Nonlinear Evolution of Ion-Cyclotron Turbulence Generated by Artificial Plasma Cloud Release

W. A. Scales,* O. Chang,† and J. J. Wang†

Center for Space Science and Engineering Research,
Virginia Polytechnic Institute and State University, Blacksburg, Virginia, 24061

G. Ganguli, L. Rudakov, and M. Mithaiwala
Plasma Physics Division, Naval Research Laboratory, Washington D.C., 20075
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Nonlinear evolution of plasma turbulence generated by a plasma cloud released into a magnetized background plasma is studied using electromagnetic hybrid (fluid electrons and particle ions) plasma simulations incorporating electron inertia. The turbulence considered is generated from free energy in an assumed ring velocity distribution in the released plasma cloud heavy ions. The turbulence initially lies near harmonics of the ring plasma ion cyclotron frequency and propagates nearly perpendicular to the background magnetic field as predicted by linear theory. If the amplitude of the turbulence is sufficiently large, the relatively short wavelength ion cyclotron waves evolve nonlinearly into much longer wavelength obliquely propagating shear Alfven waves. The results indicate that ring densities above a few percent of the background plasma density may produce wave amplitudes large enough for such an evolution to occur. The extraction of energy from the ring plasma may be in the range of 10-20% with a slight decrease in the magnitude as the ring density is increased from a few percent to several 10’s of percent of the background plasma density. Suitability of the nonlinearly generated shear Alfven waves for applications to scattering radiation belt particles is discussed.

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I. INTRODUCTION

It has recently been proposed [1] that the release of an artificial plasma cloud in the near earth space environment may induce plasma turbulence with unique properties. Of particular interest is the possibility of such turbulence providing control of space weather processes which adversely effect reliability of space assets. The scenario for generating this turbulence is the release, from a spacecraft, of an easily ionizable chemical such as lithium, cesium, etc. Upon ionization, a heavy ion component will be produced in the background plasma which has a ring distribution in velocity space. Although chemical releases in the near-earth space environment have been studied for decades, the proposed process is unique in the sense that the free energy for development of turbulence is the orbital kinetic energy of the neutral atoms released from a satellite. It has been shown that this kinetic energy may be substantial and therefore have ability to greatly impact space weathering processes for such applications as the earth’s radiation belts.

Specific wave modes of importance for interaction with radiation belt particles would be electromagnetic plasma waves which would have ability to pitch-angle scatter trapped relativistic electrons. Examples studied in the past include whistler and electromagnetic ion cyclotron EMIC waves. Generation and nonlinear evolution of an alternative type of electromagnetic plasma turbulence (shear Alfven) will be the subject of this investigation. The frequency of the turbulence considered in this investigation is initially near ion cyclotron harmonic frequencies of ring velocity plasma created by the release. These are relatively short wavelength waves that initially propagate nearly perpendicular to the magnetic field. It has been proposed that the nonlinear evolution of these waves ultimately results in longer wavelength shear Alfven waves which propagate obliquely (although further away from perpendicular to the magnetic field), and have ability to pitch angle scatter radiation belt electrons.

There are several important plasma physics questions that must be answered to access the practicality of this scenario. These questions relate directly to the nonlinear evolution of the turbulence. First, is the efficiency of extraction of the plasma cloud kinetic energy into wave energy through wave-particle processes sufficient to make a useful impact on the background plasma? Another question is what is the amplitude and characteristics of the waves in a nonlinear saturated state? Also, do these waves appear to have characteristics in frequency and wavenumber that are suitable for interactions with energetic particles? The investigation here will attempt to answer some of these important questions. The next section will describe the linear theory of these waves under a range of parameters. To study the nonlinear evolution of

*Also at The Bradley Department of Electrical and Computer Engineering, Virginia Tech; Electronic address: wscale@vt.edu
†Also at Department of Astronautical Engineering, Engineering, University of Southern California
II. LINEAR THEORY

The linear theory of the proposed ion cyclotron turbulence has been described in detail by Ref. 1 for the case of the release of Lithium into a background electron-hydrogen plasma representative of the earth’s magnetosphere. It was shown in detail how such a release produces a lithium plasma with a ring velocity distribution. Only a brief review of the key characteristics of the plasma instability will be described here. This theory describes the initial generation of highly oblique shear Alfvén waves near lithium cyclotrons. The growth rate $\gamma$ is obtained from resonance of the Lithium cyclotron frequency, $\omega_c \parallel$ is the ring plasma velocity which would be equal to the electron (hydrogen) thermal speed, $c$ is the speed of light in vacuum, $\nu_{ci}(H)$ is the electron (hydrogen) thermal speed, $\omega_{pe(H)}$ is the electron (hydrogen) plasma frequency, $\rho_H = \nu_{ci}(H)/c$ is the hydrogen gyroradius, $Z'(\zeta) = \omega/(\sqrt{2}k_v v_{te})$, $b_H = k_s \rho_H$ and $\Gamma_1(b) = I_1(b) \exp(-b)$ where $I_1$ is the modified Bessel function.

