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Energy dissipation by whistler turbulence: Three-dimensional particle-in-cell simulations

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Three-dimensional particle-in-cell simulations of whistler turbulence are carried out on a collisionless, homogeneous, magnetized plasma model. The simulations use an initial ensemble of relatively long wavelength whistler modes and follow the temporal evolution of the fluctuations as they cascade into a broadband, anisotropic, turbulent spectrum at shorter wavelengths. For relatively small levels of the initial fluctuation energy \( \varepsilon_e \), linear collisionless damping provides most of the dissipation of the turbulence. But as \( \varepsilon_e \) and the total dissipation increase, linear damping becomes less important and, especially at \( \beta_e \ll 1 \), nonlinear processes become stronger.

The PDFs and kurtoses of the magnetic field increments in the simulations suggest that intermittency in whistler turbulence generally increases with increasing \( \varepsilon_e \) and \( \beta_e \). Correlation coefficient calculations imply that the current structure dissipation also increases with increasing \( \varepsilon_e \) and \( \beta_e \), and that the nonlinear dissipation processes in these simulations are primarily associated with regions of localized current structures. © 2014 Author(s). All article content, except where otherwise noted, is licensed under a Creative Commons Attribution 3.0 Unported License. [http://dx.doi.org/10.1063/1.4875728]

I. INTRODUCTION

An unsolved problem in plasma turbulence is the dissipation of fluctuation energy at short wavelengths. The dissipation of turbulence in neutral fluids is usually described in terms of a forward cascade to short wavelengths where collisional processes such as resistivity or viscosity convert fluctuating fluid energy into thermal energy. Turbulence in collisionless magnetized plasmas is also often described in terms of a forward cascade, although the processes which can contribute to dissipation at short wavelengths are much more diverse than in fluid turbulence. Such plasma dissipation processes include the linear damping of plasma modes (i.e., Landau and/or cyclotron damping1), various higher order dissipation processes (e.g., nonlinear Landau damping), and reconnection at current sheets in the plasma. 2

This manuscript describes three-dimensional (3D) particle-in-cell (PIC) simulations of whistler turbulence with particular attention to fluctuating field dissipation. Recent PIC simulations3,4 have begun with a large-scale sheared flow configuration subject to the Kelvin-Helmholtz instability; the consequent long-wavelength turbulence undergoes a forward cascade developing electron-scale current sheets which efficiently dissipate the fluctuation energy via associated parallel electric fields. Recent gyrokinetic kinetic Alfvén turbulence simulations5 driven by an oscillating antenna also show current sheet formation at electron gyroradius scales and suggest that the current structures are damped collisionlessly by resonant wave-particle interactions.

In contrast, we here follow References 5–8 and begin with a collisionless, homogeneous, magnetized plasma upon which is imposed an initial ensemble of relatively long wavelength whistler fluctuations. At early times, the whistler modes undergo a forward cascade to a broadband, anisotropic, turbulent spectrum at shorter wavelengths. At later times, the field fluctuations are dissipated and the plasma electrons are heated, with a temperature anisotropy \( T_T > T_\parallel \) where \( \parallel \) and \( \perp \) denote directions parallel and perpendicular, respectively, to the background magnetic field \( \mathbf{B}_0 = 2B_0 \). To study the physics of whistler turbulence dissipation as functions of the initial \( \beta_e \) and the initial fluctuation energy \( \varepsilon_e \), where \( \beta_e = 8\pi n_e k_B T_e / B_0^2 \) and \( \varepsilon_e = \sum_k |\delta \mathbf{B}(k_t)|^2 / 8\pi n_e k_B T_e, \) we organized simulation runs into two sets. In one set, we varied the initial \( \beta_e = 0.01, 0.1, 1.0 \) while fixing the initial fluctuation energy at \( \varepsilon_e = 2.0 \); in the other set, we varied \( \varepsilon_e = 0.5, 1.0, 2.0, 5.0 \) while fixing the initial \( \beta_e = 0.1 \). In certain diagnoses, we also include data from variable \( \varepsilon_e \) at three different \( \beta_e \). All other plasma and computational parameters are the same as those in Ref. 6.

