Dynamic Spectrum Supply Chain Model for Cognitive Radio Networks

Peyman Setoodeh and Simon Haykin

Cognitive Systems Laboratory
McMaster University
Hamilton, Ontario, L8S 4K1, Canada
Email: setoodp@mcmaster.ca, haykin@mcmaster.ca

Keyvan Rezaei Moghadam

Ming Hsieh Department of Electrical Engineering
University of Southern California
Los Angeles, CA 90007, USA
Email: rezaeimo@usc.edu

***Abstract*—Built on the theory of supply chain networks, this paper presents a model for the spectrum market, which includes three different tiers of decision makers; legacy owners, spectrum brokers, and secondary users (cognitive radios). Behavior of various decision-makers, the governing equilibrium conditions, and the transient behavior of the network are studied. Prices are determined endogenously in the model.**

***Keywords*-Cognitive radio; projected dynamic (PD) systems; supply chain network; variational inequalities (VI).**

I. Introduction

In signal-processing terms, a feature that distinguishes cognitive radio from conventional wireless communication, is the cognitive-information-processing cycle [1], which is depicted in Figure 1. This cycle applies to a secondary (unserviced) user, where a transmitter at one location communicates with a receiver at some other location via a spectrum hole, that is, a licensed subband of the radio spectrum that is underutilized at a particular point in time and at a particular location. The cognitive cycle encompasses two basic operations; radio-scene analysis of the surrounding wireless environment at the receiver, and dynamic spectrum management/transmit-power control at the transmitter (radio-environment actuator). Information on spectrum holes and the forward channel's condition, extracted by the scene-analyzer at the receiver, is sent to the transmitter via a feedback channel.

There are two primary resources in a cognitive radio network; channel bandwidth and transmit power. Dynamic spectrum manager solves a limited-resource distribution problem and is designed to dynamically assign available channels to cognitive radio units in a fair and efficient manner [2]. The information that transmitter receives through the feedback channel enables it to adaptively adjust the transmitted signal and update its transmit power over desired channels [3].

There are two major categories for dynamic spectrum access:

- *Opportunistic spectrum access* employs open sharing of spectrum among peer secondary users.
- *Market-oriented spectrum access*, in which pricing is involved and legacy owners gain profit by leasing their idle and partially used subbands to secondary users.

Secondary users rely much more on the spectrum sensing in the opportunistic spectrum sharing regime.

TV and cellular bands are two major candidates, in which cognitive radio networks can be implemented via secondary usage of spectrum. There are two ways to build a cognitive radio network, one being *evolutionary* and the other *revolutionary*, which are suitable for cellular bands and TV bands, respectively.

In the revolutionary viewpoint, there are no communication infrastructures. The activities of TV stations and therefore, the occupancy of the TV bands by legacy owners are well-defined. TV bands provide idle spectrum, suitable for secondary usage, over specified relatively-long time intervals. In the idle periods, secondary users compete with each other for using the TV bands and their performance totally relies on the radio-scene analyzer at the receiver, which continuously monitors the radio environment to identify less-crowded subbands that are suitable for communication. In [3] the noncooperative game between secondary users in such an open spectrum regime, in which pricing is not involved, was solved using *iterative waterfilling algorithm* (IWFA). The network was viewed as a global closed loop feedback system, embodying simple models of four functional blocks of the cognitive radio; transmitter, receiver, feedback channel, and communication channel. Both equilibrium and transient behaviors of the network were studied using the theories of *variational inequalities* (VI) [4] and *projected dynamic* (PD) systems [5], respectively. The same framework can be equally applied to investigate the network behavior in license-free subbands.

In the evolutionary viewpoint, the currently established communication infrastructures can be utilized and cognitive radio networks are built around existing multi-cellular networks. A market-oriented framework is required to model and analyze the behavior of the cognitive radio networks in cellular bands. Since pricing is involved, the model should take account of decision makers from other tiers as well.

Spectrum sharing among one primary user and multiple

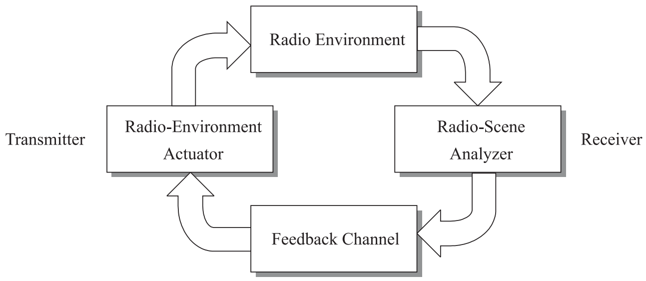

Figure 1. The cognitive-information-processing cycle in cognitive radio.