The double resonance condition of these two frequencies yields the growth rate:

$$\gamma = \frac{\ell \Omega_{Li}}{2}$$

$$\left[ -\Delta + \sqrt{\Delta^2 + \frac{n_{Li} M_{Li}}{n_H M_H} \frac{dP(\sigma_s)}{d\sigma_s} b_H \frac{\Omega^2_H - \ell^2 \Omega^2_{Li}}{\Gamma_1(b_H) \Omega_H^2}} \right]$$

where $\Delta = \zeta^3 \exp(-\zeta^3) (\Omega^2_H - \ell^2 \Omega^2_{Li})$.

Also $v_r$ is the ring plasma velocity which would be equal to the speed of the releasing spacecraft, $M_{Li(Li)}$ is the hydrogen (lithium) mass, $\rho_{Li(Li)}$ is the lithium (hydrogen) density, $\sigma_s = k_s v_r / \Omega_{Li}$, and $J_\ell$ is the Bessel function of order $\ell$.

Figure 1 shows the growth rate for parameters consistent with creation of a ring plasma generated in the earth’s radiation belts in an approximate altitude range between 3000km and 9000km. The density $n_H$ is fixed at $3 \times 10^4$ cm$^{-3}$, $T_e = T_H \approx 0.5$ eV, $B_0 \approx 0.04$ G, $V_A \approx 1.6 \times 10^8$ km/s, $v_t = 7$ km/s, and $\beta \approx 4 \times 10^{-5}$. Recall that $M_{Li} = 7m_H$, however, the effect of varying ring species mass will be discussed later. Four values of the ratio of lithium to hydrogen ion density $n_{Li}/n_H = 1\%$, 5\%, 10\%, and 30\% are shown to assess the variation of density on wave processes. This important issue will be discussed in detail later upon discussing the numerical simulation results. The first three harmonics $\ell = 1, 2, 3$ which have the largest growth rate are shown. Fixed values of $k_s$ are shown for each harmonic so the growth rate is maximized while $k_s$ is varied. It should be noted that the waves propagate nearly perpendicular to the magnetic field with $k_s/k_\parallel \sim 250$. Therefore initially these waves have short wavelengths perpendicular to the magnetic field. In the lowest density case, the $\ell = 1$ harmonic has a significantly higher growth rate than the higher harmonics. It can be observed from Fig. 1 that increasing the ring density tends to drive the harmonics to more similar amplitudes.

III. NONLINEAR SIMULATION MODEL

A magnetized two-dimensional electromagnetic hybrid computational model has been developed to study the nonlinear evolution of these waves after they grow to large amplitudes. Three species, as described in the previous section, are included in the model. Again, these are ambient hydrogen and electrons and also the ionized re-
lease species ion which will be taken to be lithium for this investigation. To model the plasma dynamics, the model uses particle ions and fluid electrons with finite mass. The hydrogen ions are initialized with a Maxwellian velocity distribution while the lithium ions are initialized with a standard ring velocity distribution in the plane perpendicular to the background magnetic field $\vec{B}_0$. The neutral lithium ionization process is not included in the current version of the model although it is relatively straightforward. It has been shown that this is a reasonable assumption [1] since the creation time of the ring plasma is shorter than the time for the ring plasma to release its energy to plasma waves. Figure 2 shows the configuration of the simulation domain.

