Truthful Online Auctions for Pricing Peer-to-Peer Services

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Abstract

We consider truthful online auctions that aim at optimizing sellers’ revenues, representing service contributors’ satisfactions, as a general model for pricing peer-to-peer services under the assumption of individual service consumer’s rationality. For services that are in unlimited supply, we design a randomized truthful online auction with guaranteed revenue based on a randomized truthful offline auction. It is shown that the expected revenue extracted by our truthful online auction over all random factors achieves a $\Theta(1)$ approximation ratio relative to the optimal single-price revenue under some reasonable assumption about the input bids. Since a peer must serve others to earn sufficient revenue that can cover its payment for being served, we argue that our online truthful auctions can be suitable schemes for incentivizing peer nodes in peer-to-peer systems to share, and thereby addressing the “free-rider” problem in peer-to-peer service sharing.

1. Introduction

One of the major characteristics of peer-to-peer systems is the convenience for peer nodes to access different services provided by other peers. Such services can be file downloading, video streaming like pay-per-view movies, remote program execution, etc. Service sharing among peer nodes increases the entire network’s utility. However, individual peer’s rationality (or, self-interest) may naturally result in the “free-riding” behavior, i.e., requesting other peers’ services as much as possible without reciprocally contributing its own service [18]. One may choose to ignore the rationality in a peer-to-peer system if the induced effect is tolerable. Yet, when a large number of nodes act on their own, the network may experience severely downgraded performance as contribution from the few nodes is out-matched by the demand of many. To address this problem, schemes that encourage sharing have been devised to reconcile the rationality of individual node and the benefit of the entire network.

Reputation systems are designed to incentivize peer nodes to share. For example, the generalized evolutionary prisoner’s dilemma model is used to study individual nodes’ incentives to cooperate with others, using the reputation of each node [11]; an admission control system has been devised to differentiate services to users based on their reputation system [10]. On the other hand, payment systems are also used to encourage sharing [13]. Besides reputation and payment systems, there are formally analyzable methods which use pricing as a mechanism to determine payment. Algorithmic Mechanism Design (AMD), which introduce game-theoretic ideas into computational systems, and Distributed Algorithmic Mechanism Design (DAMD), which is AMD in a distributed setting, help devise systems adapting to rationality while attaining a desirable overall system outcome [15][5, 3].

As a special case of AMD and DAMD, auction design has been extensively studied in microeconomics for selling goods [9]. The objectives for holding an auction often is to achieve the optimal sum of all buyers’ valuations for the goods [19] or to approximate the optimal revenue gained by the seller [7, 16]. In an online setting, often the objective is to approximate the optimal sum of valuations [12]. We observe that auctions can be good ways to model service sharing in peer-to-peer systems under the assumption of individual rationality. By regarding peers’ services as goods for sale, and considering service requesters and service contributors as buyers (bidders) and sellers (auctioners) respectively, the service sharing between any two peer nodes can be modeled as a peer’s bidding in some auction held by the other peer and the corresponding result (bid satisfied or bid rejected). In a peer-to-peer environment, each peer node will be both a seller for its own service and a buyer for others’ services at the same time. The problem of incentivizing peer nodes to share is thereby transformed into designing auctions that optimize some objective functions under the assumption of individual rationality, which further
confines these auctions have to be “incentive-compatible” mechanisms or, specifically, “truthful” auctions.

In this paper we not only want auctions to be truthful but also to be able to take and answer bids arriving at different time instants. That is, they are online auctions [12, 6], as in a peer-to-peer system service requests may come at any time and need immediate responses. In our model, identical truthful online auction algorithm is used independently at each peer to sell its service with a goal to extract as much revenue as possible. Services, which are reproducible at negligible marginal cost, can be thought as in unlimited supply (think online music), not in scarce supply that is traditionally considered in the auction literature. Therefore, a peer has unlimited units of its goods (service) for sale and faces the problem of setting a price for each unit along with determining the number of units that is sold in the whole timeline. Note in auctions for goods in scarce supply, the quantity of the sold goods is fixed in advance, not decided by the auctioning process.