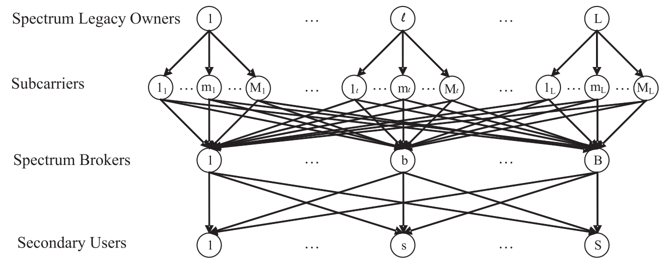

Figure 2. The spectrum supply chain network.

secondary users [6] as well as spectrum sharing among multiple primary users and one secondary user [7], [8] were studied using market-based models. In [6] two scenarios were considered for one primary and multiple secondary user case, in which secondary users compete with each other for accessing the spectrum. In the first model, it was assumed that all secondary users are completely aware of strategies and payoffs of other secondary users. In the second model, each secondary user was able to exchange information only with the primary user. Secondary users were adjusting their actions according to the price requested by the primary user. In [7], [8] for multiple primary and one secondary user case, primary users compete with each other to offer their bands to the secondary user by dynamically adjusting their prices. Three different scenarios were considered for this case. In the first model, the total profit of all the primary users was maximized by cooperative pricing. In the second model, primary users were not aware of the utility functions of other primary users but they could observe their offered prices. In the third model, the assumption that each primary user knows other primary users' prices was relaxed.

Built on the theory of *supply chain networks* [9], this paper presents a model for the spectrum market, which includes three different tiers of decision makers; spectrum legacy owners that provide service to primary users, spectrum brokers, and secondary users (cognitive radio networks). Figure 2 depicts the multitiered spectrum supply chain network [9]. In this framework, spectrum brokers are responsible for assigning channels to cognitive radios through advertising the idle bands and therefore, the radio-scene analyzer does not play a key role in identifying idle subbands as before. However, availability of subbands depends on the communication patterns of primary users in the region of interest, which is far less predictable than activities of primary users in TV bands. This makes the cellular band a more dynamic environment than TV band. The equilibrium point of the spectrum supply chain network, in which none of the decision makers has the incentive to change its policy unilaterally, is the solution of a VI problem. Also, the corresponding PD system, whose stationary points coincide with the equilibrium solutions of the VI, describes the transient behavior of the network.

There are two main sources of uncertainty in the spectrum supply chain, which may lead to disruptions and the associated risks; demand-side risk and supply-side risk. Secondary users are the cause of demand-side risk. They may freely join or leave the network and their communication patterns heavily depend on social events. Primary users are the cause of supply-side risk. Appearance and disappearance of spectrum holes (under-utilized subbands) depend on their communication patterns, which are also related to social events. Therefore, in a region, where there is a high demand for spectrum, the amount of idle spectrum available for secondary usage may be very limited. Since the impacts of supply chain disruptions may not remain locally and they may propagate globally all over the network, modeling and analysis methods, which provide system-level views, are essential for understanding the complex interactions of decision-makers. Such a global viewpoint will be very helpful for risk management. In order to reduce the impacts of disruptions and the associated risks, it is crucial to employ design methods that provide robustness from both local and global perspectives [10].

The paper is organized as follows: Sections II and III focus on the equilibrium and disequilibrium behaviours of the network. Section IV provides the stability analysis of the network. The equivalence between the spectrum supply chain network and a properly configured transportation network is presented in Section V, which allows us to benefit from the literature and ongoing research in other disciplines. Also, it facilitates studying emergent behaviors in the spectrum supply chain network similar to *Braess paradox* [11] in transportation networks, when primary users release subbands. The paper concludes in the final section.