The electromagnetic fields are treated with a low frequency approximation and thus the displacement current is neglected in Ampere’s law [2]. Therefore Faraday’s and Ampere’s law become

$$\nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t} \tag{4}$$

$$\nabla \times \vec{B} = \frac{4\pi}{c} \vec{j} \tag{5}$$

To treat the plasma ignoring effects on the electron Debye length scale, quasi-neutrality is imposed by setting:

$$n_e = n_H + n_{Li} \tag{6}$$

The highly oblique shear Alfvén waves being considered have perpendicular wavelengths, comparable to the electron inertial length ($c/\omega_{pe}$). The wave dispersion relation therefore requires finite electron inertia effects [3, 4] and in the simulation model, the finite electron mass is incorporated in the electron momentum equation:

$$m_e n_e \frac{d}{dt} \vec{v}_e = -e n_e (\vec{E} + \frac{\vec{v}_e \times \vec{B}}{c}) + e n_e \eta \vec{J} \tag{7}$$

The pressure term is neglected in Eq. (7) because the plasma considered has a very low $\beta$ and the pressure term only has insignificant impact on the physics. The last term in Eq. (7), which contains the current density $\vec{J}$ and resistivity $\eta$, represents both collisional effects between the electrons and ions and anomalous electron collisions with plasma turbulence [5]. The impact of such collisional processes will be accessed and discussed later. Eqs. (4, 5, 6, 7) can be used to obtain an expression for calculating the electric field [6, 7]:

$$\frac{c^2}{4\pi} (\nabla (\nabla \cdot \vec{E}) - \nabla^2 \vec{E}) + \frac{e^2 n_i}{m_e} \vec{E}$$

$$= \frac{n_i e^2}{cm_e} (\vec{v}_i \times \vec{B}) + \frac{e}{4\pi m_e} (\nabla \times \vec{B}) \times \vec{B} + \frac{ce^2 n_i \eta}{4\pi m_e} (\nabla \times \vec{B}) \tag{8}$$

A brief derivation of the field equation is given in Appendix A. Note that here $n_i (n_H + n_{Li})$ is the total ion density and $\vec{v}_i$ is the ion velocity, both determined from the PIC method [8]. The hydrogen and lithium fluxes, $\Gamma_H$ and $\Gamma_{Li}$, are used to calculate $\vec{v}_i = (\Gamma_H + \Gamma_{Li})/n_i$. The nonlinear terms on the right hand side of Eq. (8) are essential for modeling the evolution of shear Alfvén waves in the turbulence state. The challenge of this model comes from the fact that the dispersion relation for the waves requires $\nabla \cdot \vec{E} \neq 0$ in Eq. (8) while at the same time the quasi-neutral assumption must be maintained. Naively solving Eq. (8) explicitly may result in numerical inaccuracies due to violation of the Poisson equation. One way to overcome this problem is to separate the electric field into transverse $\vec{E}_t$ and longitudinal $\vec{E}_l = -\nabla \phi$ components, where $\phi$ is the electric scalar potential [9]. Separating Eq. (8) results in the transverse field equation:

$$-\frac{c^2}{4\pi} \nabla^2 \vec{E}_t + \frac{e^2 n_i}{m_e} \vec{E}_t$$

$$= \frac{n_i e^2}{cm_e} (\vec{v}_i \times \vec{B}) + \frac{e}{4\pi m_e} (\nabla \times \vec{B}) \times \vec{B} + \frac{ce^2 n_i \eta}{4\pi m_e} (\nabla \times \vec{B})$$

$$- \frac{e^2 n_i}{m_e} \vec{E}_t \tag{9}$$

and the longitudinal field equation which is written in terms of the potential

$$\nabla^2 \phi + \frac{1}{n_i} (\nabla n_i \cdot \nabla \phi)$$

$$= \frac{1}{c} \left( \vec{B} \cdot (\nabla \times \vec{v}_i) - \vec{v}_i \cdot (\nabla \times \vec{B}) \right) + \frac{1}{cm_i} (\vec{v}_i \times \vec{B}) \cdot \nabla n_i$$

$$+ \frac{1}{4\pi m_i} (\vec{B} \cdot \nabla^2 \vec{B} + (\nabla \times \vec{B}) \cdot (\nabla \times \vec{B}) - \frac{c}{4\pi n_i} (\nabla \times \vec{B}) \cdot \nabla n_i$$

$$+ \frac{1}{n_i} (\nabla n_i \cdot \vec{E}_t) \tag{10}$$

A standard finite difference solution method (e.g., pre-conditioned conjugate gradient method) or the pseudo-spectral method can be applied to solve Eq. (9) and Eq. (10). In addition, an iterative procedure is required for calculating $\vec{E}_t$ and $\vec{E}_l$ since the two equations are coupled.

