CSCI 271
Homework 7

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1. The set of modulus-congruencies of d is d long; therefore, by the pigeonhole principle, a set of $d + 1$ elements must contain two elemnts congruent to $\mod d$.

2. 36

3. (a) If there were fewer than 5 male and 5 female students, then the class would consist of ≤ 8 people; which contradicts that there are actually 10.

   (b) If there were fewer than 3 male and 7 female students, then the class would consist of ≤ 8 people; which contradicts that there are actually 10.

4. (a) $xxxxy + xxxyx + xxyxx + xyxxx + yxxxx + xxxyy + xxyxy + xyxxy + yxxxy + xxyyx + xyxyx + yxxyx + xyyxx + yxyxx + yyxxx + xxyyy + xyxyy + xyyxy + xyyyx + yxxyy + yxyxy + yxyyx + yyxyx + yyyxx + xyyyy + yxyyy + yyxyy + yyyxy + yyyyx + yyyyy$

   (b) $\binom{5}{0}x^5y^0 + \binom{5}{1}x^4y^1 + \binom{5}{2}x^3y^2 + \binom{5}{3}x^2y^3 + \binom{5}{4}x^1y^4 + \binom{5}{5}x^0y^5$

   (a) To choose two things $2n$ ways, is the same as choosing n things 2 twice plus a residual $n^k = n^2$ corresponding to defective subsets.

(b)

$$\begin{aligned}
\binom{2n}{2} &= \frac{(2n)!}{2!(2n-2)!} && (1)\\
&= \frac{4n^2-2n}{2} && (2)\\
&= n^2 - n + n^2 && (3)\\
&= \frac{n!}{(n-2)!} + n^2 && (4)\\
&= 2\frac{n!}{2!(n-2)!} + n^2 && (5)\\
&= 2\binom{n}{2} + n^2 && (6)
\end{aligned}$$

5.
```
procedure prod(m, n: m, n natural numbers)
  if n = 0 then
    prod(m, n) := 0
  else
    prod(m, n) := m + prod(m, n - 1)
end { prod(m, n) is the product of m and n }
```

6.
```
procedure flatten(L: list)
  F := empty list
  extract(L)
  procedure extract(m: member)
    if m is an atom
      F := F + m
    else if m is a list
      extract(m)
  end
end { F is the flattened list }
```