Distributed Storage Codes Meet Multiple-Access Wiretap Channels

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Overview

- MDS Storage Codes
- Minimize Repair Bandwidth
  - Interference Alignment
  - $R$: Rank Constrained sumRank Minimization (over field)

- Multiple-Access Compound Wiretap Channel
- Maximize S-DoF
  - Interference Alignment
  - $\mathcal{V}$: Rank Constrained maxRank Minimization

We establish a connection.
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\(\mathcal{R}:\) Rank Constrained sumRank Minimization (over field)

Multiple-Access Compound Wiretap Channel

Maximize S-DoF

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\(\mathcal{V}:\) Rank Constrained maxRank Minimization

We establish a connection.
Connection

- For *good* storage MDS codes and *good* multiple-access wiretap channels.

\[
\min(\text{repair BW}) \equiv \max(\text{S-DoF})
\]

i.e. if I can solve one, I can solve the other.

- Good storage codes $\leftrightarrow$ Good multiple-access wiretap channels
- Good repair strategies $\leftrightarrow$ Good beamforming strategies
  (over the same field)
Connection

- For good storage MDS codes and good multiple-access wiretap channels.

\[
\min(\text{repair BW}) \equiv \max(\text{S-DoF})
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- Good storage codes \(\leftrightarrow\) Good multiple-access wiretap channels
- Good repair strategies \(\leftrightarrow\) Good beamforming strategies

(over the same field)
Byproduct: We characterize the S-DoF of the SISO multiple-access compound wiretap channel.

How? By using as beamforming matrices, the repair matrices of a diagonal code by [CJM10], [SR10].

[Khisti], [Bagherikaram, Motahari, Khandani], [Koyluoglu, El Gamal, Lai, Poor], [Kobayashi, Piantanida, Yang, Shamai], [He, Yener], [Bassily, Ulukus]
- Minimizing the Repair BW
- Maximizing the S-DoF
- Establishing the Connection
- The S-DoF of SISO Multiple-Access Compound Wiretap Channel
- Extensions and Conclusions
The Repair Problem

- Cut a file into 3 parts $a, b, c$. Store it across 5 nodes, rate $= \frac{3}{5}$. Each part has length $2N$.
- We use $(5, 3)$ MDS codes.
  - 1. Each node stores $2N$.
  - 2. Any 3 nodes know everything.

\[
\begin{align*}
1 & \quad A_1^T a + B_1^T b + C_1^T c \\
2 & \quad A_2^T a + B_2^T b + C_2^T c
\end{align*}
\]

- $A_i, B_i, C_i: 2N \times 2N$

[Dimakis, Wu, Suh, Ramchandran], [Rashmi, Shah, Kumar]
The Repair Problem

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[Dimakis, Wu, Suh, Ramchandran], [Rashmi, Shah, Kumar]
Q: If a disk fails? A: Exactly repair what was lost!

1. $A_1^T a + B_1^T b + C_1^T c$
2. $A_2^T a + B_2^T b + C_2^T c$

- # unknowns in $a$: $2N$
The Repair Problem

Q: If a disk fails? A: Exactly repair what was lost!

# unknowns in \( a \): 2\( N \)

\( R_i \) : 2\( N \times N \)

Q: Cost?
The Repair Problem

Q: If a disk fails? A: Exactly repair what was lost!

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2. $A_2^T a + B_2^T b + C_2^T c$

- $\#$ unknowns in $a$: $2N$
- $R_i : 2N \times N$
- Q: Cost? A: $[\text{Size of lost } a] + [\# \text{ dimensions of } \square \text{ and } \blacksquare]$. 

$\square$ and $\blacksquare$ not specified in the image.
Q1: How to find the minimum cost?
A: Choose $R_i$s so that interference spaces collapse to the smallest spaces possible $\Rightarrow$ **Interference Alignment**.

Q2: Can you formalize this?
A:

$$
\mathcal{R}:
\begin{align*}
\min_{R_1, R_2} & \left( \text{rank}(\begin{bmatrix} a \\ b \\ c \end{bmatrix}) + \text{rank}(\begin{bmatrix} a \\ b \\ c \end{bmatrix}) \right) \\
\text{s.t.} & \text{rank}(\begin{bmatrix} a \\ b \\ c \end{bmatrix}) = 2N
\end{align*}
$$
The Repair Problem

- Q1: How to find the minimum cost?
- A: Choose $R_i$s so that interference spaces collapse to the smallest spaces possible $\Rightarrow$ *Interference Alignment*.