II. Variational Inequality Formulation of the Spectrum Supply Chain Network

A three dimensional resource space is considered, with time, frequency, and power as different dimensions. *Virtual power cube* (VPC), which is the minimum transmit power required to transmit an information unit, is used as the resource unit [12]. Consider L spectrum legacy owners, each of which owns spectrum subbands including M_ℓ subcarriers ($\ell = 1, \cdots, L$) and provides service to a number of primary users. Assume that there are B spectrum brokers and S

secondary users. Each legacy owner tries to maximize its profit by leasing its idle and partially used subcarriers to spectrum brokers. Spectrum brokers purchase the right for using those subcarriers from legacy owners and sell them to secondary users. They compete with each other in a noncooperative manner for gaining and trading that right. In the following subsections, the behavior of subsystems in each tier of the network as well as their optimality conditions are studied and a VI formulation for finding the network equilibrium is presented.

A. Legacy Owners

Let us assume that legacy owner ℓ charges the spectrum broker b for subcarrier m_ℓ with the unit price $\rho^*_{m_\ell b}$. The optimal values of prices $\rho^*_{m_\ell b}$ for $\ell = 1, \cdots, L$; $b = 1, \cdots, B$; $m_\ell = 1_\ell, \cdots, M_\ell$; ($\sum_{\ell=1_\ell}^{L} M_\ell = M$) are determined by finding the equilibrium point of the spectrum supply chain network. Legacy owners compete in a noncooperative manner and each legacy owner ℓ solves the following optimization problem in order to maximize its own profit.

$$
\begin{aligned}
max \quad & \sum_{m_\ell=1_\ell}^{M_\ell} \sum_{b=1}^{B} \left[\rho^*_{m_\ell b} p_{m_\ell b} - c_{m_\ell b}(p_{m_\ell b})\right] \quad (1)\\
& - \sum_{m_\ell=1_\ell}^{M_\ell} f_{m_\ell}(p_{m_\ell})\\
subject\ to \quad : \quad & \sum_{b=1}^{B} p_{m_\ell b} \le C_{m_\ell},\ \forall m_\ell = 1_\ell, \cdots, M_\ell\\
& p_{m_\ell b} \ge 0,\ \forall m_\ell = 1_\ell, \cdots, M_\ell,\\
& \qquad \forall b = 1, \cdots, B
\end{aligned}
$$

The first term in the objective function is the difference between revenue and transaction costs. The second term is a penalty function, which reflects the effect of cross-channel interference and the effect of co-channel interference if legacy owners let the secondary users use the nonidle subcarriers in their frequency bands as well. It will also help to encourage users to use less-utilized subcarriers. The transaction- and interference-cost functions, $c_{m_\ell b}(p_{m_\ell b})$ and $f_{m_\ell}(p_{m_\ell})$, are assumed to be continuously differentiable and convex. The first set of constraints guarantees that the *permissible interference power level* limit, C_{m_ℓ}, will not be violated in any subcarrier.

The Nash equilibrium of the noncooperative game between legacy owners, $(\mathbf{p}^{L*}, \mathbf{p}^{B*}) \in K^1$, coincides with the solution of the following VI problem.

$$
\begin{aligned}
&\sum_{\ell=1}^{L} \sum_{m_\ell=1_\ell}^{M_\ell} \sum_{b=1}^{B} \left[\frac{\partial c_{m_\ell b}(p^*_{m_\ell b})}{\partial p_{m_\ell b}} - \rho^*_{m_\ell b}\right] \times \left[p_{m_\ell b} - p^*_{m_\ell b}\right]\\
&\qquad + \sum_{\ell=1}^{L} \sum_{m_\ell=1_\ell}^{M_\ell} \frac{\partial f_{m_\ell}(p^*_{m_\ell})}{\partial p_{m_\ell}} \times \left[p_{m_\ell} - p^*_{m_\ell}\right] \ge 0 \quad (2)
\end{aligned}
$$

$\forall(\mathbf{p}^L, \mathbf{p}^B) \in K^1$, where $\mathbf{p}^L$ and $\mathbf{p}^B$ are respectively the LM- and LMB-dimensional power vectors, whose elements are p_{m_ℓ} and $p_{m_\ell b}$.

$$
\begin{aligned}
&K^1 = \{(\mathbf{p}^L, \mathbf{p}^B) | (\mathbf{p}^L, \mathbf{p}^B) \in \mathbb{R}_+^{LM+LMB};\\
&\sum_{b=1}^{B} p_{m_\ell b} \le C_{m_\ell}, \forall \ell = 1, \cdots, L,\ \forall m_\ell = 1_\ell, \cdots, M_\ell\} \quad (3)
\end{aligned}
$$