We determine the total dissipation rate \( \Gamma_{\text{total}} \) from the simulations by computing the overall rate of change of fluctuating magnetic and electric field energy densities:

\[
\Gamma_{\text{total}} = \frac{d[\delta B^2(t)] + \delta E^2(t)]/8\pi K_{t=0}}{dt},
\]

where \( \delta B^2(t) = \sum_k |\delta \mathbf{B}(k, t)|^2 \) and \( \delta E^2(t) = \sum_k |\delta \mathbf{E}(k, t)|^2 \). The normalization factor is the initial electron kinetic energy density \( K_{t=0} \). The instantaneous \( \Gamma_{\text{total}} \) can be calculated from the slope of the curve of total fluctuating energy time history shown in Fig. 1 of Ref. 7 and Fig. 1 of Ref. 6. A small smoothing factor is used to calculate \( \Gamma_{\text{total}} \), i.e., an

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average over every 6 electron plasma periods to reduce the noise.

We also define the linear damping rate of the fluctuating fields as \( \Gamma_{\text{linear}} \). Linear theory assumes the fluctuation amplitude is sufficiently small that a normal mode of the plasma is proportional to \( \exp(\mathbf{k} \cdot \mathbf{x} - i \omega t) \) where \( \omega = \omega_r + i \gamma \) so that
\[
|\delta \mathbf{B}(\mathbf{k}, t)|^2 = \delta^2 \mathbf{B}(\mathbf{k}) \exp(2i\gamma t)
\]
where \( \gamma \) is the imaginary part of the frequency computed from linear kinetic dispersion theory [e.g., Ref. 1] based on plasma conditions at \( t = 0 \). Therefore, the instantaneous \( \Gamma_{\text{linear}} \) can be approximated by an integral over all wavevectors \( \mathbf{k} \):
\[
\Gamma_{\text{linear}} \equiv \frac{1}{\mathcal{V}} \int \frac{2\gamma(\mathbf{k})}{\mathcal{V}} |\delta \mathbf{B}(\mathbf{k})|^2 + |\delta \mathbf{E}(\mathbf{k})|^2 d\mathbf{k}/8\pi K_c t = 0.
\] (2)

The integral is computed as \( \sum \frac{2\gamma(\mathbf{k})}{\mathcal{V}} |\delta \mathbf{B}(\mathbf{k})|^2 + |\delta \mathbf{E}(\mathbf{k})|^2 /8\pi K_c t = 0 \) on the computational grid.

The fluctuating magnetic field increment as a function of spatial separation length \( r \) is \( \delta_r \mathbf{B}_i = \mathbf{B}_i(r - r) - \mathbf{B}_i(j) \), and \( i \) and \( j \) are either \( x \), \( y \), or \( z \). The kurtosis of field increments is then defined as \( \langle \sigma^4 \rangle \):
\[
\langle \sigma^4 \rangle \equiv \frac{\langle \delta_r B_i \rangle^4}{\sigma^4},
\] (3)

where \( \sigma_i \equiv \langle |\delta_r B_i |^2 \rangle^{1/2} \) is the dimensional standard deviation and \( \langle \cdot \rangle \cdot \cdot \cdot \) denotes an ensemble average over three dimensional space. A Gaussian distribution of field increments yields a kurtosis of 3; departures from a Gaussian distribution correspond to increasing values of the kurtosis and are a measure of what has been called turbulent intermittency.

The correlation coefficient of two quantities \( X \) and \( Y \) is denoted as
\[
\rho(X, Y) = \frac{\text{cov}(X, Y)}{\sigma_X \sigma_Y},
\] (4)

where \( \text{cov} \) denotes covariance, and \( \sigma \) is the standard deviation. The correlation coefficient is a normalized value between \(-1\) and \(1\). Values close to 1 indicate that there is a positive linear relationship between \( X \) and \( Y \), while values close to \(-1\) indicate a negative linear relationship. Values close to or equal to 0 suggest there is no linear relationship between the two variables.