- Q2: Can you formalize this?
- A:

$$\mathcal{R} : \begin{align*}
\min_{R_1, R_2} & \text{rank}(\mathbf{a}) + \text{rank}(\mathbf{b}) \\
\text{s.t.} & \text{rank}(\mathbf{c}) = 2N
\end{align*}$$
The Repair Problem

- A special class of MDS codes: **Optimal** MDS codes. (a.k.a MSR codes)

Minimum Repair BW = \([\text{size of lost piece}] + 2 \times N\)

- Any interference space can be squeezed into \(N\) dimensions.
The Repair Problem

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- Minimizing the Repair BW
- **Maximizing the S-DoF**
- Establishing the Connection
- The S-DoF of SISO Multiple-Access Compound Wiretap Channel
- Extensions and Conclusions
The S-DoF Problem

Multiple-access compound wiretap channel.

- System Model:
  - 2 users, each transmits $N$ symbols,
  - 1 desired receiver, 2 wiretappers.

$$
H_1 V_1 x_1 + H_2 V_2 x_2 + n = \begin{bmatrix} H_1 V_1 & H_2 V_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n
$$

- $H_i, H_i^{w1}, H_i^{w2} : 2N \times 2N$,
- $V_i : 2N \times N$

Objective?

$$
\text{S-DoF} = \text{[rank of ]-max}{\{[\text{rank of }], [\text{rank of }]} \}
$$

(outerbound = $2N - N$)
The S-DoF Problem

Multiple-access compound wiretap channel.

- **System Model:**
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$$
H_1 V_1 x_1 + H_2 V_2 x_2 + n = [H_1 V_1 \ H_2 V_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n
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- **Objective?**
  - $S$-DoF = [rank of ■]-max{[rank of ■], [rank of ■]}

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The S-DoF Problem

Multiple-access compound wiretap channel.

- System Model:
  2 users, each transmits $N$ symbols,
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\begin{align*}
H_1 V_1 x_1 + H_2 V_2 x_2 + n &= [H_1 V_1 \quad H_2 V_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n \\
&= \begin{bmatrix} H_1^{(w1)} V_1 & H_2^{(w1)} V_2 \\ H_1^{(w2)} V_1 & H_2^{(w2)} V_2 \end{bmatrix}
\end{align*}
\]

- $H_i, H_i^{w1}, H_i^{w2} : 2N \times 2N$,
- $V_i : 2N \times N$
- Objective?
  
  $S\text{-DoF} = \text{[rank of ]-max\{[rank of ], [rank of ]\}}$

  (outerbound $= 2N - N$)
Q1: How to maximize the S-DoF?

A: Choose $\mathbf{V}_i$s so that the best wiretapper listens to the smallest space possible $\Rightarrow$ **Interference Alignment**.

Q2: Can you formalize this?

A:

\[
\mathcal{V} : \\
\min_{\mathbf{V}_1, \mathbf{V}_2} \max \{ \text{rank } (\text{■}) , \text{rank } (\text{■}) \} \\
\text{s.t. } \text{rank } (\text{■}) = 2N
\]
Q1: How to maximize the S-DoF?

A: Choose $V_i$s so that the best wiretapper listens to the smallest space possible $\Rightarrow$ **Interference Alignment**.

Q2: Can you formalize this?

A:

\[
V : \\
\min_{V_1, V_2} \max \{ \operatorname{rank} (\text{ }, \text{blue} ), \operatorname{rank} (\text{ }, \text{red} ) \} \\
\text{s.t. } \operatorname{rank} (\text{ }, \text{red} ) = 2N
\]
A special class of such channels: those that achieve the S-DoF outerbound

\[ S\text{-DoF} = 2N - N \]

Any wiretaper’s space can be squeezed into \( N \) dimensions.
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A special class of such channels: those that achieve the S-DoF outerbound

$$\text{S-DoF} = 2N - N$$

Any wiretaper’s space can be squeezed into \( N \) dimensions.
- Minimizing the Repair BW
- Maximizing the S-DoF
- **Establishing the Connection**
- The S-DoF of SISO Multiple-Access Compound Wiretap Channel
- Extensions and Conclusions
The Connection Overview

Storage:

- 1 Desired space: \([A_1R_1 A_2R_2]: 2N \times 2N\)
- 2 Harmful spaces: \([B_1R_1 B_2R_2], [C_1R_1 C_2R_2]: 2N \times 2N\)
- 6 Coding matrices (human made): \(A_i, B_i, C_i: 2N \times 2N\)
- 2 Repair matrices: \(R_i: 2N \times N\)

Wireless:

- 1 Desired space: \([H_1V_1 H_2V_2]: 2N \times 2N\)
- 2 Harmful places: \([H_{w1}^1V_1 H_{w1}^2V_2], [H_{w2}^1V_1 H_{w2}^2V_2]: 2N \times 2N\)
- 6 Channel matrices (nature made): \(H_i, H_{w1}^i, H_{w2}^i: 2N \times 2N\)
- 2 Beamforming matrices: \(V_i: 2N \times N\)

hmm..

Q: What about the two optimizations \(R\) and \(V\)?
The Connection Overview

Storage:
- 1 Desired space: \([A_1 R_1 \ A_2 R_2]\): \(2N \times 2N\)
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Wireless:
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- 2 Beamforming matrices: \(V_i\): \(2N \times N\)

hmm..

Q: What about the two optimizations \( \mathcal{R} \) and \( \mathcal{V} \)?
The Connection Overview

Storage:

- 1 Desired space:
  \[[A_1 R_1 A_2 R_2]: 2N \times 2N\]
- 2 Harmful spaces:
  \[[B_1 R_1 B_2 R_2], [C_1 R_1 C_2 R_2]: 2N \times 2N\]
- 6 Coding matrices
  (human made):
  \[A_i, B_i, C_i: 2N \times 2N\]
- 2 Repair matrices:
  \[R_i: 2N \times N\]

Wireless:

- 1 Desired space:
  \[[H_1 V_1 H_2 V_2]: 2N \times 2N\]
- 2 Harmful spaces:
  \[[H^{w1}_1 V_1 H^{w1}_2 V_2], [H^{w2}_1 V_1 H^{w2}_2 V_2]: 2N \times 2N\]
- 6 Channel matrices
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  \[H_i, H^{w1}_i, H^{w2}_i: 2N \times 2N\]
- 2 Beamforming matrices:
  \[V_i: 2N \times N\]

hmm..

Q: What about the two optimizations \( \mathcal{R} \) and \( \mathcal{V} \)?
The Connection Overview

Theorem

For any optimal storage code \( A_i, B_i, C_i \), minimizing the repair BW \( \equiv \) maximizing the S-DoF of \( H_i = A_i, H_i^w1 = B_i, H_i^w2 = C_i \).

Lemma

If \( H_i, H_i^w1, H_i^w2 \) is a full S-DoF channel, then it is also a code with minimum repair BW for node 1.

Lemma

If \( A_i, B_i, C_i \) is an optimal MDS code, then it is also a full S-DoF channel.
The Connection Overview

Theorem

For any optimal storage code $A_i, B_i, C_i$, minimizing the repair BW $\equiv$ maximizing the S-DoF of $H_i = A_i, H_{i1}^w = B_i, H_{i2}^w = C_i$

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Lemma

If $A_i, B_i, C_i$ is an optimal MDS code, then it is also a full S-DoF channel.
Sketch:

Optimal Codes:

\[
\min_{R_1, R_2, \text{rank}(\mathbf{A})=2N} \left( \text{rank}(\mathbf{A}) + \text{rank}(\mathbf{B}) \right) = 2 \min_{R_1, R_2, \text{rank}(\mathbf{A})=2N} \text{max} \left\{ \text{rank}(\mathbf{A}), \text{rank}(\mathbf{B}) \right\} = 2N
\]

Full S-DoF channels:

\[
\min_{V_1, V_2, \text{rank}(\mathbf{A})=2N} \text{max} \left\{ \text{rank}(\mathbf{A}), \text{rank}(\mathbf{B}) \right\} = \frac{1}{2} \min_{V_1, V_2, \text{rank}(\mathbf{A})=2N} \left( \text{rank}(\mathbf{A}) + \text{rank}(\mathbf{B}) \right) = N
\]

Solving one is solving the other

Good Codes = Good Channels

(Good Repair Matrices = Good Beamforming Matrices)

(over the same field)

Any practical examples?
**The Connection Overview**

**Sketch:**

**Optimal Codes:**

\[
\min_{R_1, R_2, \text{rank}(\square) = 2N} \left( \text{rank}(\square) + \text{rank}(\square) \right) = 2 \quad \min_{R_1, R_2, \text{rank}(\square) = 2N} \left\{ \text{rank}(\square), \text{rank}(\square) \right\} = 2N
\]

