 and 3 due to lack of space.

Theorem 1. *The set* $\mathbf{U}$ *of feasible instances of the reliability metric is convex and compact.*

Theorem 2. *The Nash bargaining solution for our bargaining problem is unique.*

Theorem 3. *The Nash bargaining solution for our bargaining problem is optimal.*

Theorem Implications. Theorem 1 (proved in Section IV-B) gives mathematical properties of $\mathbf{U}$, which are necessary and sufficient for an instance in $\mathbf{U}$ to be a Nash bargaining solution. Theorems 2 and 3 state that the solution to our problem is unique and optimal. The optimal solution (bandwidth allocation) drives the working of various system parameters at the optimal level, thereby maximizing reliability.

V. Conclusion

In this paper, we have investigated the important problem of *unified reliability characterization* and *optimization* in multi-hop wireless body sensor networks. We have derived a unified reliability metric for individual WBSN nodes with bandwidth as the limiting resource. The metric will be able to jointly reflect performance parameters like throughput, network delay, quality of audio/video, network life-time, and application fairness, for a network with time-varying channel conditions. In addition, we have designed an efficient distributed mechanism (bargaining) based on co-operative game theory to optimize our proposed reliability metric.

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