Whistler turbulence at variable electron beta: Three-dimensional particle-in-cell simulations

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Three-dimensional particle-in-cell (PIC) simulations of whistler turbulence at three different initial values of $\beta_e$ are carried out on a collisionless, homogeneous, magnetized plasma model. The simulations begin with an initial ensemble of relatively long-wavelength whistler modes and follow the temporal evolution of the fluctuations as wave-wave interactions lead to a forward cascade into a broadband, turbulent spectrum at shorter wavelengths with a wave vector anisotropy in the sense of $k_\perp \gg k_\parallel$. Here $\perp$ and $\parallel$ denote directions perpendicular and parallel to the background magnetic field, respectively. In addition, wave-particle interactions lead to fluctuating field dissipation and electron heating with a temperature anisotropy in the sense of $T_\parallel > T_\perp$. At early times, the wave-wave cascade dominates energy transport, whereas wave-particle Landau damping dominates at late simulation times. Larger values of $\beta_e$ correspond to a faster forward cascade in wave number and to a faster rate of electron heating, as well as to a less anisotropic wave vector distribution and to a less anisotropic electron velocity distribution.

1. Introduction

We define plasma turbulence as a broadband ensemble of incoherent field fluctuations in an ionized medium. The solar wind is an almost collisionless, magnetized plasma which bears magnetic and electric field fluctuation spectra that exhibit turbulent properties over several orders of magnitude in the observed frequency $f$. From $f \sim 10^{-4}$ Hz down to $f \sim 0.2$–$0.5$ Hz, magnetic spectra are observed to scale approximately as $f^{-5/3}$; this is called the “inertial range” of solar wind turbulence. Near 0.2–0.5 Hz, observations of magnetic fluctuation spectra show a break to a steeper frequency dependence, implying a change in the plasma physics from that of the inertial range [Smith et al., 2006, and references therein].

It is generally agreed that kinetic Alfvén waves are likely the predominant constituent of solar wind turbulence at wavelengths immediately shorter than those of the inertial range spectral break [Leamon et al., 1998; Bale et al., 2005; Sahraoui et al., 2010; He et al., 2012; Salem et al., 2012; TenBarge et al., 2012]. At still shorter wavelengths, of order of the electron inertial length and/or the thermal electron gyroradius, magnetic fluctuations show another change to still steeper spectra. These very short wavelength spectra have been fit with both a power law in frequency [Sahraoui et al., 2009, 2010] and an exponential wave number dependence [Alexandrova et al., 2009, 2012]. There is currently substantial debate about the properties of the very short wavelength turbulence which extends down to electron scales [Smith et al., 2012]. Some have suggested that kinetic Alfvén waves remain the primary constituent of this electron-scale turbulence [Howes et al., 2008, 2011; Sahraoui et al., 2009; Kiyani et al., 2013]. An alternative point of view is that solar wind turbulence near electron scales primarily consists of broadband whistler fluctuations [Ghosh et al., 1996; Stawicki et al., 2001; Krishan and Mahajan, 2004; Galtier, 2006; Gary et al., 2008; Alexandrova et al., 2008; Shaikh and Zank, 2009]. This manuscript describes new computer simulations of whistler turbulence, providing fresh information to help determine whether whistler fluctuations contribute substantially to electron-scale turbulence in the solar wind.

Recent particle-in-cell (PIC) simulations, which compute velocity-space dynamics of ion and electrons, have considered the evolution of whistler turbulence in homogeneous, collisionless, magnetized plasmas. Gary et al. [2008], Saito et al. [2008, 2010], and Saito and Gary [2012] describe two-dimensional (2-D) PIC simulations of whistler turbulence in which the background magnetic field $B_o$ lies in the simulation plane. Svidzinski et al. [2009] carried out 2-D PIC simulations of longer wavelength magnetosonic turbulence, a likely source of whistler turbulence, also with $B_o$ in the simulation plane. Ganguli et al. [2010] carried out two-dimensional simulations of whistler turbulence in which $B_o$ lies at an angle to the simulation plane. Fully three-dimensional (3-D) PIC simulations of whistler turbulence have been described by Chang et al. [2011] and Gary et al. [2012].
In many of these whistler simulations, a spectrum of relatively long-wavelength waves ($0.10 < k_c/\omega_e < 0.50$) is imposed as an initial condition, and the fluctuations are allowed to freely decay in time. These simulations exhibit the development of a broadband spectrum of turbulence via a forward cascade, in which relatively long-wavelength fluctuations transfer their energy via nonlinear processes to shorter wavelength modes. Furthermore, the short wavelength fluctuations preferentially develop quasi-perpendicular propagation, that is, with $k_\perp \gg k_\parallel$. Here the subscripts refer to directions relative to $B_0$. The 2-D simulations of Saito and Gary [2012] have been conducted for different values of initial $\beta_e$ and show that the wave vector anisotropy (defined in Appendix A) decreases with increasing $\beta_e$.