B. Spectrum Brokers

Spectrum brokers are involved in transactions with both legacy owners and secondary users. Let us assume that spectrum broker b charges the secondary user s with the unit price ρ^*_{bs}. This price is determined by finding the equilibrium point of the spectrum supply chain network. Each spectrum broker b maximizes its own profit by solving the following optimization problem.

$$
\begin{aligned}
max \quad & \sum_{s=1}^{S} \left[\rho^*_{bs} p_{bs} - c_{bs}(p_{bs})\right] \quad (4)\\
& - \sum_{m_\ell=1_\ell}^{M_\ell} \sum_{b=1}^{B} \left[\rho^*_{m_\ell b} p_{m_\ell b} + \hat{c}_{m_\ell b}(p_{m_\ell b})\right]\\
subject\ to \quad : \quad & \sum_{s=1}^{S} p_{bs} = \sum_{\ell=1}^{L} \sum_{m_\ell=1_\ell}^{M_\ell} p_{m_\ell b}\\
& p_{m_\ell b} \ge 0,\ \ \forall \ell = 1_\ell, \cdots, L;\\
& \qquad \forall m_\ell = 1, \cdots, M_\ell\\
& p_{bs} \ge 0,\ \ \forall s = 1, \cdots, S
\end{aligned}
$$

The objective function includes the revenue, payment to legacy owners, and the respective transaction costs. The first constraint expresses that the total amounts of purchased and sold spectra are equal for each spectrum broker. It is assumed that transaction costs $c_{bs}(p_{bs})$ and $\hat{c}_{m_\ell b}(p_{m_\ell b})$ are continuously differentiable and convex.

The Nash equilibrium of the noncooperative game between spectrum brokers, $(\mathbf{p}^{B*}, \mathbf{p}^{S*}) \in K^2$, coincides with the solution of the following VI problem.

$$
\begin{aligned}
&\sum_{\ell=1}^{L} \sum_{m_\ell=1_\ell}^{M_\ell} \sum_{b=1}^{B} \left[\frac{\partial \hat{c}_{m_\ell b}(p^*_{m_\ell b})}{\partial p_{m_\ell b}} + \rho^*_{m_\ell b}\right] \times \left[p_{m_\ell b} - p^*_{m_\ell b}\right]\\
&\qquad + \sum_{b=1}^{B} \sum_{s=1}^{S} \left[\frac{\partial c_{bs}(p^*_{bs})}{\partial p_{bs}} - \rho^*_{bs}\right] \times \left[p_{bs} - p^*_{bs}\right] \ge 0 \quad (5)
\end{aligned}
$$

$\forall(\mathbf{p}^B, \mathbf{p}^S) \in K^2$, where $\mathbf{p}^B$ and $\mathbf{p}^S$ are respectively the LMB- and BS-dimensional power vectors, whose elements are $p_{m_\ell b}$ and p_{bs}.

$$
\begin{aligned}
&K^2 = \{(\mathbf{p}^B, \mathbf{p}^S) | (\mathbf{p}^B, \mathbf{p}^S) \in \mathbb{R}_+^{LMB+BS};\\
&\sum_{s=1}^{S} p_{bs} = \sum_{\ell=1}^{L} \sum_{m_\ell=1_\ell}^{M_\ell} p_{m_\ell b},\ \ \forall b = 1, \cdots, B\} \quad (6)
\end{aligned}
$$

C. Secondary Users

Secondary users compete with each other for resources in a noncooperative manner. The secondary users' priority is to use license-free subbands. In license-free subbands they compete for limited resources till the network reaches an equilibrium in the sense described in [3]. Since those subbands are usually too crowded, in the equilibrium condition some of the users may not gain enough resources and they will have to lease the licensed subbands. Each secondary user s tries to maximize its share of resources, d_s, subject to its power and budget constraints by solving the following optimization problem.

$$
\begin{aligned}
max \qquad & d_s - \sum_{b=1}^{B} [\rho_{bs}^* + \hat{c}_{bs}]\, p_{bs} && (7)\\
subject\ to \quad : \qquad & d_s = \sum_{b=1}^{B} p_{bs} \\
& \rho_{bs}^* + \hat{c}_{bs} \leq \frac{M_s^{max}}{P_s^{max}} \\
& 0 \leq d_s \leq P_s^{max} \\
& p_{bs} \geq 0, \quad \forall b = 1, \cdots, B
\end{aligned}
$$

where $\hat{c}_{bs}$ is the unit transaction cost, M_s^{max} is the secondary user's maximum budget that can be dedicated to leasing spectrum, and P_s^{max} is the difference between the secondary user's power budget and the amount of transmit power that it uses in license-free subbands. The first inequality constraint set puts a limit on the price that the secondary user will pay for a resource unit.