II. PARTICLE-IN-CELL SIMULATIONS

First, we quantify the fluctuating field energy dissipation contributed from linear damping and compare that against the total dissipation in the simulations. Figure 1 compares the total damping rate \( \Gamma_{\text{total}} \) (solid lines) versus the linear damping rate \( \Gamma_{\text{linear}} \) (dashed lines) for each of our simulations. The difference between the total and linear damping rates corresponds by definition to the dissipation due to nonlinear mechanisms which may come from various higher order wave-particle processes and/or reconnection. At early times of some cases, the linear damping rate computed from Eq. (2) is larger than the total dissipation rate computed from Eq. (1). A possible explanation for this unphysical result may be that the phases of the initial fluctuations in each simulation have been chosen at random and are therefore not fully self-consistent. We assume that nonlinear interactions eventually produce appropriate phase angles and yield a fully self-consistent ensemble of turbulent field fluctuations as the simulations evolve. We believe that the demonstration of clearly turbulent properties at late times in our simulations as described in our earlier papers, as well as the consistent result of \( \Gamma_{\text{linear}} \leq \Gamma_{\text{total}} \) at late times in Figs. 1(a) and 1(b), support the appropriateness of this assumption.

Figure 1(a) illustrates the time histories of \( \Gamma_{\text{total}} \) and \( \Gamma_{\text{linear}} \) for three different values of \( \beta_e \) at \( \epsilon_e = 2.0 \). For this value of \( \epsilon_e \), linear damping accounts for most of the dissipation at both \( \beta_e = 1.0 \) and \( \beta_e = 0.1 \), whereas nonlinear processes clearly dominate \( \Gamma_{\text{total}} \) at \( \beta_e = 0.01 \).

Figure 1(b) shows the time histories of \( \Gamma_{\text{total}} \) and \( \Gamma_{\text{linear}} \) for four different values of \( \epsilon_e \) at \( \beta_e = 0.1 \). For this value of \( \beta_e \), it is clear that the nonlinear processes contribute a monotonically increasing percentage of the total dissipation as the initial fluctuation energy increases.

Figure 1(c) compares \( \Gamma_{\text{total}} \) and \( \Gamma_{\text{linear}} \) as functions of both \( \epsilon_e \) and \( \beta_e \) at relatively early times. For each value of \( \beta_e \), the ratio \( \Gamma_{\text{linear}}/\Gamma_{\text{total}} \) diminishes as \( \epsilon_e \) increases, indicating the increasing importance of nonlinear processes. In contrast, for fixed \( \epsilon_e \), \( \Gamma_{\text{linear}}/\Gamma_{\text{total}} \) increases as \( \beta_e \) becomes larger, consistent with the prediction of linear theory that damping of whistler fluctuations monotonically increases with increasing \( \beta_e \) as in Fig. 8 of Ref. 6. For example, at \( \epsilon_e = 5.0 \), for \( \beta_e = 0.01 \), \( \Gamma_{\text{linear}}/\Gamma_{\text{total}} \approx 2\% \); for \( \beta_e = 0.1 \), \( \Gamma_{\text{linear}}/\Gamma_{\text{total}} \approx 5\% \); and for \( \beta_e = 1.0 \), \( \Gamma_{\text{linear}}/\Gamma_{\text{total}} \approx 77\% \). In summary, for relatively small levels of the initial fluctuation energy, linear collisionless damping provides most of the turbulent dissipation. But as \( \epsilon_e \) and the total dissipation increase, linear damping becomes less important and, especially at \( \beta_e \approx 1 \), nonlinear processes become stronger.

We next investigate intermittency in our simulations. Intermittency can be framed in terms of the probability density function (PDF) of the fluctuating magnetic field increments \( \delta_r \mathbf{B}_i \) as functions of spatial separation length \( r \) with greater departures from a Gaussian distribution corresponding to a more intermittent ensemble of fluctuations. Figure 2 shows the normalized PDFs for \( r = 0.01 \lambda_e, 1.0 \lambda_e, 2.5 \lambda_e, 5.0 \lambda_e, 10 \lambda_e, 25 \lambda_e \), as labeled from the \( \beta_e = 0.1 \) and \( \epsilon_e = 2.0 \) simulation at \( |\Omega_r| = 134.2 \), where \( \lambda_e = c/\omega_e \) is the electron inertial length, \( \omega_e \) is the electron plasma frequency, and \( \Omega_r \) is the electron cyclotron frequency. Panel (a) illustrates PDFs for one of the perpendicular magnetic field increments \( \delta_r \mathbf{B}_x \) whereas Panel (b) illustrates PDFs for the parallel magnetic field increments \( \delta_r \mathbf{B}_x \). As the spatial separation \( r_y \) is decreased, both PDFs become increasingly non-Gaussian, especially in the tails of the PDFs. However, there is a noticeable anisotropy between the PDFs of the perpendicular and parallel field increments. PDFs of \( \delta_r \mathbf{B}_x \) have much larger non-Gaussian tails than those of \( \delta_r \mathbf{B}_x \), indicating stronger intermittency in the \( \delta B_x \) component. This anisotropy could be explained by the fact that whistler turbulence cascades lead to strongly oblique propagation, which results in more contribution from \( \delta B_x \) than \( \delta B_y \), in the probability...
density function calculation. Reference 9 finds that the PDFs of $\delta B_L$ and $\delta B_\parallel$ are similar for kinetic Alfvén waves. Therefore, this difference might help in distinguishing between whistlers and kinetic Alfvén waves in short-wavelength turbulence.