Truthful auctions for selling goods in unlimited supply was first considered by Goldberg et al. in an offline setting [7] and then by Bar-Yossef et al. in an online setting [1]. The randomized truthful offline auction proposed in [7] is shown to approximate the revenue of the optimal (offline) single-price auction by a constant factor with high probability under some assumption on the highest bid $h$, while no deterministic truthful (offline) auction can approximate the optimal single-price revenue within a constant factor. Let an approximation ratio of some auction relative to some benchmark auction’s revenue such as the optimal single-price revenue be defined as the reciprocal of such a factor (that is obtained via dividing the auction’s (expected) revenue by the benchmark auction’s revenue). Furthermore, the lower bounds for the approximation ratios of randomized and deterministic truthful online auctions have been shown to be $\Omega(1)$ and $\Omega(h)$, respectively, relative to the optimal single-price revenue in [1]. However, the randomized truthful online auction proposed in [1] does not achieve a $\Theta(1)$ ratio relative to the optimal single-price revenue under the same standard assumption on $h$ as in [7]. We are therefore motivated to give a truthful online auction for goods in unlimited supply with a $\Theta(1)$ ratio relative to the optimal single-price revenue under some reasonably stronger assumption about the input bids. Note that a truthful online auction with an $O(1)$ ratio relative to the optimal revenue, i.e., the optimal multiple-price revenue, is impossible. This is due to the lower bound $\Omega(\log h)$ for a randomized truthful online auction’ approximation ratio relative to the optimal revenue, which has been shown in [1]. That is why we also choose the optimal single-price auction as the benchmark auction, instead of the optimal multi-price auction.

Based on the results in [7], in particular the Random Sampling Optimal Threshold Auction (RSOTA), we develop an online truthful auction, the online Random Sampling Optimal Threshold Auction (online RSOTA), to provide guarantee about individual peer’s revenue under some reasonable assumption, which is somewhat a stronger assumption than the standard one in [7]. It is analytically shown that the expected revenue gained by the online RSOTA over all random factors (including the independently identical distribution of each input bid) achieves a $\Theta(1)$ ratio relative to the optimal single-price revenue. Considering the network as a whole, under individual rationality, i.e., each peer’s utility maximization from bidding in other peers’ auctions for services, the expected total revenues that are gained by all peers’ auctions for their own services can approximate the optimal sum of single-price revenues in a constant factor. Intuitively, each peer can maximize its utility from being served and is guaranteed a competitive revenue, which can be used as payments when requesting others’ services, by serving others. Therefore, a peer cannot request others’ services unless it has enough revenue from serving others. Finally, as for the revenue that each peer as a seller will extract in practice, we use experimental results to show that it can be fair under many circumstances.

In Section 2, we introduce the system model and notations, including those for offline auctions in [7] and these auctions, on which our work is based. In Section 3, our truthful online RSOTAs are designed and analyzed. In Section 4, we consider model extensions to the model in Section 2. In Section 5, the revenue of the online RSOTA is evaluated using simulations, assuming certain bid value distributions and incoming orders. In Section 6, conclusions and future work are given.

2. System Model and Problem Formulation

Consider a peer-to-peer network consisting of $n$ peer nodes, each of which provides its service. Each peer holds its own auction to sell its service and at the same time wants to buy each other peer’s service by bidding in each of their auctions. We first consider the case where each peer node bids once in each other peer’s auction. In Section 4 we extend the model to let each peer bid in each other peer’s auction several times whenever it need to. Node $i$ has its valuation $v_{ij}$ for one unit (i.e., one time) of each other node $j$’s service. Thus, node $i$’s non-negative utility for bidding in node $j$’s auction, which node $i$ wants to maximize in this auction, will be $u_{ij} = v_{ij} - p_{ij}$ where $p_{ij}$ is its payment to node $j$ if it gets node $j$’s service once. We assume that $u_{ij} = 0$ with $p_{ij} = 0$ if node $i$ is not satisfied in the auction. Node $i$’s utility for participating in all others’ auctions is therefore $u_i = \sum_{j \neq i} u_{ij}$, which node $i$ wants to maximize. Notice that node $i$’s bid for getting node $j$’s service, $b_{ij}$, does not necessarily equal its true valuation $v_{ij}$ because
it is allowed to behave rationally by not truthfully revealing its valuation.

2.1 Truthful Auctions using Revenues as Goals

We assume individual rationality and want to design auctions to accommodate this assumption. We have known that \( b_{ij} \) is node \( i \)'s declared value for its true valuation \( v_{ij} \). Node \( i \) will use \( b_{ij} \) as a strategy to interact with other competing buyers in node \( j \)’s auction to affect the auction outcome, and to maximize its own utility \( u_{ij} \). This is so-called nodes’ rationality. Thus, for each node \( i \), if the strategy of making \( b_{ij} = v_{ij} \) will result in \( u_{ij} \geq u_{ij}^{*} \) for each \( u_{ij}^{*} \) produced by making \( b_{ij} \neq v_{ij} \), it will truthfully reveal \( v_{ij} \) because deception about \( v_{ij} \) does not pay off. Such auction is said to be incentive-compatible or truthful, and \( b_{ij} = v_{ij} \) is called a dominant strategy. Peers’ rationality confines us to consider only truthful auctions.