After the electric field is calculated, the magnetic field is updated by using Faraday’s law. A 4th-order Runge-Kutta scheme with subcycled time interval is applied to time advance the magnetic field equation. The updated fields are then used to push the ion particles. This closes the computational cycle.
IV. SIMULATION RESULTS

A number of simulations of the nonlinear evolution of the ion-cyclotron waves generated by the ring velocity instability have been made. A description of the basic nonlinear evolution is described first. Parameters used for the background plasma are those stated in the previous section. The case described here uses a lithium ring plasma to background plasma density ratio $n_L/n_H = 0.1$. Important effects of varying the ring plasma density will be considered later. There are 400 background hydrogen simulation particles per cell as well as 400 lithium simulation particles per cell. The velocity of the ring is the same as section II with $v_r/v_{H} \approx 1$ which corresponds to a release speed of approximately 7 km. The lithium ring is taken to be cold with $T_L \approx 0$. The end time of the simulations are $\Omega_H t = 700$ which provides enough time to observe the nonlinear evolution. The 2D simulation is $128 \times 128$ grid cells with the background magnetic field in the simulation plane and in the z direction. The domain is elongated along the background magnetic field with length along the magnetic field $L_z \approx 56c/\omega_{BH}$. The length across the magnetic field $L_x \approx 1c/\omega_{BH}$. Such a simulation domain is required to follow the evolution of waves with long wavelengths along the magnetic field and relatively short wavelengths across the magnetic field as described in the previous section. For the parameters of interest, the simulation domain corresponds to $L_z \approx 200$ km and $L_x \approx 5$ km.

A. Temporal Evolution

Figure 3 shows the time evolution of the energy in the magnetic field. During the initial evolution, linear growth in the field is observed. The linear growth rate from Fig. 1 is observed to describe the growth of the electric and magnetic fields well particularly since the growth of the first three harmonics is predicted to be similar. At time $\Omega_H t \approx 150$ the linear wave growth saturates. The higher frequency oscillations during the linear growth period are observed to be in the lithium cyclotron frequency range as pointed out in the figure. In the saturated state, lower frequency oscillations gradually begin to be observed and can be shown to be in the range of shear Alfven waves that can be shown to satisfy the dispersion relation in Eq. (2) quite well. Figure 4 shows the power spectrum of the dominant magnetic field $B_x$ component taken during three time periods. The first time period $0 < \Omega_H t < 200$ is during the linear growth. The spectrum shows power at harmonics of the lithium cyclotron frequency. This power is particularly enhanced at the first three harmonics which is in agreement with the predictions of linear theory in Fig. 1. The next period shown $0 < \Omega_H t < 300$ is taken during the initial saturation of the waves. Lower frequency waves are observed as well as the waves near the lithium cyclotron harmonics. The low frequency waves are found to satisfy the dispersion relation of shear Alfven waves quite well. The final spectrum is taken over the entire time of the simulation $0 < \Omega_H t < 700$. The whole time period spectrum shows that the low frequency waves near the shear Alfven frequency dominate the spectrum. Therefore in general a evolution of the initially generated waves near the lithium cyclotron harmonics to shear Alfven waves is observed.

To examine the wave interactions in more detail, wavenumber spectra are shown in Fig. 5 at three fixed times which are $\Omega_H t = 100, 200, 700$. During the linear growth period, power is observed at three distinct regions in wavenumber space which correspond to the first three lithium cyclotron harmonics as predicted by linear theory. After nonlinear saturation, in addition to the cyclotron harmonic waves, a mode with much longer perpendicular wavelength is observed which is interpreted as a shear Alfven wave. The spectrum at the end of the simulation shows a wave propagating nearly perpendicular to the magnetic field with a substantially longer wavelength (more than a factor of 30) interpreted as a shear Alfven wave. Comparison of the frequency and wavenumber of this low frequency wave to the dispersion relation of oblique shear Alfven waves Eq. (2) shows quite good agreement. This lower frequency longer wavelength wave dominates the spectrum after the wave amplitude has saturated. Therefore the results indicate an initial generation of waves near lithium cyclotron harmonic frequencies and the subsequent development of these higher frequency shorter wavelength into obliquely propagating shear Alfven waves with much longer wavelengths both parallel and perpendicular to the magnetic field. Although the waves in the saturated state still propagate nearly perpendicular to the magnetic field by the end of the simulation, they propagate less perpendicular than the initially generated cyclotron waves. The size of the simulation domain of course plays a critical role in restricting the development of longer wavelengths. In the saturated state, it is observed that the perpendicular
The previous results indicate important nonlinear properties involving nonlinear wave-wave processes. The results to be presented next provide insight on the anomalous transport processes. Figure 9 shows the phase space of the lithium and hydrogen ions at three fixed times during the simulation which are $\Omega_H t = 0, 150, 500$. It is observed that there is relatively little wave-particle heating of the hydrogen ions. The lithium particles with a ring velocity distribution show slowing down and heating of the lithium particles. Distribution functions in $v_y$ for the ions are shown in Fig. 7. It is observed that the background hydrogen ions experience only some tail heating while the lithium ring ions slow and exhibit bulk heating.

