CSCI 271
Homework 2

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Section 1.3

8. (a) All rabbits hop.
   (b) All animals are rabbits and all animals hop.
   (c) There is a rabbit that hops, or a non-rabbit that hops, or a non-rabbit that hops not.
   (d) There is a rabbit that hops.

16. (a) T
    (b) F
    (c) T
    (d) F

20. (a) $P(-5) \vee P(5) \vee P(-3) \vee P(-1) \vee P(1) \vee P(3) \vee P(5)$
    (b) $P(-5) \wedge P(5) \wedge P(-3) \wedge P(-1) \wedge P(1) \wedge P(3) \wedge P(5)$
    (c) $P(-5) \wedge P(5) \wedge P(-3) \wedge P(-1) \wedge P(3) \wedge P(5)$
    (d) $P(5) \vee P(1) \vee P(3) \vee P(5)$
    (e) $(\neg P(-5) \vee \neg P(5) \vee \neg P(-3) \vee \neg P(-1) \vee \neg P(1) \vee \neg P(3) \vee \neg P(5)) \wedge$ $(P(-5) \wedge P(-3) \wedge P(-1))$

30. (a) Let $P(x)$ be "is a koala," and $Q(x)$, "can climb."

$$\forall x (P(x) \rightarrow Q(x))$$

$$\exists\neg x(P(x) \to Q(x))$$

No koala can climb.

(d) $P(x)$: "is a monkey;" $Q(x)$: "can climb."

$$\neg\exists x(P(x) \land Q(x))$$

$$\exists x(P(x) \land Q(x))$$

Some monkey can speak French.

(e) $P(x)$: "is a pig;" $Q(x)$: "can swim;" $R(x)$: "can catch fish."

$$\exists x(P(x) \land Q(x) \land R(x))$$

$$\forall x\neg(P(x) \land Q(x) \land R(x))$$

No pigs can swim or catch fish.

46. Solution given.

Section 1.4

6. (a) Randy Goldberg is enrolled in CS 252.
   (b) There is a student enrolled in Math 695.
   (c) Carol Sitea is enrolled in a class.
   (d) There is a student enrolled in both Math 222 and CS 252.
   (e) One student enrolls in the same classes as another student.
   (f) There are two students who enroll in exactly the same classes.

10. (a) $\forall x F(x, Fred)$
   (b) $\forall x \forall y F(x, y)$
   (c) $\forall x \exists y F(x, y)$
   (d) $\forall x \exists y \neg F(x, y)$
   (e) $\forall x \neg(F(x, Fred) \land F(x, Jerry))$
   (g) $\exists! x \exists! y (x \neq y \land F(\text{Nancy}, x) \land F(\text{Nancy}, y))$, where $\exists!$ is the **uniqueness quantifier** (see pg. 37).

(h) $\exists! x \forall y (F(x, y))$

(i) $\forall x \neg F(x, x)$

(j) $\exists x \exists! y (x \neq y \land F(x, y))$

22. $\exists x > 0\, (y^2 + z^2 + w^2 \neq x)$

38. $x = -100$

44. In the four cases that $x, y = \{\{T, T\}, \{T, F\}, \{F, T\}, \{F, F\}\}$ for all instances, $\forall x P(x) \lor \forall x Q(x)$ and $\forall x \forall y (P(x) \lor Q(y))$ show the same truth tables.

Section 1.5

4. p "It's raining."

   q "It's foggy."

   r "The sailing race will be held."

   s "The life-saving-demonstration will go on."

   t "The will be awarded."

$$\begin{array}{l} \neg p \lor \neg q \rightarrow r \land s \\ r \rightarrow t \\ \underline{\neg t} \\ \therefore p \end{array}$$

$r \rightarrow t$	hypothesis	(1)
$\neg t$	hypothesis	(2)
$\neg r$	modus tollens (1), (2)	(3)
$\neg p \lor \neg q \rightarrow r \land s$	hypothesis	(4)
$\neg p \lor \neg q \rightarrow r$	simplification (4)	(5)
$\neg(\neg p \lor \neg q)$	modus tollens (3), (5)	(6)
$p \land q$	double negative (6)	(7)
p	simplification (7)	(8)

12. (a) $p(x)$ "x is enrolled in the university."
$q(x)$ "x has lived in the dormitory."

$$\begin{array}{rl} & \forall x(P(x) \to Q(x)) \\ & \underline{\neg Q(\text{Mia})} \\ \therefore & P(\text{Mia}) \end{array}$$

$\forall x(P(x) \to Q(x))$	hypothesis	(1)
$P(\text{Mia}) \to Q(\text{Mia})$	universal instantiation (1)	(2)
$\neg Q(\text{Mia})$	hypothesis	(3)
$\neg P(\text{Mia})$	modus tollens (2), (3)	(4)

(b) $p(x)$ "x is a convertible."
$q(x)$ "x is fun to drive."

$$\begin{array}{rl} & \forall x(P(x) \to Q(x)) \\ & \underline{\neg P(\text{Isaac's car})} \\ \therefore & \neg Q(\text{Isaac's car}) \end{array}$$

$\forall x(P(x) \to Q(x))$	hypothesis	(1)
$P(\text{Isaac's car}) \to Q(\text{Isaac's car})$	universal instantiation (1)	(2)
$\neg P(\text{Isaac's car})$	hypothesis	(3)
$\neg Q(\text{Isaac's car})$	FALLACY: denying the hypothesis (2), (3)	(4)

36. Assume that $n = 2k + 1$ for some k; $5(2k + 1) = 2(5k + 5) + 1$ is odd. For the converse, assume that $n = 2k$ for some k; $5 \cdot 2k + 6 = 2(5k + 3)$ is even.

Thus, $5n + 6$ is odd iff n is odd.