The 3-D PIC computations of Chang et al. [2011] and Gary et al. [2012] were carried out only with initial $\beta_e = 0.10$. Here we describe 3-D simulations of the forward cascade of whistler turbulence for three separate cases: the initial conditions of $\beta_e = 0.01$, 0.10, and 1.0 with the latter case representing the condition most typical of solar wind plasmas. This parametric variation embraces a very broad range of plasma physics effects; here we describe consequent variations in the forward cascade rate, the wave vector anisotropy, the electron heating rate, and the electron anisotropy.

We define forward cascade to mean a transfer of fluctuating magnetic field energy from longer to shorter wavelengths, that is, from smaller $k$ values to larger $k$ values. We choose our initial spectrum of whistler waves to have the longest wavelengths that fit into the simulation box; therefore, by construction, the cascade processes here are necessarily in the forward direction.

2. Particle-in-Cell Simulations

Our simulations use the three-dimensional electromagnetic particle-in-cell (3-D-EMPIC) code derived from the computations described by Wang et al. [1995]. In this code, plasma particles are pushed using a standard relativistic particle algorithm; currents are deposited using a rigorous charge conservation scheme [Villasenor and Buneman, 1992]; and the self-consistent electromagnetic field is solved using a local finite difference time domain solution to the full Maxwell’s equations.

For all our simulations, the plasma is collisionless and homogeneous with periodic boundary conditions. The initial plasma conditions are mostly the same as those used in Chang et al. [2011] and Gary et al. [2012]: $m_p/m_e = 1836$, $T_e = T_p$, and $v_e^2/c^2 = 0.01$. To study the consequences of $\beta_e$ variations, we vary the magnitude of $B_0$ to obtain three cases: $\omega_e^2/\Omega_e^2 = 50$ implying $\beta_e = 1.0$ [Saito and Gary, 2012], $\omega_e^2/\Omega_e^2 = 5$ implying $\beta_e = 0.10$ [Chang et al., 2011; Gary et al., 2012], and $\omega_e^2/\Omega_e^2 = 0.5$ implying $\beta_e = 0.01$. The computational parameters are also the same as in our previous 3-D simulations; that is, the grid spacing is $\Delta = 0.10c/\omega_e$, where $c/\omega_e$ is the electron inertial length, the time step is $\delta t = 0.05$, and the number of superparticles per cell is 64. The system has a spatial length of $51.2c/\omega_e$ in each direction.
Figure 2. Reduced magnetic fluctuation energy spectra, $\log[|\delta\mathbf{B}(k_x, k_z)|^2/8\pi n_e k_B T_e(t = 0)]$, at four times as labeled for the three simulations with initial values of $\beta_e$ as labeled.

[10] The wave vectors of the initial spectrum of whistler fluctuations imposed upon the system are, again, those used in Chang et al. [2011] and Gary et al. [2012]: There are 42 modes in the $y$-$z$ plane corresponding to $k_x/c/\omega_e = 0.1227 n_x$ ($n_x = 0, \pm 1, \pm 2, \pm 3$) and $k_z/c/\omega_e = 0.1227 n_z$ ($n_z = \pm 1, \pm 2, \pm 3$); 36 modes in the $x$-$z$ plane corresponding to $k_x/c/\omega_e = 0.1227 n_x$ ($n_x = \pm 1, \pm 2, \pm 3$) and $k_z/c/\omega_e = 0.1227 n_z$ ($n_z = \pm 1, \pm 2, \pm 3$); and an additional 72 modes corresponding to rotations of the $x$-$z$ modes through $45^\circ$ and $135^\circ$ about the $z$-axis. Of course, these wave vectors normalized in terms of the electron gyroradius vary with $\beta_e$ because $k_x/c/\omega_e = \sqrt{2/\beta_e k_v e/|B_e|}$.