An important consequence of the wave-particle interactions is of course the extraction of free energy of the ring particles into the generation of the waves. The amount of energy extracted can be quantified by considering the temporal evolution of the lithium kinetic energy which is shown in Fig. 8. Also for comparison the energy that goes into heating the background hydrogen ions is also shown. It can be observed that roughly 12% of the energy is extracted from the lithium ring ions in steady state for the background hydrogen ions experience only some tail heating while the lithium ring ions slow and exhibit bulk heating.

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FIG. 4: Frequency power spectrum of the electric field $E_x$ component and magnetic field $B_y$ component. Electric field spectrum is taken for $0 < \Omega_H t < 200$. The magnetic field spectrum is taken for $0 < \Omega_H t < 500$. (Note change in vertical scale for the two spectra.)

FIG. 5: Wavenumber spectrum of $B_y$ showing initial development of shorter wavelength lithium cyclotron harmonic and subsequent development of longer wavelength Alfvén waves.
the case of 10% lithium ring ions. There is actually more extraction of energy at the time of saturation (∼20%). This is due to the fact that after saturation of the waves, there is further heating of the lithium ions as observed in the phase space and distribution functions. Possibilities of controlling the amount of energy reabsorbed by the ring particles will be discussed shortly. It is also observed from Fig. 8 that there is a relatively small percentage increase in the background hydrogen kinetic energy from wave-particle heating.

B. Variation with Ring Density

An important question relating to the nonlinear evolution of the ring plasma instability generated ion cyclotron waves is the condition under which they evolve nonlinearly into shear Alfvén waves. It has been predicted [1] that if the waves are of sufficient amplitude that the process observed in the previous section may occur. From a practical standpoint, it is useful to consider how the strength of the perturbation, i.e. ring plasma density, is related to the nonlinear evolution. Linear theory calculations in the previous section for ring plasma densities from 1% to 30% of the background density predict initial generation of the ion cyclotron waves, however, it is of importance to see if the long wavelength shear Alfvén waves are generated after the nonlinear saturation upon varying the ring density.

A series of simulations were performed with varying ring plasma density to consider this question. Figure 10 shows the time evolution of the magnetic field energy and energy extraction from the ring plasma for four values of ring density, 30%, 10%, 5%, and 1%. The high 30% density case shows that the ion cyclotron waves quickly evolve into the long wavelength shear Alfvén waves after the nonlinear saturation. The 10% ring density case (described in detail in the previous section) shows a somewhat slower evolution of the ion cyclotron waves into the shear Alfvén waves. As the ring density is further decreased to 5%, it becomes clear that it takes much longer for the shear Alfvén waves to appear. A power spectrum shows that the Alfvén waves are much smaller in amplitude than in the previous cases and comparable to the cyclotron harmonic waves. The low 1% density case shows no generation of long wavelength Alfvén waves. The short wavelength $\ell = 1$ harmonic is dominant in this case for most of the simulation and observed in detail in a frequency and wavenumber spectrum (not shown). This is consistent with the linear growth predictions in Fig. 1 which shows this harmonic dominates for 1% ring density. In general, the linear growth predictions of Fig. 1 are in good agreement with the simulations during the initial growth phase. It can be noted in the 1% den-
This investigation has considered the nonlinear evolution of plasma turbulence created by a ring distribution in the perpendicular velocity of ions injected via chemical release into a background plasma typical of the earth’s magnetosphere at altitudes between 3000 km and 9000 km.