From the set of bids \( A_{j} = \{ b_{ij} \mid i \neq j \} \), node \( j \) decides which nodes to sell and how to charge them using its auction algorithm. In the case where goods are in unlimited supply, the quantity of goods that are actually sold is also decided by the auction algorithm. Node \( j \)'s revenue from its auction is \( R(A_{j}) = \sum_{i \neq j} b_{ij} \), which is the goal that node \( j \), as a seller, want to maximize. Therefore, our way to encourage node \( j \) to share is, under the assumption of individual rationality, to make node \( j \) aim at maximizing its own revenue. Our goal considering the whole network is to try to maximize the sum of revenues that are gained by all peers’ auctions for their own services \( R(A) = \sum_{j} R(A_{j}) \) where \( A = \bigcup_{j} A_{j} \). In addition, we want \( R(A) \) to be efficiently computable.

2.2 Offline Auctions: Optimal Single-Price Auctions and Random Sampling Optimal Threshold Auctions (RSOTAs)

Our truthful online auctions in Section 3 and 4 are based on and varied from the (truthful) Random Sampling Optimal Threshold Auctions [7], which here we call as offline RSOTAs. In auction theory, optimal auctions maximize the seller’s revenue but optimal auctions that are truthful are hard to compute [17]. For truthful auctions to be efficiently computable, approximating auctions are needed. The offline RSOTA is just one of them. Nevertheless, the revenue of the offline RSOTA will be compared to the optimal single-price revenue, which is the revenue of the optimal single-price auction (that is untruthful). We will not compare the revenue of the offline RSOTA to the optimal revenue extracted from a multiple-price auction, i.e., the revenue of the optimal multiple-price auction. Here in this section, we introduce the optimal single-price auction first, then the offline RSOTA.

The optimal single-price auction at each node \( j \) takes bids \( A_{j} = \{ b_{ij} \mid i \neq j \} \) and extracts the optimal single-price revenue \( F(A_{j}) = \text{opt}(A_{j}) \cdot x \) for node \( j \): The optimal threshold price \( \text{opt}(A_{j}) \) is the bid returned from the optimal threshold function denoted by \( \text{opt} \). The \( x \) highest bids that are greater than or equal to \( \text{opt}(A_{j}) \) are satisfied and charged with \( \text{opt}(A_{j}) \) as the payment price, while the other bids are rejected. Formally, that is \( \text{opt}(A_{j}) = \arg \max_{b_{ij} \in A_{j}} b_{ij} \cdot x \). By THEOREM 4.1 of [7], the penalty for auctions to be single-price is bounded, so we can bound the revenue of other truthful auctions by comparing their revenues to \( F(A_{j}) \) without much overestimating these revenues. THEOREM 4.1 of [7] shows that \( F(A_{j}) \geq T(A_{j})/(2 \log h(A_{j})) \), where \( T(A_{j}) \) is the optimal revenue at node \( j \) and \( h(A_{j}) \) the highest bid at node \( j \)'s auction.

The offline RSOTA at each node \( j \) randomly selects a sample \( S(A_{j}) \) of size \( \lfloor n/2 \rfloor \) from \( A_{j} = \{ b_{ij} \mid i \neq j \} \) to compute \( \text{opt}(S(A_{j})) \) as a threshold for the bids in the non-sample \( A_{j} - S(A_{j}) \). The bids in \( A_{j} - S(A_{j}) \) that are greater than or equal to \( \text{opt}(S(A_{j})) \) are satisfied and each charged the payment price \( \text{opt}(S(A_{j})) \); the other bids in \( A_{j} - S(A_{j}) \) are rejected. The sample is used to get a good guess for the optimal threshold price, and such guess is applied to the non-sample. Let the revenue of the offline RSOTA for node \( j \) be \( R(A_{j}) \). Assume that \( F(A_{j}) \) is significantly larger than \( h(A_{j}) \), and at least \( \alpha(A_{j}) \) units of goods are required to be sold to obtain \( F(A_{j}) \). By THEOREM 6.1 of [7] with notation substitutions, \( R(A_{j}) \geq F(A_{j})/6 \) with probability of at least \( 1 - e^{\alpha(A_{j})/36 - 40e^{\alpha(A_{j})}/72} \) under the assumption \( \alpha(A_{j})h(A_{j}) \leq F(A_{j}) \).

3. Online Random Sampling Optimal Threshold Auctions (online RSOTAs)

Our truthful online auctions, which will be developed and analyzed in this section, are based on and varied from the offline RSOTAs but are adapted to the online requirement. We design our online RSOTAs for the model in Section 2 and will modify the online RSOTAs for the model extension in Section 4. Our sampling method differs from the sampling method used in the offline RSOTA, which randomly selects a sample as a whole from the set of input bids. As bids now comes at anytime, we may want to avoid resampling each time the set of input bids is incremented by a newly coming bid. We want to decide if a bid is in the sample at once without affecting the sample that has been selected until now. Also, for the performance analysis we need to assume that the value of each bid is independently identically distributed. Though this assumption is not necessary in the offline setting, it is not uncommon in auction theory.