The PDFs in Figure 2 have enhanced tails at large increments, but fit well to Gaussians at small increments. This is in contrast to the PDFs of Refs. 2, 9, and 10, which show strong departures from Gaussians at both large and small increments. We believe that the enhanced PDF tails are due to the intermittency in short-wavelength turbulence, but that the steepening of the PDFs at small increments is due to the presence of large-scale inhomogeneities. When summing over Gaussians with somewhat different standard deviations, a so-called Castaing distribution is formed, which usually has a more peaked distribution at small increments. This implies that observations and simulated turbulence driven by long-wavelength instabilities may sum over many different small-scale measurements in inhomogeneous plasmas, so that their PDFs represent several different local Gaussians which result in the more peaked distributions at small increments, whereas Figure 2’s relatively Gaussian PDFs at small increments confirms that our simulations are carried out on a homogeneous medium. Similar PDF characteristics with non-Gaussian tails but Gaussian-like shapes at small increments are reported in Refs. 12 and 13.

Another quantity which can be an indicator of intermittency and current structures is kurtosis as defined in Eq. (3). Figure 3 shows results for several different values of $\epsilon_e$ for three cases: the kurtoses of $\delta_x B_z$ as functions of $r_x$ at $\beta_e = 0.01$; the kurtoses of $\delta_y B_z$ as functions of $r_y$ at $\beta_e = 0.10$; and the kurtoses of $\delta_y B_z$ as functions of $r_y$ at $\beta_e = 1.0$. In most cases, the kurtosis grows from near-Gaussian values at scales larger than 10$\lambda_e$ to strongly non-Gaussian values near and below electron scales. The kurtosis increases with increasing $\epsilon_e$ and $\beta_e$ and also generally increases with decreasing spatial separation length $r$ until reaching constant values at $r < \lambda_e$. This behavior clearly signifies the statistical intermittency of the turbulence which shows the same trend as the current and electron pressure correlation coefficient $\rho(n, T_e, E)$. The noticeable jump of the kurtosis values near $\lambda_e$ between $\epsilon_e = 5.0$ cases and other lower $\epsilon_e$ cases at $\beta_e = 0.01$ and 0.1 indicates the point when intermittent current structures become relatively dominant in these whistler turbulence simulations. Note that the kurtosis values in our simulations are much smaller than those in the recent large-scale
two-dimensional (2D) PIC simulation of sheared-flow-driven turbulence\(^3\) which yields values up to 10.5. This indicates again that our simulations remain relatively homogeneous.

We also examined whether current structures are formed in our simulations by plotting parallel current density \(J_k\) in a \(x\)-\(y\) plane with spatial band-pass filters applied in a similar fashion as in Fig. 2 of Ref. 4. These figures in Ref. 7 (Figs. 6.2–6.4) suggest that current structures form primarily at \(j_k < k_0/C_21\) for all three \(\beta_e = 0.01, 0.1, 1.0\) runs. The current structures in our simulations form self-consistently from the cascade of decaying turbulence without any driving force and are not seeded or otherwise initialized. Therefore, we conclude that electron-scale current structures form as a natural consequence of whistler turbulence, even in weakly perturbed, relatively homogeneous plasmas such as these 3D PIC simulations.

To investigate the relationship between energy dissipation associated with current structures and the total electron heating measured in the simulation, we compute the correlation coefficient, \(\rho(n_e T_e, J^2)\), between the current density squared \(J^2\) and the electron pressure \(n_e T_e\) in the simulations as functions of both \(\beta_e\) and \(\epsilon_e\). The values are listed in Table I. Note that, in general, \(\rho(n_e T_e, J^2)\) does not reflect Landau/cyclotron damping’s correlation with electron pressure. The current density \(J\) on its own does not necessarily correspond to Landau damping, because in the microscopic picture, \(J\) means a velocity shift of all the electrons relative to the ions.