C. Effect of Electron Resistivity on Energy Extraction

It is observed that the extraction of energy from the ring ions maximizes near the time of wave saturation. After this time some of the wave energy is reabsorbed by the ring ions. Therefore after wave saturation the efficiency of ring ion kinetic energy extraction is reduced. The simulation model used for this investigation incorporates fluid electrons for the sake of computational efficiency. The influence of anomalous resistivity $\eta^*$ on the fluid electrons is included and may be modeled to consider the effects of transfer of energy to other nonlinear processes or convection out of the system. Such effects may have influence on the extraction of energy from the ring particles. To consider these effects, the resistivity in the electron momentum Equation (7) is modeled as

$$\eta = \eta_0 + \eta^* \exp(|E|^2/|E_{\text{max}}|^2)$$  \hspace{1cm} (11)

where $\eta_0$ is the numerical resistivity and $\eta^*$ is the anomalous resistivity $\eta_0 < \eta^*$. The electric field amplitude is denoted by $|E|$ and the electric field amplitude at saturation is $|E_{\text{max}}|$ which is determined a priori by performing a simulation without the effects of anomalous resistivity. Such a model approximately describes the effects of wave-particle interactions with the fluid electrons in the simulation with an anomalous resistivity. Figure 11 shows the results of varying the anomalous resistivity for ring plasma density $n_1/n_H = 30\%$. The anomalous resistivity for the conditions under consideration is $\eta^* \sim 10^{-12}$ s. Simulation results for three values $\eta^* = 10^{-12}$, $2 \times 10^{-12}$, and $4 \times 10^{-12}$ are shown. It is observed that anomalous resistivity increases the efficiency of extraction from the ring ions after saturation compared to the case with no anomalous resistivity. In this case the energy from the ring is not appreciably reabsorbed by the ring ions after saturation. There is also less heating of the background ions. The importance of this result is that most likely the estimates of energy extraction efficiency after saturation from the hybrid fluid electron model used in this investigation are underestimates. Therefore the instability process described here may be even more effective at generating plasma turbulence. The model for the resistivity is rather simplified and future work will consider a more realistic model.

V. SUMMARY AND CONCLUSIONS

This investigation has considered the nonlinear evolution of plasma turbulence created by a ring distribution in the perpendicular velocity of ions injected via chemical release into a background plasma typical of the earth’s magnetosphere at altitudes between 3000 km and 9000 km.
FIG. 9: Effect of varying ring plasma density ratio $n_{Li}/n_H$ on wave frequency evolution. Long wavelength shear Alfven waves result from sufficiently large density ratios. (a) and (b) is 50% density ratio case. (c) and (d) is 10% case. (e) and (f) is 1% case. Electric field spectra are taken for the linear growth period. Magnetic field spectra are taken for the whole simulation period.

FIG. 10: Effect of varying ring plasma density ratio $n_{Li}/n_H$ on ring kinetic energy.

FIG. 11: Impact of varying anomalous electron resistivity $\eta^*$ on lithium kinetic energy extraction after wave saturation.

km. The specific plasma wave modes under consideration here are obliquely propagating shear Alfven waves due to their predicted unique suitability for interaction with relativistic electrons in the earth’s radiation belts. The investigation here has validated a number of important qualitative predictions of the theoretical scenario for
generation of such turbulence as well as provided further important quantitative details of the initial nonlinear evolution. The results provide estimates of the extraction of energy from the ring plasma kinetic energy into plasma wave turbulence and the processes that impact this extraction efficiency, characteristics of the turbulence in a saturated state, and ring ion density levels that are predicted to efficiently initiate the production of such plasma turbulence. The current model is restricted to investigation of the initial generation of the plasma turbulence. Currently it does not allow study of further nonlinear processes predicted to occur on much longer time and spatial scales than considered here. Also it does not allow study of the efficiency of pitch angle scattering of relativistic electrons which would also occur on much longer timescales. However, the characterization of the generation of the seed electromagnetic turbulence provided here will make such investigations possible in ongoing and future studies. Larger domain sizes will be used in future investigations and may provide further insight into the processes under investigation.