The \( n \) input bids \( A_{j} = \{ b_{ij} \mid i \neq j \} \) for an online auction at each node \( j \) are described as a sequence of bids denoted
by \( \sigma(A_j) \), a permutation of \( A_j \) representing the actual incoming order of the bids in \( A_j \). The optimal single-price revenue and the revenue of the online RSOTA on the first \( i \) bids at each node \( j \), denoted as \( \sigma’(A_j) \), are respectively denoted by \( F(\sigma’(A_j)) \) and \( R(\sigma’(A_j)) \). Therefore, the optimal single-price revenue is \( F(A_j) = F(\sigma(A_j)) = F(\sigma^n(A_j)) \), and the revenue of the online RSOTA is \( R(\sigma(A_j)) = R(\sigma^n(A_j)) \). Let \( r(\sigma(A_j)) \) denote the revenue from the \( i \)th bid (the payment price for the \( i \)th bid). Note that \( r(\sigma(A_j)) \) is actually a payment \( p_{ij} \) from some node \( l \). Therefore, \( R(\sigma(A_j)) = \sum_{i=1}^{n} r(\sigma(A_j)) \) can also be expressed as \( \sum_{l \in j} p_{ij} \), and the revenues of the whole network is \( R(\{\sigma(A_j)\}) = \sum_{j} R(\sigma(A_j)) \) where \( \{\sigma(A_j)\} \) is a vector \( \{\sigma(A_1), ..., \sigma(A_n)\} \).

At any time, \( \sigma_i(A_j) \) are kept as two sets, namely \( S(\sigma_i(A_j)) \) and \( \sigma’(A_j) - S(\sigma_i(A_j)) \). All the bids are answered one by one when arriving. When the \( i \)th bid arrives, it will be selected into \( S(\sigma_i(A_j)) \) with probability of 1/2 or into \( \sigma’(A_j) - S(\sigma_i(A_j)) \) if it is not in \( S(\sigma_i(A_j)) \). If the \( i \)th bid is in \( S(\sigma_i(A_j)) \), the optimal threshold on the other set \( \sigma’(A_j) - S(\sigma_i(A_j)) \), i.e., \( \text{opt}(\sigma’(A_j) - S(\sigma_i(A_j))) \), is processed; the \( i \)th bid will be satisfied with a payment price \( r(\sigma_i(A_j)) = \text{opt}(\sigma’(A_j) - S(\sigma_i(A_j))) \) if it is greater than or equal to \( \text{opt}(\sigma’(A_j) - S(\sigma_i(A_j))) \), or rejected with \( r(\sigma_i(A_j)) = 0 \) otherwise. Likewise, if the \( i \)th bid is in \( \sigma’(A_j) - S(\sigma_i(A_j)) \), then \( \text{opt}(S(\sigma_i(A_j))) \) is used.

That is, the \( i \)th bid will be satisfied with a payment price \( r(\sigma_i(A_j)) = \text{opt}(S(\sigma_i(A_j))) \) if it is greater than or equal to \( \text{opt}(S(\sigma_i(A_j))) \), or rejected with \( r(\sigma_i(A_j)) = 0 \) otherwise. Actually, \( r(\sigma_i(A_j)) \) is decided like conducting the offline RSOTA on \( \sigma_i(A_j) \) just for the \( i \)th bid (in the following analysis we call it an “imaginary” offline RSOTA) with \( S(\sigma_i(A_j)) \) or \( \sigma’(A_j) - S(\sigma_i(A_j)) \) as a sample. This procedure of incremental sampling and optimal threshold price computing is repeated on the arrival of each newly coming bid.

**Performance Analysis.** The performance analysis for the offline RSOTA in [7] basically relies on the following lemma, which is used to bound the number of the bids in some sample that contribute to the optimal single-price revenue.

**Lemma 6.1 of [7].** Consider a set \( A \) and its subset \( B \subset A \). Suppose we pick an integer \( k \) such that \( 0 < k < |A| \) and a random subset (sample) \( S \subset A \) of size \( k \). Then for \( 0 < \delta \leq 1 \) we have \( \Pr[|S \cap B| < (1 - \delta)|B| \cdot k/|A|] < e^{-|B| \delta^2/(2|A|)} \).

The above lemma is actually a variation of the Chernoff inequality [2]. By our sampling method of each bid being independently selected into \( S(\sigma_i(A_j)) \) with probability of 1/2, the Chernoff inequality can be directly used. The Chernoff inequality ensures that the number of successes of several independent Bernoulli variables is not far from the expected number of successes. Consider the first \( i \) bids at node \( j \), \( \sigma_i(A_j) \), and its subset \( B_j \) that contribute to \( F(\sigma_i(A_j)) \). An independent Bernoulli variable is successful with probability of 1/2 if one bid in \( B_j \) is selected into \( S(\sigma_i(A_j)) \); there are \(|B_j| \) such variables. Therefore, the following lemma is obtained.

