CSCI 271
Homework 1

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1. (a) $r \wedge \neg q$
   (b) $p \wedge q \wedge r$
   (c) $r \rightarrow p$
   (d) $p \wedge \neg q \wedge r$
   (e) $(p \wedge q) \rightarrow r$
   (f) $r \leftrightarrow (q \vee p)$
2. (a) $T \leftrightarrow T \equiv T$
   (b) $T \leftrightarrow F \equiv F$
   (c) $T \leftrightarrow \neg(F \vee F \vee F) \equiv T \leftrightarrow \neg F \equiv T \leftrightarrow T \equiv T$
   (d) $F \leftrightarrow F \equiv T$
   (e) $F \leftrightarrow T \equiv F$

3. $((p \to q) \to r) \to s$

p	q	r	s	$p \to q = t$	$t \to r = u$	$u \to s$
1	1	1	1	1	1	1
1	1	1	0	1	1	0
1	1	0	1	1	0	1
1	1	0	0	1	0	1
1	0	1	1	0	1	1
1	0	1	0	0	1	0
1	0	0	1	0	1	1
1	0	0	0	0	1	0
0	1	1	1	1	1	1
0	1	1	0	1	1	0
0	1	0	1	1	0	1
0	1	0	0	1	0	1
0	0	1	1	1	1	1
0	0	1	0	1	0	1
0	0	0	1	1	1	1
0	0	0	0	1	0	1

4. (a) $\neg(p \wedge q) \equiv \neg p \vee \neg q$

p	q	$p \wedge q = r$	$\neg r$	$\neg p$	$\neg q$	$\neg p \vee \neg q = s$	$\neg r \leftrightarrow s$
1	1	1	0	0	0	0	1
1	0	0	1	0	1	1	1
0	1	0	1	1	0	1	1
0	0	0	1	1	1	1	1

(b) $\neg(p \vee q) \equiv \neg p \wedge \neg q$

p	q	$p \vee q = r$	$\neg r$	$\neg p$	$\neg q$	$\neg p \wedge \neg q = s$	$\neg r \leftrightarrow s$
1	1	1	0	0	0	0	1
1	0	1	0	0	1	0	1
0	1	1	0	1	0	0	1
0	0	0	1	1	1	1	1

5. Let $p =$ Motion, $q =$Noise, $r =$Intruder.

$$(p \to r) \lor (q \to r) \equiv (p \land q) \to r \qquad \text{(equivalence)}$$

p	q	r	$p \land r = s$	$s \to r$
1	1	1	1	1
1	1	0	1	0
1	0	1	0	1
1	0	0	0	1
0	1	1	0	1
0	1	0	0	1
0	0	1	0	1
0	0	0	0	1

6. $p \to q \neq q \to p \equiv \neg p \to \neg q$

p	q	$p \to q = r$	$q \to p = s$	$\neg p \to \neg q = t$	$s \leftrightarrow t = u$	$r \leftrightarrow t = v$
1	1	1	1	1	1	1
1	0	0	1	1	1	0
0	1	1	0	0	1	1
0	0	1	1	1	1	1

u is tautological and v is not.

7. $(\neg q \land (p \to q)) \to \neg p$

$$\begin{aligned}
(\neg q \land (p \to q)) \to \neg p &\equiv (\neg q \land (\neg p \lor q)) \to \neg p && \text{(equivalence)}\\
&\equiv ((\neg q \land \neg p) \lor (\neg q \land q)) \to \neg p && \text{(distribution)}\\
&\equiv ((\neg q \land \neg p) \lor F) \to \neg p && \text{(negation)}\\
&\equiv (\neg q \land \neg p) \to \neg p && \text{(identity)}\\
&\equiv \neg (\neg q \land \neg p) \lor \neg p && \text{(equivalence)}\\
&\equiv (q \lor p) \lor \neg p && \text{(De Morgan's)}\\
&\equiv q \lor (p \lor \neg p) && \text{(association)}\\
&\equiv q \lor T && \text{(negation)}\\
&\equiv T && \text{(domination)}
\end{aligned}$$

8. $\neg(p \leftrightarrow q) \equiv (p \leftrightarrow \neg q)$

$$\begin{aligned}
\neg(p \leftrightarrow q) &\equiv \neg((p \wedge q) \vee (\neg p \wedge \neg q)) && \text{(equivalence)} \\
&\equiv \neg(p \wedge q) \wedge \neg(\neg p \wedge \neg q) && \text{(De Morgan's)} \\
&\equiv (\neg p \vee \neg q) \wedge (q \vee p) \\
& \qquad \text{(De Morgan's, double negative, commutivity)} \\
&\equiv (p \to \neg q) \wedge (\neg q \to p) && \text{(equivalence)} \\
&\equiv (p \leftrightarrow \neg q) && \text{(equivalence)}
\end{aligned}$$

9. (a) $q \to p$
   (b) $q \wedge p$
   (c) $q \to p$
   (d) $\neg q \to \neg p$

10. (a) **Antecedent** Sufficient water
    Consequent Healthy plant growth
    (b) **Antecedent** Availability of information
    Consequent Technological advances
    (c) **Antecedent** Modification to the program
    Consequent Error introduced
    (d) **Antecedent** Fuel savings
    Consequent Good insulation or storm windows

11. (a) The food is good, but the service is not excellent.
    (b) The food is not good, and the service is not excellent.

12. (a) p = Prices go up, q = Housing is plentiful, r = Housing is expensive.

$$(p \to (q \wedge r)) \wedge (\neg p \to r)$$

    (b) p = Going to bed, q = Going swimming, r = Changing clothes.

$$((p \vee q) \to r) \wedge \neg(q \to r)$$

(c) $p =$ It rains, $q =$ It snows.

$$(p \land \neg q) \lor (\neg p \land q) \equiv p \oplus q$$

(d) $p =$ Janet wins, $q =$ Janet loses, $r =$ Janet is tired.

$$(p \lor q) \to r$$

(e) $p =$ Janet wins, $q =$ Janet loses, $r =$ Janet is tired.

$$p \lor (q \to r)$$

13. Let $p =$ "The bill was sent," and $q =$ "You will be paid tomorrow;" the proposition corresponds to $p \to q$ and the truth table to:

p	q	$p \to q$
1	1	1
1	0	0
0	1	1
0	0	1

Therefore, one can conclude (if the conversant is trustworthy) that "You will be paid tomorrow;" not, however, that "The bill was sent."

14. $\neg a \land (a \lor b) \to b$

$$\begin{aligned}
\neg a \land (a \lor b) \to b &\equiv (\neg a \land a) \lor (\neg a \land b) \to b && \text{(distribution)}\\
&\equiv F \lor (\neg a \land b) \to b && \text{(negation)}\\
&\equiv (\neg a \land b) \to b && \text{(domination)}\\
&\equiv \neg (\neg a \land b) \lor b && \text{(equivalence)}\\
&\equiv (\neg\neg a \lor \neg b) \lor b && \text{(De Morgan's)}\\
&\equiv \neg\neg a \lor \neg b \lor b && \text{(association)}\\
&\equiv a \lor \neg b \lor b && \text{(double negation)}\\
&\equiv a \lor T && \text{(negation)}\\
&\equiv T && \text{(domination)}
\end{aligned}$$