

CSCI 271

Homework 3

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Problem 1. p “It raineth.”

q “Yvette hath her umbrella.”

r “Yvette getteth wet.”

$$\begin{array}{l} \neg p \vee q \\ \neg q \vee \neg r \\ \hline p \vee \neg r \\ \therefore \neg r \end{array}$$

$(q \vee \neg p) \wedge (\neg q \vee \neg r)$	hypotheses	(1)
$\neg p \vee \neg r$	resolution (1)	(2)
$(p \vee \neg r) \wedge (p \vee \neg r)$	hypothesis and (2)	(3)
$\neg r \wedge \neg r$	resolution (3)	(4)
$\neg r$	idempotency (4)	(5)

Problem 2.

$(p \vee q) \wedge (\neg p \vee q) \wedge (p \vee \neg q) \wedge (\neg p \vee \neg q)$	hypothesis	(1)
$(q \vee q) \wedge (\neg q \vee \neg q)$	resolution (1)	(2)
$q \wedge \neg q$	idempotency (2)	(3)
F	negation (3)	(4)

Problem 3. It sufficeth that an unbroken chain connecteth each proposition; see figure 1.

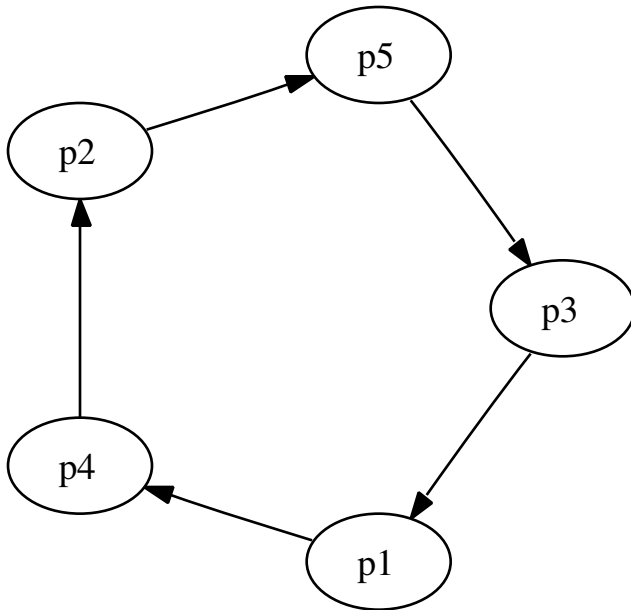


Figure 1: Chained progression of propositions.

Problem 4. $\exists!k \in \mathbb{Z} \exists! \epsilon \in \mathbb{R} \forall x \in \mathbb{R} (0 \leq \epsilon < 1 \wedge x = n - \epsilon)$. Proceed through proof by cases.

Case (i). $x \in \mathbb{Z}$, where $n = x$ and $\epsilon = 0$ (the existential proof); that this solution is unique: $n > x$ would cause $\epsilon \geq 1$; and $n < x$, $\epsilon < 0$ (both of which contradict $0 \leq \epsilon < 1$).

Case (ii). $x \notin \mathbb{Z}$, where $n = \lceil x \rceil$ and $\epsilon = n - x$; by definition of an integer, ϵ is constrained by $\mathbb{Z}_{n+1} - \mathbb{Z}_n = 1$ and 0 (the limit of (i)), thus $0 \leq \epsilon < 1$.

Problem 5. By a constructive existential proof, there are 101 imperfect squares between 50 and 51; the bc script follows:

```

# Prove that there are 100 consecutive positive
# integers that are not perfect squares. Is your
# proof constructive or non-constructive?
  
```

```
span = 100
i = 0
dist = 0

while (dist < span) {
    dist = (++i)^2 - (prev = i - 1)^2
}

print "Between ", prev, " and ", i, " there are ", dist
print " consecutive imperfect squares."
```