**Lemma 1** Consider the first \( i \) bids at node \( j \), \( \sigma_i(A_j) \), and its subset \( B_j \) that contribute to \( F(\sigma_i(A_j)) \) for \( i \leq n \). Then for \( 0 < \delta < 1 \) we have \( \Pr[|S(\sigma_i(A_j)) \cap B_j| < (1 - \delta)/2 \cdot |B_j|] < e^{-|B_j| \delta^2/4} \).

We can use the above lemma to obtain a result that is actually Theorem 6.1 of [7]. The following lemma is implied by Theorem 6.1 of [7] with an expectation bound instead of a high probability bound. Let \( R(\sigma_i(A_j)) \) denote the revenue of the offline RSOTA on the first \( i \) bids of \( \sigma(A_j) \). Note that \( h(\sigma(A_j)) \) is the highest bid in \( \sigma(A_j) \) and at least \( \alpha(\sigma(A_j)) \) units must be sold to obtain \( F(\sigma(A_j)) \). Notice that even the assumption of \( \alpha(\sigma(A_j)) \) makes \( h(\sigma(A_j)) \) is reasonable, because, though the standard assumption is only \( \alpha(\sigma(A_j)) \) units \( h(\sigma(A_j)) \leq F(\sigma(A_j)) \) in [7], \( n \) is variable.

**Lemma 2** \( E[R(\sigma_i(A_j))'] \geq F(\sigma_i(A_j))/c \) for some constant \( c > 1 \).

\((R(\sigma_i(A_j))') \geq F(\sigma_i(A_j))/c \) with high probability for some constant \( c > 1 \) if assuming \( \alpha(\sigma(A_j)) h(\sigma(A_j)) \leq F(\sigma(A_j)) \) for all \( i \in \{1, ..., n\} \).

Let \( F(\sigma_i(A_j))' \) be the part of \( F(\sigma_i(A_j)) \) that are from bids in \( \sigma_i(A_j) \). For the \( i \)th bid, the expected revenue from it is \( E[r(\sigma_i(A_j))] = \Pr[r(\sigma_i(A_j)) \neq 0] \cdot E[r(\sigma_i(A_j)) | r(\sigma_i(A_j)) \neq 0] \geq 2E[R(\sigma_i(A_j))']/i \) because the “imaginary” offline RSOTA extracts twice the expected revenue of the offline RSOTA. Lemma 2 makes \( E[r(\sigma_i(A_j))] \geq F(\sigma_i(A_j))'/c' \cdot i \) for some constant \( c' > 1 \). Then, \( E[r(\sigma_i(A_j))] \geq F(\sigma_i(A_j))'/c' \cdot i \) for \( F(\sigma_i(A_j)) \geq F(\sigma(A_j))' \).

We want to relate \( F(\sigma_i(A_j))' \) to \( F(A_j) \). Because each bid is independently identically distributed, each bid contributes to \( F(A_j) \) with the same probability. Thus, \( E_{\sigma(A_j)}[F(\sigma_i(A_j))'] \geq \frac{1}{n} \cdot F(A_j) \). We now get \( E_{\sigma(A_j)}[E[r(\sigma_i(A_j))] \geq E_{\sigma(A_j)}[F(\sigma_i(A_j))']/c' \cdot i \geq F(A_j)/c' \cdot n \). By the linearity of expectations, \( F(\sigma(A_j)) \geq E[R(\sigma(A_j))] = \sum_{i=1}^{n} E_{\sigma(A_j)}[E[r(\sigma_i(A_j))]] \geq F(A_j)/c' \).

**Theorem 1** Assume that the value of each bid is independently identically distributed. Then \( E_{\sigma(A_j)}[E[R(\sigma(A_j))]] \geq F(A_j)/c' \) for some constant \( c' > 1 \).
Truthfulness. We assume that each node submits a bid exactly when it really wants service and has formed its valuation. That is, we preclude a node’s delayed or advanced bidding in the auction for lower payment price. Therefore, there is no deception about bidding time. Then, in the online RSOTA at each node, given the first bids $\sigma^i(A_j)$ we claim the truthful revelation about the valuation $v_{ij}$ of some peer node $l$ that gives its bid $b_{ij}$ as the $(i+1)$th bid.