Acknowledgments

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APPENDIX A: DERIVATION OF THE ELECTRIC FIELD EQUATIONS

The derivation of the field Eqs. (8, 9, 10) will now be discussed. The basic electron momentum equation including electron inertia term is given in Eq. (7). It can be rewritten as:

\[ E = -\frac{\vec{v}_e \times \vec{B}}{e} + \frac{1}{4\pi e n_c}(\vec{\nabla} \times \vec{B}) \times \vec{B} + \frac{c n_e}{4\pi}(\vec{\nabla} \times \vec{B}) \]

(A1)

With the quasi-neutrality condition Eq. (6), the electron velocity can be expressed as:

\[ n_e \vec{v}_e = n_i \vec{v}_i - \frac{\vec{J}}{e} \]

(A2)

Using Eq. (A2) and Eq. (5) to substitute \( \vec{v}_e \) and \( \vec{J} \) in Eq. (A1) respectively, the electron momentum equation becomes:

\[ E = -\frac{\vec{v}_i \times \vec{B}}{e} + \frac{1}{4\pi e n_c}(\vec{\nabla} \times \vec{B}) \times \vec{B} + \frac{c n_e}{4\pi}(\vec{\nabla} \times \vec{B}) \]

(A3)

Since the electron continuity equation is:

\[ \frac{\partial}{\partial t} n_e + \vec{\nabla} \cdot (n_e \vec{v}_e) = 0 \]  

(A5)

it can then be used to substitute the time derivative term \( \frac{\partial n_e}{\partial t} \) in Eq. (A4). Then we could use the ion momentum equation:

\[ \frac{\partial \vec{v}_i}{\partial t} = -(\vec{v}_i \cdot \vec{\nabla}) \vec{v}_i + \frac{e}{m_i} E + \frac{e\vec{v}_i \times \vec{B}}{cm_i} - \frac{e n_i}{m_i} \vec{J} \]

(A6)

to get rid of time derivative of ion flow velocity \( \frac{\partial v_i}{\partial t} \). Here \( n_i \) is the PIC ion mass and \( v_i \) is ion flow velocity. Finally, using Eq. (A2) to eliminate \( \vec{v}_i \), and substituting \( n_i \) back with \( n_i \), the final expression for electric field equation is:

\[ \frac{c^2}{4\pi}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E} + \frac{e^2 n_i(1 + \frac{m_e}{m_i})}{m_e} E \]

\[ = -\frac{n_i e^2(1 + \frac{m_e}{m_i})}{cm_e} (\vec{v}_i \times \vec{B}) + \frac{e}{4\pi m_e}(\vec{\nabla} \times \vec{B}) \times \vec{B} \]

\[ + \frac{ce^2 n_i \eta(1 + \frac{m_e}{m_i})}{4\pi m_e}(\vec{\nabla} \times \vec{B}) \]

\[ + \frac{c}{4\pi}(\vec{\nabla} \cdot \vec{v}_i)(\vec{\nabla} \times \vec{B}) - \frac{e^2}{16\pi^2 e n_i}(\vec{\nabla} \times \vec{B}) \cdot (\vec{\nabla} \times \vec{B}) \]

\[ + \frac{c}{4\pi}(\vec{\nabla} \times \vec{v}_i)(\vec{\nabla} \times \vec{B}) + \frac{e^2}{16\pi^2 e n_i^2}(\vec{\nabla} \times \vec{B}) \cdot (\vec{\nabla} \times \vec{B}) n_i \]

(A7)

Since \( \frac{m_e}{m_i} \ll 1 \) which indicates the term \( \frac{m_e}{m_i} \) can be neglected. Throwing all the small terms, Eq. (A7) then becomes Eq. (8) in section III. The advantage of Eq. (A7) is that it incorporates the finite electron inertia without containing an explicit time derivative.

However, since the dispersion relation requires \( \nabla \cdot \vec{E} = 0 \), it can not be assumed that \( \nabla \cdot \vec{E} = 0 \) in Eq. (A7). However, at the same time the quasi-neutral assumption which indicates \( \nabla \cdot \vec{E} = 4\pi e (n_i - n_e) \approx 0 \) must be maintained. Solving Eq. (A7) explicitly, numerical inaccuracy would be introduced into the simulation due to the direct violation of Poisson’s equation \( \nabla \cdot \vec{E} = 4\pi e (n_i - n_e) \). Thus, solving the field equation with the \( \nabla \cdot \vec{E} \) term must be avoided. The method presented here is to separate electric field into transverse and longitudinal components and take advantage of their identities.