Table I shows that an increase in the initial fluctuation energy density translates into a monotonic increase in the coefficient \(\rho(n_e T_e, J^2)\). The increasing correlation between current density and electron pressure with increasing \(\epsilon_e\) is consistent with the hypothesis that nonlinear processes in whistler turbulence play increasing roles in energy dissipation with increasing \(\epsilon_e\) as shown in Figure 1(c). Furthermore, two-dimensional (2D) PIC simulation of sheared-flow-driven turbulence\(^3\) which yields values up to 10.5. This indicates again that our simulations remain relatively homogeneous.

![PDF of fluctuating magnetic field increments](image1)

**FIG. 2.** Normalized PDFs of fluctuating magnetic field increments from the simulation starting with \(\beta_e = 0.1\) and \(\epsilon_e = 2.0\) at \(|\Omega| t = 134.2\) for different spatial separation lengths \(r\) as labeled. (a) A perpendicular component \(\delta_r B_x\); (b) the parallel component \(\delta_r B_z\). Here, the PDFs are normalized to the standard deviation of their respective magnetic field increments. The black circled lines indicate Gaussian distributions.

![PDF of fluctuating magnetic field increments](image2)

**TABLE I.** Correlation coefficients between \(n_e T_e\) and \(J^2\). All grid points in the 3D simulation space are taken into account.

<table>
<thead>
<tr>
<th>(\rho(n_e T_e, J^2))</th>
<th>(\beta_e = 0.01)</th>
<th>(\beta_e = 0.1)</th>
<th>(\beta_e = 1.0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\epsilon_e = 1.0)</td>
<td>0.138</td>
<td>0.245</td>
<td>0.326</td>
</tr>
<tr>
<td>(\epsilon_e = 2.0)</td>
<td>0.219</td>
<td>0.304</td>
<td>0.336</td>
</tr>
<tr>
<td>(\epsilon_e = 5.0)</td>
<td>0.247</td>
<td>0.368</td>
<td>0.412</td>
</tr>
</tbody>
</table>

![Kurtoses of the magnetic field increments](image3)

**FIG. 3.** Kurtoses of the magnetic field increments \(\delta_r B_i\) as functions of spatial separation length \(r\) for different \(\epsilon_e\) and \(\beta_e\) as labeled. The left panel is from \(\beta_e = 0.01\) cases at \(|\Omega| t = 707.1\); the middle panel is from \(\beta_e = 0.1\) cases at \(|\Omega| t = 134.12\); and the right panel is from \(\beta_e = 1.0\) cases at \(|\Omega| t = 35.4\). Data are low pass filtered at \(k_{x/y/z} = 5\) to remove the statistical noise due to finite superparticle number in the PIC simulations.
this correlation coefficient increases with increasing $\beta_e$. In the limit of zero $\beta_e$, the magnetic field is rigid and there are no current sheets. As $\beta_e$ increases, the magnetic field becomes weaker and the plasma currents act to change the magnetic field, forming current structures. Therefore, current structure formation and dissipation should increase with increasing $\beta_e$, at least for $\beta_e \leq 1$.

Figure 4 shows the spatial contours of $j_j^2$, $n_e T_e$ and $\delta E^2$ from two typical cases, that is, the upper panels represent $\beta_e = 0.1$ and $\epsilon_e = 2.0$ case at $|\Omega_e| \tau = 134.2$, and the lower panels represent $\beta_e = 1.0$ and $\epsilon_e = 5.0$ case at $|\Omega_e| \tau = 14.1$. The relatively strong correlation between $j_j^2$ and $n_e T_e$ indicates that current structure formation is playing an important role in whistler turbulence simulations here. The spatial structures of $j_j^2$ resemble the electron pressure $n_e T_e$ structures shown in Figure 4, providing a qualitative evidence of dissipation in regions of coherent intermittent structures.