We discuss the case that $b_{ij}$ is put into $\sigma^i(A_j) - S(\sigma^i(A_j))$. When $v_{ij} \geq \text{opt}(S^{i+1}(\sigma_j))$, then $v_{ij} = v_{ij} - \text{opt}(S^{i+1}(\sigma_j))$ with $b_{ij} = v_{ij}$. That is because if $b_{ij} \neq v_{ij}$ and $b_{ij} \geq \text{opt}(S^{i+1}(\sigma_j))$, then an equal utility is obtained; if $b_{ij} < \text{opt}(S^{i+1}(\sigma_j))$, then $\sigma_{ij} = 0$. Therefore, $b_{ij} = v_{ij}$ is a dominant strategy for node $l$. When $v_{ij} < \text{opt}(S^{i+1}(\sigma_j))$, then $\sigma_{ij} = 0$ with $b_{ij} = v_{ij}$. That is because if $b_{ij} \neq v_{ij}$ and $b_{ij} \geq \text{opt}(S^{i+1}(\sigma_j))$, then a negative utility $v_{ij} = v_{ij} - \text{opt}(S^{i+1}(\sigma_j))$ is obtained; if $b_{ij} < \text{opt}(S^{i+1}(\sigma_j))$, then still $\sigma_{ij} = 0$. Analogously, $b_{ij} = v_{ij}$ is a dominant strategy for node $l$. The analysis for case that $b_{ij}$ is in $S(\sigma^i(A_j))$ can be similarly derived.

Note that we can ensure that a peer only bids when it really wants service and forms its valuation, by the above argument with $v_{ij} = 0$, which means that node $l$ is not interested in bidding right now. Also, a peer will not choose to avoid bidding once it really wants service and has its non-zero valuation.

4. Model Extension

A peer node may want to try to bid again after its previous bids are rejected. Even if a peer is satisfied in its previous bid, it may still want to be served again. Therefore, the basic model defined in Section 2, which only allows a peer node to bid once in each other peer’s auction, and the online RSOTA built on such model in Section 3 both need to be further extended into a more general model. This new model will allow a peer to bid several times in each other peer’s auction whenever it really wants to be served, and we need a correspondingly modified online RSOTA.

Node $j$ has its valuations $v_{ij}^1, v_{ij}^2, ..., v_{ij}^k, ...,$ each for one time of each other node $j$’s service. Node $i$’s non-negative utility for bidding the $k$th time in node $j$’s auction, which node $i$ wants to maximize this time in this auction, will be $u_{ij}^k = v_{ij}^k - p_{ij}^k$ where $p_{ij}^k$ is its payment to node $j$ if it gets node $j$’s service this time. We assume that $u_{ij}^k = 0$ with $p_{ij}^k = 0$ if node $i$ is not satisfied in the auction this time. Node $i$’s non-negative utility for bidding in node $j$’s auction is $u_{ij} = \sum_k u_{ij}^k$, and its total payment to node $j$ is $p_{ij} = \sum_k p_{ij}^k$. Node $i$’s utility for participating in all others’ auctions is therefore $u_i = \sum_{j \neq i} u_{ij}$, which node $i$ wants to maximize. Notice that node $i$’s bid the $k$th time for getting node $j$’s service, $b_{ij}^k$, does not necessarily equal its true valuation $v_{ij}^k$ this time.

We must clarify our notion of truthfulness (incentive-compatibility) in this extended model. Node $i$ will use a vector of its bids $b_{ij}^1, ..., b_{ij}^k, ...$ as a strategy to interact with other competing buyers in node $j$’s auction to affect the auction outcome, and to maximize its own utility $u_{ij}$ in this auction. We give a strong notion of truthfulness that does not allow deception about any $v_{ij}^k$. Thus, for each node $i$, if the strategy of making $b_{ij}^k = v_{ij}^k$ will result in $u_{ij}^k \geq \sum_k v_{ij}^k$ for all $k$ and therefore $u_{ij} \geq \sum_k v_{ij}^k$ for each $u_{ij}$ produced by making $b_{ij}^k \neq v_{ij}^k$, it will truthfully reveal $v_{ij}^k$ for all $k$ because deception about any $v_{ij}^k$ does not pay off. We call such auction incentive-compatible or truthful, and $b_{ij}^k = v_{ij}^k$ for all $k$ a dominant strategy.

We can modify the online RSOTA in Section 3 to suit our extended model. The set of the input bids for an online auction at each node $j$ does not have to be of size $n$ but is $A_j = \{b_{ij}^k | i \neq j \forall k \geq 1, 2, ... \}$. Still, $\sigma(A_j)$ is a permutation of $A_j$ representing the actual incoming order of the bids in $A_j$. The optimal single-price revenue and the revenue of the online RSOTA on the first $i$ bids at each node $j$, $\sigma(A_j)$, are respectively denoted by $F(\sigma(A_j))$ and $R(\sigma(A_j))$. Then, the optimal single-price revenue is now $F(A_j) = F(\sigma(A_j)) = F(\sigma^m(A_j))$ where the most newly incoming bid is the $m$th bid and the revenue of the online RSOTA is $R(\sigma(A_j)) = R(\sigma^m(A_j))$. Note $r(\sigma(A_j))$ is the revenue from the $i$th bid (the payment price for the $i$th bid); $r(\sigma(A_j))$ is a payment for some node $i$’s $k$th bid $p_{ij}^k$. Thus, $R(\sigma(A_j)) = \sum_{i=1}^m r(\sigma(A_j)) = \sum_{i \neq j} \sum p_{ij}^k$ and $R(\sigma(A_j)) = \sum_{i \neq j} R(\sigma(A_j))$.