\[ \cdot \vec{E} = \vec{E}_t + \vec{E}_l, \vec{\nabla} \cdot \vec{E}_t = 0, \vec{\nabla} \cdot \vec{E}_l = 0 \]

\[ \cdot \vec{E}_t \vec{\nabla} \cdot \vec{E}_t = \vec{\nabla} \cdot (\vec{\nabla} \times \vec{E}_t) \]

\[ = \vec{\nabla} \times (\vec{\nabla} \times \vec{E}_t) = \vec{\nabla} \times (\vec{\nabla} \times (\vec{E}_t + \vec{E}_l)) \]

\[ = \vec{\nabla} \vec{\nabla} \vec{\nabla} \vec{E}_t = \vec{\nabla} (\vec{\nabla} \cdot \vec{E}_t) - \nabla^2 \vec{E}_l \]

\[ = -\nabla^2 \vec{E}_t \]

(A8)

Applying the above identity to Eq. (A7), the transverse

The electron and ion momentum equations can be summed (assuming \(n_e = n_i\)) to obtain:

\[
\frac{\partial \mathbf{J}}{\partial t} + n_i (\mathbf{v}_i \cdot \nabla) \frac{\mathbf{J}}{n_i} + (\mathbf{J} \cdot \nabla) (\mathbf{v}_i - \frac{\mathbf{J}}{en_i}) + \frac{\mathbf{J}}{n_i} \cdot \nabla (n_i \mathbf{v}_i) = e^2 n_i (\frac{1}{m_i} + \frac{1}{m_e}) \mathbf{J}
\]

(A10)

Here \(\mathbf{J}\) is total current density, equal to \(en_i (\mathbf{v}_i - \mathbf{v}_e)\). Next \(\mathbf{E}\) is substituted with \(-\nabla \phi\), where \(\phi\) is a scalar potential, and then the divergence \(\nabla \cdot \mathbf{J}\) on both sides of Eq. (A10) is taken. Note that the divergence of the first left hand side term will then vanish as a result of quasi-neutrality \(n_e = n_i\). Using the same approach in the derivation of transverse component to get rid of electron velocity \(\mathbf{v}_e\), the longitudinal field equation which is written in terms of the electric scalar potential can be obtained as:

\[
\nabla^2 \phi + \frac{1}{en_i} (\nabla \mathbf{n}_i \cdot \nabla \phi) = \frac{1}{c} (\mathbf{B} \cdot (\nabla \times \mathbf{v}_i) - \mathbf{v}_i \cdot (\nabla \times \mathbf{B})) + \frac{1}{en_i} (\mathbf{v}_i \times \mathbf{B}) \cdot \nabla \mathbf{n}_i
\]

\[
+ \frac{c^2}{4 \pi e_m (1 + \frac{m_e}{m_i})} (\mathbf{B} \cdot \nabla \mathbf{v}_i + (\nabla \times \mathbf{B}) \cdot (\nabla \times \mathbf{v}_i)) - \frac{en_i}{4 \pi n_i} (\nabla \times \mathbf{B}) \cdot \nabla \mathbf{n}_i + \frac{1}{n_i} (\nabla \mathbf{n}_i \cdot \mathbf{E})
\]

\[
- \frac{en_i c}{4 \pi e_m (1 + \frac{m_e}{m_i})} (\mathbf{v}_i \cdot (\nabla \times \mathbf{B}) + (\nabla \times \mathbf{v}_i) \cdot (\nabla \times \mathbf{B}))
\]

\[
+ \frac{c^2 m_e}{16 \pi^2 e^2 n_i^3 (1 + \frac{m_e}{m_i})} (\nabla \times \mathbf{B}) \cdot (\mathbf{v}_i \times (\nabla \times \mathbf{B}))
\]

\[
- \frac{c^2 m_e}{16 \pi^2 e^2 n_i^3 (1 + \frac{m_e}{m_i})} ((\nabla \times \mathbf{B}) \cdot (\mathbf{v}_i \times (\nabla \times \mathbf{B})) + (\mathbf{v}_i \cdot (\nabla \times \mathbf{B})) (\nabla \times \mathbf{v}_i) - (\nabla \mathbf{n}_i \cdot (\nabla \times \mathbf{B})) (\nabla \times \mathbf{v}_i) + (\mathbf{v}_i \cdot (\nabla \times \mathbf{B})) (\nabla \times \mathbf{v}_i))
\]

\[
+ \frac{c^2 m_e}{8 \pi^2 e^2 n_i^3 (1 + \frac{m_e}{m_i})} ((\nabla \times \mathbf{B}) \cdot (\nabla \times \mathbf{n}_i))^2
\]

(A11)

Neglecting the terms proportional to \(m_e\), Eq. (A11) then becomes Eq. (10) in section III.