The Nash equilibrium of the noncooperative game between secondary users, $(\mathbf{p}^{S*}, \mathbf{d}^*) \in K^3$, coincides with the solution of the following VI problem.

$$
\sum_{b=1}^{B} \sum_{s=1}^{S} [\rho_{bs}^* + \hat{c}_{bs}] \times [p_{bs} - p_{bs}^*] - \sum_{s=1}^{S} [d_s - d_s^*] \geq 0 \quad (8)
$$

$\forall (\mathbf{p}^S, \mathbf{d}) \in K^3$, where $\mathbf{p}^S$ and $\mathbf{d}$ are respectively the BS-and S-dimensional vectors, whose elements are p_{bs} and d_s.

$$
\begin{aligned}
K^3 = \{(\mathbf{p}^S, \mathbf{d}) | (\mathbf{p}^S, \mathbf{d}) \in \mathbb{R}_+^{BS+S}; \\
\rho_{bs}^* + \hat{c}_{bs} \leq \frac{M_s^{max}}{P_s^{max}}, \forall b = 1, \cdots, B, \forall s = 1, \cdots, S; \\
d_s = \sum_{b=1}^{B} p_{bs} \leq P_s^{max}, \ \ \forall s = 1, \cdots, S\} \quad (9)
\end{aligned}
$$

D. Spectrum Supply Chain Network Equilibrium

The network reaches an equilibrium point if the optimality conditions of all decision makers in the network including the legacy owners, the spectrum brokers, and the secondary users are satisfied in a way that none of them has the incentive to change its policy.

Theorem 1: The equilibrium of the spectrum supply chain network, $(\mathbf{p}^{L*}, \mathbf{p}^{B*}, \mathbf{p}^{S*}, \mathbf{d}^*) \in K$, coincides with the solution of the following VI problem.

$$
\begin{aligned}
& \sum_{\ell=1}^{L} \sum_{m_\ell=1_\ell}^{M_\ell} \frac{\partial f_{m_\ell}(p_{m_\ell}^*)}{\partial p_{m_\ell}} \times \left[p_{m_\ell} - p_{m_\ell}^*\right] - \sum_{s=1}^{S} [d_s - d_s^*] \\
& + \sum_{b=1}^{B} \sum_{s=1}^{S} \left[\frac{\partial c_{bs}(p_{bs}^*)}{\partial p_{bs}} + \hat{c}_{bs}\right] \times [p_{bs} - p_{bs}^*] \\
& + \sum_{\ell=1}^{L} \sum_{m_\ell=1_\ell}^{M_\ell} \sum_{b=1}^{B} \left[\frac{\partial c_{m_\ell b}(p_{m_\ell b}^*)}{\partial p_{m_\ell b}} + \frac{\partial \hat{c}_{m_\ell b}(p_{m_\ell b}^*)}{\partial p_{m_\ell b}}\right] \\
& \times \left[p_{m_\ell b} - p_{m_\ell b}^*\right] \geq 0, \\
& \forall (\mathbf{p}^L, \mathbf{p}^B, \mathbf{p}^S, \mathbf{d}) \in K \quad (10)
\end{aligned}
$$

where

$$
\begin{aligned}
K = \{(\mathbf{p}^L, \mathbf{p}^B, \mathbf{p}^S, \mathbf{d}) | (\mathbf{p}^L, \mathbf{p}^B, \mathbf{p}^S, \mathbf{d}) \in \mathbb{R}_+^{(LM+S)(B+1)}; \\
\sum_{b=1}^{B} p_{m_\ell b} \leq C_{m_\ell}, \forall \ell = 1, \cdots, L; \ \forall m_\ell = 1_\ell, \cdots, M_\ell, \\
\sum_{s=1}^{S} p_{bs} = \sum_{\ell=1}^{L} \sum_{m_\ell=1_\ell}^{M_\ell} p_{m_\ell b}, \ \ \forall b = 1, \cdots, B; \\
d_s = \sum_{b=1}^{B} p_{bs} \leq P_s^{max}, \ \ \forall s = 1, \cdots, S; \\
\rho_{bs}^* + \hat{c}_{bs} \leq \frac{M_s^{max}}{P_s^{max}}, \forall b = 1, \cdots, B, \forall s = 1, \cdots, S\} \quad (11)
\end{aligned}
$$