We keep the payment price for a peer’s bid each time in some auction to be independent from those of its previous bids to maintain the revenue and to ensure truthfulness. To identify the bids from the same peer, we need some authentication mechanism, which may be externally available. Like in the online RSOTA in Section 3, at any time, $\sigma^i(A_j)$ are still kept as two sets, namely $S(\sigma^i(A_j))$ and $\sigma^i(A_j) - S(\sigma^i(A_j))$. When arrives the $i$th bid that is not some peer’s 1st bid in node $j$’s auction, it will be put into $S(\sigma^i(A_j))$ if the previous bids from the node submitting this bid are in $S(\sigma^i(A_j))$, put into $\sigma^i(A_j) - S(\sigma^i(A_j))$ if they are in $\sigma^i(A_j) - S(\sigma^i(A_j))$. A peer’s 1st bid in node $j$’s auction is put into $S(\sigma^i(A_j))$ or $\sigma^i(A_j) - S(\sigma^i(A_j))$ as before. The remaining steps of optimal threshold processing on $S(\sigma^i(A_j))$ or $\sigma^i(A_j) - S(\sigma^i(A_j))$ is conducted as before, and the procedure of incremental sampling and optimal threshold price computing is again repeated on the arrival of each newly coming bid.

Because a peer’s revenue is used as its payment to any auction, a peer’s bidding in any auction each time can only be done with having earned sufficient revenue that at least
covers its own valuation this time (since the payment price is at most equal to its valuation). A peer will not be able to abusively bid too many times than other bidders in some auction, since other nodes do not necessarily pay it an enough revenue to do so. Therefore, each peer’s expected bidding behavior, i.e., the between the largest number of biddings from a peer and the smallest number is constantly bounded, will still make Theorem 1 hold under our modified online RSOTA by maintaining the following lemma (a generalized Lemma 1), on which Lemma 2 relies. See Appendix for using a variant of the Chernoff inequality to obtain Lemma 3. Note that \( |B_j|/m \geq \alpha (\sigma^{i/m}(A_j)) \) because \( \alpha (\sigma^{i/m}(A_j))h(\sigma^{i/m}(A_j)) \leq F(\sigma^{i/m}(A_j)) = (|B_j|/m)\cdot \text{opt}(\sigma^{i/m}(A_j)) \). Also note that the high probability still grows with \( \alpha (\sigma^{i/m}(A_j)) \) so as to derive Lemma 2.

**Lemma 3** Consider the first \( i \) bids at node \( j \), \( \sigma^i (A_j) \), and its subset \( B_j^i \) that contribute to \( F(\sigma^i (A_j)) \). Then for the expected number of biddings from each node, \( m \), and \( 0 < \delta < 1 \) we have

\[
Pr(|S(\sigma^i (A_j))\cap B_j^i| < (1-\delta)/2 \cdot |B_j^i| < e^{-|B_j^i| \cdot \delta^2 / (4m)}. \]

Considering all the \( n \) online RSOTAs held in the network. While each peer’s utility maximizes from bidding in other peers’ auctions for services, by Theorem 1, the expected total revenues that are gained by all peers’ online RSOTAs for their own services can approximate the optimal sum of single-price revenues by a constant factor. That is, \( E_{|\sigma(A_j)|}[E[R(\sigma(A_j))] = \sum_j E_{\sigma(A_j)}[E[R(\sigma(A_j)))] \geq (\sum_j F(A_j))/c \) for some constant \( c \).

**5. Experimental Results**

The \( \Theta(1) \) approximation ratio relative to the optimal single-price revenue obtained in Theorems 1 may not be sufficient to demonstrate the actual revenue generated in practice by the online RSOTAs. Therefore, simulations is used to experiment the online RSOTAs with the value of each bid independently ranging within \([1, 100]\). This implies \( h(\sigma^n(A_j)) = h(\sigma(A_j)) = 100 \). Bids are in two different distributions, uniform distribution and some bell-shaped distribution, e.g., normal distribution. Also, the bids may arrive in three different orders, which are random, non-decreasing, and non-increasing orders. We set \( n = 5000 \) in the simulations and take the average result of 10 runs. With the optimal single-price revenue as a benchmark, the revenues of the online and offline RSOTA, which are both with a \( \Theta(1) \) approximation ratio, are compared under six different conditions with respect to different assumptions of bid distributions and incoming orders.