**Full S-DoF channels:**

\[
\min_{V_1, V_2, \text{rank}(\square) = 2N} \left\{ \text{rank}(\square), \text{rank}(\square) \right\} = \frac{1}{2} \quad \min_{V_1, V_2, \text{rank}(\square) = 2N} \left( \text{rank}(\square) + \text{rank}(\square) \right) = N
\]

Solving one is solving the other

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Any practical examples?
Sketch:
Optimal Codes:

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\min_{R_1, R_2, \text{rank}(\square) = 2N} (\text{rank}(\square) + \text{rank}(\square)) = 2 \quad \min_{R_1, R_2, \text{rank}(\square) = 2N} \max \{\text{rank}(\square), \text{rank}(\square)\} = 2N
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Full S-DoF channels:

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\min_{V_1, V_2, \text{rank}(\square) = 2N} \max \{\text{rank}(\square), \text{rank}(\square)\} = \frac{1}{2} \min_{V_1, V_2, \text{rank}(\square) = 2N} (\text{rank}(\square) + \text{rank}(\square)) = N
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**Sketch:**

Optimal Codes:

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Full S-DoF channels:

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\min_{V_1, V_2, \text{rank}(\square) = 2N} \left\{ \text{rank}(\square), \text{rank}(\square) \right\} = \frac{1}{2} \quad \min_{V_1, V_2, \text{rank}(\square) = 2N} \left( \text{rank}(\square) + \text{rank}(\square) \right) = N
\]

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(over the same field)

Any practical examples?
Sketch:

Optimal Codes:

\[ \min_{R_1, R_2, \text{rank}(\mathbb{S}) = 2N} (\text{rank}(\mathbb{S}) + \text{rank}(\mathbb{S})) = 2N \]

Full S-DoF channels:

\[ \min_{V_1, V_2, \text{rank}(\mathbb{S}) = 2N} \max \{\text{rank}(\mathbb{S}), \text{rank}(\mathbb{S})\} = \frac{1}{2} \min_{V_1, V_2, \text{rank}(\mathbb{S}) = 2N} (\text{rank}(\mathbb{S}) + \text{rank}(\mathbb{S})) = N \]

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Any practical examples?
The Connection Overview

**Sketch:**

Optimal Codes:

\[
\min_{R_1, R_2, \text{rank}(\mathbf{M}) = 2N} (\text{rank}(\mathbf{M}) + \text{rank}(\mathbf{N})) = 2N
\]

\[
\min_{R_1, R_2, \text{rank}(\mathbf{M}) = 2N} \max_{\mathbf{R}} \{\text{rank}(\mathbf{M}), \text{rank}(\mathbf{N})\} = 2N
\]

Full S-DoF channels:

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\min_{V_1, V_2, \text{rank}(\mathbf{M}) = 2N} \max_{\mathbf{V}} \{\text{rank}(\mathbf{M}), \text{rank}(\mathbf{N})\} = \frac{1}{2} \text{rank}(\mathbf{M}) + \text{rank}(\mathbf{N}) = N
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\min_{V_1, V_2, \text{rank}(\mathbf{M}) = 2N} \max_{\mathbf{V}} \{\text{rank}(\mathbf{M}), \text{rank}(\mathbf{N})\} = N
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Any practical examples?
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The Connection Overview

Example:

Let

\[ A_i = \begin{bmatrix} a_i(1) & \cdots & 0 \\ \vdots \\ 0 & \cdots & a_i(2N) \end{bmatrix}, \quad B_i = \begin{bmatrix} b_i(1) & \cdots & 0 \\ \vdots \\ 0 & \cdots & b_i(2N) \end{bmatrix}, \quad C_i = \begin{bmatrix} c_i(1) & \cdots & 0 \\ \vdots \\ 0 & \cdots & c_i(2N) \end{bmatrix}. \]

Elements drawn iid.

This code is asymptotically optimal: interference dimensions = \( N' \) [CJM10], [SR10], with \( \lim_{N \to \infty} \frac{N'}{N} = 1 \). (Symbol Extension).

Q: Does this map to any interesting channel?