The prices $\rho_{m_\ell b}^*$ can be recovered from (2) and (3) as

$$
\rho_{m_\ell b}^* = \frac{\partial f_{m_\ell}(p_{m_\ell}^*)}{\partial p_{m_\ell}} + \frac{\partial c_{m_\ell b}(p_{m_\ell b}^*)}{\partial p_{m_\ell b}} \quad (12)
$$

for any ℓ, m_ℓ, b such that $p_{m_\ell b}^* > 0$ and the prices ρ_{bs}^* can be recovered from (8) and (9) as

$$
\rho_{bs}^* = \frac{M_s^{max}}{P_s^{max}} - \hat{c}_{bs} \quad (13)
$$

for any b, s such that $p_{bs}^* > 0$.

The state vector of the spectrum supply chain network is defined as

$$
\mathbf{x} = \left(\mathbf{p}^L, \mathbf{p}^B, \mathbf{p}^S, \mathbf{d}\right)^T \quad (14)
$$

By concatenating the corresponding terms in (10), we will have

$$
\mathbf{F}(\mathbf{x}) = \begin{bmatrix}
\left[\left[\frac{\partial f_{m_\ell}(p_{m_\ell}^*)}{\partial p_{m_\ell}}\right]_{m_\ell=1_\ell}^{M_\ell}\right]_{\ell=1}^{L} \\
\left[\left[\left[\frac{\partial c_{m_\ell b}(p_{m_\ell b}^*)}{\partial p_{m_\ell b}} + \frac{\partial \hat{c}_{m_\ell b}(p_{m_\ell b}^*)}{\partial p_{m_\ell b}}\right]_{b=1}^{B}\right]_{m_\ell=1_\ell}^{M_\ell}\right]_{\ell=1}^{L} \\
\left[\left[\frac{\partial c_{bs}(p_{bs}^*)}{\partial p_{bs}} + \hat{c}_{bs}\right]_{s=1}^{S}\right]_{b}^{B} \\
-\mathbf{1}_S
\end{bmatrix} \quad (15)
$$

where $\mathbf{1}_S$ is an S-dimensional vector, whose elements are all 1. The VI problem in (10) and (11) can be rewritten in the compact form VI(K,$\mathbf{F}$):

The vector $\mathbf{x}^*$ is a Nash equilibrium point of the VI(K,$\mathbf{F}$) if, and only if, $\mathbf{x}^* \in K$ and $\forall \mathbf{x} \in K$

$$(\mathbf{x} - \mathbf{x}^*)^T \mathbf{F}(\mathbf{x}^*) \geq 0 \tag{16}$$

III. Projected Dynamic System

The PD system theory [5] can be utilized to associate an ordinary differential equation (ODE) to the obtained VI. A projection operator, which is discontinuous, appears in the right-hand side of the ODE to incorporate the feasibility constraints of the VI problem into the dynamics. This ODE provides a dynamic model for the competitive system whose equilibrium behavior is described by the VI. Also, the stationary points of the ODE coincide with the set of solutions of the VI, which are the equilibrium points [5].

Theorem 2: Assume that *K* is a convex polyhedron. Then the equilibrium points of the PDS(*K*,F) coincide with the solutions of VI(*K*,F).

Thus, the equilibrium problem can be studied in a dynamic framework. This dynamic model enables us not only to study the transient behavior of the network, but also to predict it. Before we proceed, we need to recall some mathematical definitions from [5]. The set of inward normals at $\mathbf{x} \in K$ is defined as

$$S(\mathbf{x}) = \{\gamma : \ \|\gamma\| = 1 \ , \ \langle \gamma, \mathbf{x} - \mathbf{y} \rangle \leq 0, \ \ \forall \mathbf{y} \in X\} \tag{17}$$