The relative contribution to electron heating from linear damping could be studied by calculating the coefficient $\rho(n_e T_e, \delta E^2)$. For Landau/cyclotron damping to be involved, there must be a microscopic quantity involved, such as $\delta E_z$ or $\delta E$. We note that $\rho(n_e T_e, \delta E^2)$ does not uniquely represent the correlation between linear damping and electron heating, but just involves linear dissipation effects. The correlation coefficient ratio $\frac{\rho(n_e T_e, j_j^2)}{\rho(n_e T_e, \delta E^2)}$ implies the relative contribution between current structure dissipation and linear damping. In Figure 4, the ratio from $\beta_e = 0.1$ case is 4.85, whereas $\frac{\rho(n_e T_e, j_j^2)}{\rho(n_e T_e, \delta E^2)}$ from $\beta_e = 1.0$ case is 1.92, as listed in Table II. The ratio in Table II decreases with increasing $\beta_e$, indicating the relatively smaller importance of current structure dissipation and greater importance of linear damping in higher $\beta_e$ plasmas, and this trend is consistent with that of Figure 1(c).

### III. CONCLUSIONS

Using results from the three-dimensional PIC simulations of Refs. 6 and 8 we have for the first time examined dissipation properties of the forward cascade of homogeneous whistler turbulence in a collisionless, magnetized plasma. By computing the ratio of dissipation rates $\Gamma_{linear}/\Gamma_{total}$, we find that linear damping constitutes the majority of the total dissipation at sufficiently small initial amplitudes, that nonlinear processes increase in relative importance as $\epsilon_e$ increases, and that for fixed $\epsilon_e$ linear damping increases as $\beta_e$ becomes larger, consistent with the predictions of linear dispersion theory. These conclusions are consistent with the strongly driven results of Ref. 2, who found current sheet heating to be several orders of magnitude more efficient than damping by kinetic Alfvén waves, and with the less strongly driven results of Ref. 4, who concluded that
collisionless damping of kinetic Alfvén waves is sufficient to account for all of the heating in their simulations. Reference 2 indicates that electron heating from reconnection should be parallel to \( \mathbf{B}_0 \) and scale as \( 1/\beta_e^{1/2} \). Our PIC simulations show that electron heating from whistler turbulence is also primarily parallel to \( \mathbf{B}_0 \) [Gary et al., 2012, Fig. 1(b), and Chang et al., 2013, Fig. 4(b)] and that this parallel heating scales as a weaker inverse power of \( \beta_e \). So there is a consistent trend between the electron heating results of our simulations and those of Ref. 2, although this consistency does not necessarily demonstrate that reconnection plays a role in whistler turbulence.

We further quantify the intermittency in our simulations by showing that the PDFs of magnetic field increments depart more strongly from Gaussian distributions as the separation length decreases. The PDFs at short spatial separations are anisotropic between \( \delta \mathbf{B}_\perp \) and \( \delta \mathbf{B}_\parallel \) with the latter having much larger values in the tails of the distribution, which is probably due to the strongly oblique propagation induced by the whistler cascade. The kurtosis of the magnetic field increments similarly increases from the value of three, corresponding to a Gaussian PDF at much larger than electron scales, to non-Gaussian values ranging from 3.5 \textasciitilde 5.5 near and below electron scale lengths. Furthermore, the kurtosis in whistler turbulence generally increases with increasing \( \epsilon_e \) and \( \beta_e \).

The correlation coefficient \( \rho(n,T_e,J^2) \) shows monotonic increases as either \( \epsilon_e \) or \( \beta_e \) increase, showing that current structure dissipation is positively correlated with both of these parameters. We estimate the relative contribution to the overall electron heating from current dissipation and linear damping by computing the correlation coefficient ratio \( \frac{\rho(n,T_e,J^2)}{\rho(n,T_e,\delta \mathbf{B}_\parallel \delta \mathbf{B}_\parallel)} \). The ratio decreases with increasing \( \beta_e \), implying a lesser importance of current dissipation and a greater importance of linear damping in higher \( \beta_e \) plasmas. Because the relative contribution from linear damping generally increases with \( \beta_e \) and decreases with \( \epsilon_e \), the decreasing ratio \( \frac{\rho(n,T_e,J^2)}{\rho(n,T_e,\delta \mathbf{B}_\parallel \delta \mathbf{B}_\parallel)} \) with increasing \( \beta_e \) and \( \epsilon_e \) also suggests that the relative contribution between current structure dissipation and linear damping is ultimately determined by \( \beta_e \) rather than \( \epsilon_e \). These properties maintain a consistent correlation with the trends of the relative contributions from linear and nonlinear dissipation described in Figure 1. Therefore, we conclude that the nonlinear dissipation processes involved in whistler turbulence are primarily at current structures.

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