A: The single antenna multiple-access compound wiretap channel.

\[ H_i = \begin{bmatrix} h_i(1) & \cdots & 0 \\ \vdots \\ 0 & \cdots & h_i(2N) \end{bmatrix}, \quad H_i^{w1} = \begin{bmatrix} h_i^{w1}(1) & \cdots & 0 \\ \vdots \\ 0 & \cdots & h_i^{w1}(2N) \end{bmatrix}, \quad H_i^{w2} = \begin{bmatrix} h_i^{w2}(1) & \cdots & 0 \\ \vdots \\ 0 & \cdots & h_i^{w2}(2N) \end{bmatrix}. \]

Normalized per user S-DoF = \( \frac{2N-N'}{2N} \to \frac{1}{2} \ (\frac{L-1}{L} \text{ in general}) \).
The Connection Overview

Example:

- Let

\[
A_i = \begin{bmatrix}
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\vdots & \ddots & \vdots \\
0 & \ldots & h_i^{w1}(2N)
\end{bmatrix}, \quad H_i^{w2} = \begin{bmatrix}
h_i^{w2}(1) & \ldots & 0 \\
\vdots & \ddots & \vdots \\
0 & \ldots & h_i^{w2}(2N)
\end{bmatrix}.
\]

- normalized per user S-DoF = \(\frac{2N-N'}{2N} \to \frac{1}{2} \left( \frac{L-1}{L} \right) \text{ in general}\)
The Connection Overview

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Example:

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Q: Does this map to any interesting channel?

A: The single antenna multiple-access compound wiretap channel.

\[ H_i = \begin{bmatrix} h_i(1) & \ldots & 0 \\ \vdots \\ 0 & \ldots & h_i(2N) \end{bmatrix}, \quad H_i^{w1} = \begin{bmatrix} h_i^{w1}(1) & \ldots & 0 \\ \vdots \\ 0 & \ldots & h_i^{w1}(2N) \end{bmatrix}, \quad H_i^{w2} = \begin{bmatrix} h_i^{w2}(1) & \ldots & 0 \\ \vdots \\ 0 & \ldots & h_i^{w2}(2N) \end{bmatrix}. \]

Normalized per user S-DoF = \( \frac{2N-N'}{2N} \to \frac{1}{2} (\frac{L-1}{L} \text{ in general}) \)
Example:

Let

\[
A_i = \begin{bmatrix}
  a_i(1) & \ldots & 0 \\
  \vdots & \ddots & \vdots \\
  0 & \ldots & a_i(2N)
\end{bmatrix},
B_i = \begin{bmatrix}
  b_i(1) & \ldots & 0 \\
  \vdots & \ddots & \vdots \\
  0 & \ldots & b_i(2N)
\end{bmatrix},
C_i = \begin{bmatrix}
  c_i(1) & \ldots & 0 \\
  \vdots & \ddots & \vdots \\
  0 & \ldots & c_i(2N)
\end{bmatrix}.
\]

Elements drawn iid.

This code is asymptotically optimal: interference dimensions = N’ [CJM10], [SR10], with \( \lim_{N \to \infty} \frac{N'}{N} = 1 \). (Symbol Extension).

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\[
H_i = \begin{bmatrix}
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  0 & \ldots & h_i(2N)
\end{bmatrix},
H_i^{w1} = \begin{bmatrix}
  h_i^{w1}(1) & \ldots & 0 \\
  \vdots & \ddots & \vdots \\
  0 & \ldots & h_i^{w1}(2N)
\end{bmatrix},
H_i^{w2} = \begin{bmatrix}
  h_i^{w2}(1) & \ldots & 0 \\
  \vdots & \ddots & \vdots \\
  0 & \ldots & h_i^{w2}(2N)
\end{bmatrix}.
\]

normalized per user S-DoF = \( \frac{2N-N'}{2N} \to \frac{1}{2} \left( \frac{L-1}{L} \right) \) in general)
- Minimizing the Repair BW
- Maximizing the S-DoF
- Establishing the Connection
- The S-DoF of SISO Multiple-Access Compound Wiretap Channel
- Extensions and Conclusions
For codes and channels that are not that good we use

\[
\frac{1}{n} \sum_{i=1}^{n} r_i \leq \max_i r_i \leq \sum_{i=1}^{n} r_i \leq n \max_i r_i
\]

to derive bounds.
Conclusions

- IA is used in both the Repair and S-DoF maximization problems.
- We can formulate them as Rank Constrained Rank Minimizations,
- and establish a connection between the two, with mappings and reductions.
- Then, using a repair code we characterized the S-DoF of the SISO multiple-access compound wiretap channel.
Thank you!