The following ODE

$$\dot{\mathbf{x}} = \Pi_X \left(\mathbf{x}, -\mathbf{F}(\mathbf{x})\right) \tag{18}$$

with the initial condition

$$\mathbf{x}(t_0) = \mathbf{x}_0 \in K \tag{19}$$

is called a projected dynamic system. When $\mathbf{x}(t)$ is in the interior of the feasible set, $\mathbf{x}(t) \in$ *intK*, the projection operator in the right hand side of (18) is

$$\Pi_K \left(\mathbf{x}, -\mathbf{F}(\mathbf{x})\right) = -\mathbf{F}(\mathbf{x}) \tag{20}$$

If $\mathbf{x}(t)$ reaches the boundary of the feasible set, $\mathbf{x}(t) \in \partial K$, we have

$$\Pi_X \left(\mathbf{x}, -\mathbf{F}(\mathbf{x})\right) = -\mathbf{F}(\mathbf{x}) + z(\mathbf{x})\mathbf{s}^*(\mathbf{x}) \tag{21}$$

where

$$\mathbf{s}^*(\mathbf{x}) = \underset{\mathbf{s} \in S(\mathbf{x})}{argmax} \langle -\mathbf{F}(\mathbf{x}), -\mathbf{s} \rangle \tag{22}$$

and

$$z(\mathbf{x}) = max \left(0, \langle -\mathbf{F}(\mathbf{x}), -\mathbf{s}^*(\mathbf{x}) \rangle\right) \tag{23}$$

Simply, by this projection operator, a point in the interior of X is projected onto itself, and a point outside of K is projected onto the closest point on the boundary of K.

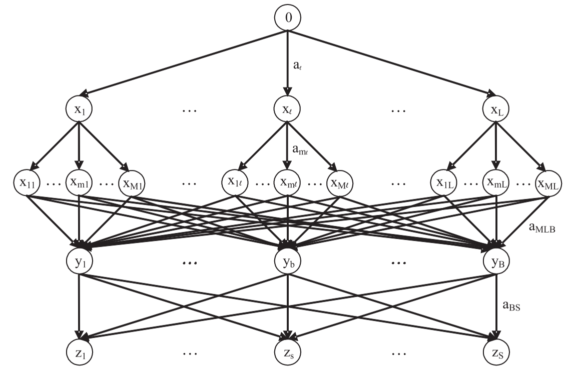

Figure 3. The transportation network representation of the spectrum supply chain network.

The associated dynamic model to the equilibrium problem will be realistic only if there is a unique solution path from a given initial point. The following theorem addresses the existence and uniqueness of the solution path for the above ODE are studied in [5].

Theorem 3: If $\mathbf{F}$ in the initial value problem (18) and (19) is *Lipschitz* continuous. Then, for any $\mathbf{x}_0 \in K$, there exists a unique solution $\mathbf{x}_0(\tau)$ to the initial value problem (18) and (19).

Definition 1: A mapping $F : K \subseteq \mathbb{R}^n \to \mathbb{R}^n$ is said to be Lipschitz continuous if there is an $L > 0$, such that

$$\|\mathbf{F}(\mathbf{x}) - \mathbf{F}(\mathbf{y})\| \leq L\|\mathbf{x} - \mathbf{y}\|, \ \ \forall \mathbf{x}, \mathbf{y} \in K; \tag{24}$$

IV. Network Stability

The following theorem guarantees the stability of the spectrum supply chain network [5].

Theorem 4: If $\mathbf{F}$ in the initial value problem (18) and (19) is *monotone*. Then, the dynamic system (18) and (19) underlying the spectrum supply chain network is stable.

Definition 2: A mapping $F : K \subseteq \mathbb{R}^n \to \mathbb{R}^n$ is said to be monotone on K if

$$\left(\mathbf{F}(\mathbf{x}) - \mathbf{F}(\mathbf{y})\right)^T (\mathbf{x} - \mathbf{y}) \geq 0, \ \ \forall \mathbf{x}, \mathbf{y} \in K; \tag{25}$$

V. Equivalent Transportation Network Equilibrium Formulation

In [9] it was shown that the electric power supply chain network equilibrium model is isomorphic to a properly configured transportation network equilibrium model. Following the approach of [9], in this section, equivalence between the spectrum supply chain network equilibrium model and transportation network equilibrium model is established. This allows us to benefit from the literature of other fields.

The corresponding transportation network is depicted in Figure 3. It has five tiers with a single origin node 0 at the top tier and S destination nodes at the bottom tier. There are $1 + L + M + b + S$ nodes, $L + M + MB + BS$ links, S origin/destination (O/D) pairs, and MBS paths.