From the experimental results, it can be found that, in both the uniform distribution and bell-shaped distribution cases, online RSOTAs basically generate more revenues than offline RSOTAs. The improving rate between the online RSOTA’s revenue and the offline RSOTA’s revenue is most significant when bids come in a non-decreasing order, moderate when in a random order, and least significant when in a non-increasing. See Figure 1 for detail. Hence, under many circumstances our online RSOTA is practical for extracting a fair revenue.

![Figure 1. Experimental results](image)

**6. Conclusions and Future Work**

We have proposed and demonstrated online truthful auctions as suitable schemes for incentivizing nodes in a peer-to-peer system to contribute their services. We have designed truthful online auctions, the online RSOTA, for pricing peer-to-peer services. In particular, the expected revenue gained by our online RSOTA over all random factors achieves a \( \Theta(1) \) ratio relative to the optimal single-price revenue under some reasonable assumption about the input bids. Therefore, while peers as buyers maximize their own utilities by truthfully revealing their needs, they as sellers can still guarantee each peer’s expected revenue competitively. A peer must serve others to gain sufficient revenue that can cover its payment for being served. This online RSOTA is finally compared with the offline RSOTA in experiments to evaluate performances in practice under different conditions.

We are further interested in devising online truthful auctions that are less computational demanding and generating more revenues. Other than using the seller’s revenue or the sum of buyers’ valuations as the goal, we are also interested in auctions that maximize the sum of buyers’ valuations and approximating the optimal seller’s revenue at the same time. Besides, other goals representing overall benefits can be further defined. Individual rational behavior can be dealt with by truthful auctions, but group collusive behavior conducted
by some subset of the network nodes will be an issue to be carefully studied in auction design. Also, though we preclude the possibility of a peer’s deception about bidding time, this preclusion may be removable (i.e., a peer may deceive about its bidding time for maximizing its utility in some auction) and bidding time should be considered when making truthfulness argument, as in [6].

As for a general model for peer-to-peer services, there are still much to be considered. Currently, we consider peer-to-peer services as goods in unlimited supply, following many of the previous results. However, if the limit of a peer node’s capacity is taken into consideration, peer-to-peer services become goods in limited supply (or, bounded supply [7]), which further will complicate the design of online auctions. In an offline setting, an optimal truthful approximation auction for limited supply has been proposed [8]. For an online setting, the design problem seems to be more interesting and open. Besides, the current model does not consider the cost for distributing services and simply assumes a zero-cost effort for acquiring services. This is not the case when transmission cost is to be shared, as in multicasting network [4, 14]. We are therefore motivated to model the cost for distributing services, which can offset a peer node’s revenue, in auction design.

Appendix

A VARIANT OF THE CHERNOFF INEQUALITY. Let $X_i^j$’s for some $i \in \{1, \ldots, n\}$ and all $j \in \{1, \ldots, m\}$ be dependent Bernoulli variables, i.e., $X_i^j \in \{0, 1\}$ and $X_i^1 = \ldots = X_i^m$, and $X_i^j$’s for all $j$ are independent from $X_i^k$’s for all $l$ if $i \neq k$. Let $X$ be the number of $X_i^j$’s that are 1. Therefore, $p_i = Pr[X_i^j = 1] = \ldots = Pr[X_i^m = 1] = Pr[X_i^1 = 1, \ldots, X_i^m = 1]$, $\mu = E[\sum_i^n X_i^j] = \sum_i^n p_i$, and $\mu' = m\mu = E[X]$. As in the proof of the original Chernoff inequality, one computes the probability that $X$ deviates significantly from $\mu'$:

$$Pr[X > (1 + \delta)\mu'] \leq e^{-\delta^2 \mu'/2}.$$ 

When $t = \frac{\ln(1+\delta)}{m}$, it minimizes the bound so we obtain, for all $\delta \geq 0$, $Pr[X > (1 + \delta)\mu'] \leq e^{\delta^2 (1 + \delta) \ln(1 + \delta) / (2m)} \mu$. Using the Taylor series expression of $\ln(1 + \delta)$ and ignoring the higher order terms, we can get a simpler form of the above bound and a similar calculation can show that $Pr[X < (1 - \delta)\mu'] \leq e^{-\delta^2 \mu'/2}$ for $0 < \delta < 1$.

Our modified online RSOTA satisfies the assumption of this variant of the Chernoff inequality. By setting $p_i = 1/2$, $n = |B_i^j|/m$, and $m$ as the expected number of biddings from each peer node, Lemma 3 is thus obtained.

References