Let a_ℓ denote the link from node 0 to node x_ℓ with link flow f_{a_ℓ}, $\forall \ell = 1, \cdots, L$; a_{m_ℓ} denote the link from node x_ℓ to node x_{m_ℓ} with link flow $f_{a_{m_\ell}}$, $\forall \ell = 1, \cdots, L$ and $\forall m_\ell = 1_\ell, \cdots, M_\ell$; $a_{m_\ell b}$ denote the link from node x_{m_ℓ} to node y_b with link flow $f_{a_{m_\ell b}}$, $\forall \ell = 1, \cdots, L$, $\forall m_\ell = 1_\ell, \cdots, M_\ell$ and $\forall b = 1, \cdots, B$; a_{bs} denote the link from node y_b to node z_s with link flow $f_{a_{bs}}$, $\forall b = 1, \cdots, B$ and $\forall s = 1, \cdots, S$.

A typical path, $q_{m_\ell bs}$, that connects the O/D pair $w_s = (0, z_s)$ consists of five links: a_ℓ, a_{m_ℓ}, $a_{m_\ell b}$, and a_{bs}. The associated flow on the path is $x_{q_{m_\ell bs}}$. Let d_{w_s} denote the demand associated with O/D pair w_s and λ_{w_s} denote the travel disutility for w_s. The following conservation of flows must be satisfied for the corresponding transportation network.

$$f_{a_\ell} = \sum_{m_\ell=1}^{M_\ell} \sum_{b=1}^{B} \sum_{s=1}^{S} x_{q_{m_\ell bs}}, \ \ \forall \ell = 1, \cdots, L \tag{26}$$

$$f_{a_{m_\ell}} = \sum_{b=1}^{B} \sum_{s=1}^{S} x_{q_{m_\ell bs}}, \ \ \forall \ell = 1, \cdots, L, \ \ \forall m_\ell = 1, \cdots, M_\ell \tag{27}$$

$$f_{a_{m_\ell b}} = \sum_{s=1}^{S} x_{q_{m_\ell bs}}, \ \ \forall \ell = 1, \cdots, L, \ \ \forall m_\ell = 1, \cdots, M_\ell, \ \ \forall b = 1, \cdots, B \tag{28}$$

$$f_{a_{bs}} = \sum_{\ell=1}^{L} \sum_{m_\ell=1}^{M_\ell} x_{q_{m_\ell bs}}, \ \ \forall b = 1, \cdots, B, \ \ \forall s = 1, \cdots, S \tag{29}$$

Also, we have

$$d_{w_s} = \sum_{\ell=1}^{L} \sum_{m_\ell=1}^{M_\ell} \sum_{b=1}^{B} \sum_{b=1}^{B} x_{q_{m_\ell bs}}, \ \ \forall s = 1, \cdots, S \tag{30}$$

A feasible path flow pattern will be achieved if there are nonnegative path flows that satisfy equations (26)-(30). A feasible path flow pattern induces a feasible link flow pattern [9]. A feasible link flow pattern can be constructed based on the corresponding feasible transmit power vectors in the spectrum supply chain network model as:

$$p_\ell \equiv f_{a_\ell}, \ \ \forall \ell = 1, \cdots, L \tag{31}$$

$$p_{m_\ell} \equiv f_{a_{m_\ell}}, \ \ \forall \ell = 1, \cdots, L, \ \ \forall m_\ell = 1, \cdots, M_\ell \tag{32}$$

$$p_{m_\ell b} \equiv f_{a_{m_\ell b}}, \ \ \forall \ell = 1, \cdots, L, \ \ \forall m_\ell = 1, \cdots, M_\ell, \ \ \forall b = 1, \cdots, B \tag{33}$$

$$p_{bs} \equiv f_{a_{bs}}, \ \ \forall b = 1, \cdots, B, \ \ \forall s = 1, \cdots, S \tag{34}$$

where

$$p_\ell = \sum_{m_\ell=1}^{M_\ell} p_{m_\ell}, \ \ \forall \ell = 1, \cdots, L \tag{35}$$

VI. Conclusion

This paper presented a market-oriented model for the spectrum supply chain network with three tiers including legacy owners, spectrum brokers, and secondary users. The decision-makers in each tier are involved in a noncooperative game with their peers, while they are directly or indirectly interacting with decision makers in other tiers.

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