[11] As in our earlier PIC simulations of whistler turbulence, each initial mode is given the same amplitude, and $\beta_e$ is increased through a decrease in $B_o$. However, in the 2-D simulations of Saito and Gary [2012], the dimensionless parameter characterizing the initial magnetic fluctuation energy was $\epsilon_o$, defined by $\Sigma_k |\delta\mathbf{B}(t = 0)|^2/8\pi = \epsilon_o B_o^2/8\pi$. At constant $\epsilon_o$, an increase in $\beta_e$ due to a reduction of $B_o$ implies a decrease in the initial dimensional magnetic fluctuation energy; as a consequence, the dimensional electron heating rate necessarily decreases, as shown by the 2-D simulations of Saito and Gary [2012]. Here we take a different approach, which we believe is a better way to compare results of turbulent heating due to $\beta_e$ variations. We choose the initial dimensional magnetic fluctuation energy density to be fixed as $B_o$ and $\beta_e$ are varied. Thus, we define $\epsilon_e$ by $\Sigma_k |\delta\mathbf{B}(t = 0)|^2/8\pi = \epsilon_e n_e k_B T_e(t = 0)$. For the simulations described here, $\epsilon_o = 2.0$, so if $\beta_e = 0.01$, then $\epsilon_e = 0.02$; if $\beta_e = 0.10$, then $\epsilon_e = 0.20$; and if $\beta_e = 1.0$, then $\epsilon_e = 2.0$.

[12] For all three of our runs, the initial ensemble of relatively long-wavelength whistlers at $0.12 \leq k_x/c/\omega_e \leq 0.36$, $0.12 \leq k_x/c/\omega_e \leq 0.36$, and $0.12 \leq k_x/c/\omega_e \leq 0.36$ drives a forward cascade of fluctuating field energy toward shorter wavelengths which, as shown in Figure 2 of Chang et al. [2011], has a gyrotrropic wave vector distribution. Figure 1a shows the total energy density (dashed lines) and the total magnetic fluctuation energy density (solid lines); Figure 1b illustrates the cascaded magnetic fluctuation energy (which corresponds to wave numbers satisfying $0.65 \leq k_x/c/\omega_e \leq 3.0$) as functions of time. Figure 1a demonstrates that the total energy is well conserved in each of the simulations, that the total fluctuating field energy is dissipated with time as in Figure 1a of Gary et al. [2012], and that the rate of dissipation increases dramatically with increasing initial $\beta_e$. The total energy variation is less than 0.4% throughout each of the three simulations.

[13] Figure 1b illustrates the cascaded fluctuating magnetic field energy, showing that there are two distinct stages for each simulation. During early times, wave-wave interactions carry magnetic fluctuation energy to shorter wavelengths, where there is initially only a low level of thermal fluctuations present. The rate of forward cascade in wave number is a sensitive function of $\beta_e$, increasing rapidly with this parameter. The early-time stage ends at the maximum value of the cascaded energy; at this time, the overall damping rate becomes faster than the overall cascade rate so that dissipation dominates the late-time stage of the simulations, as indicated by the temporal decrease in the cascaded magnetic energy density.

[14] Figure 2 illustrates representative two-dimensional reduced spectra from all three runs at several times. The three left-hand panels of this figure show relatively early-time spectra with successively decreasing wave vector...
anisotropies (in the sense of $k_{\perp} > k_{||}$) with increasing $\beta_e$. In section 3, we argue that wave-wave coupling of three whistlers should yield a relatively isotropic spectrum at $\beta_e = 1.0$, whereas wave-wave interactions at smaller values of $\beta_e$ should yield more anisotropic spectra in the sense of $k_{\perp} \gg k_{||}$, consistent with these three panels. The other nine panels of Figure 2 show that the wave vector anisotropies continue to increase during those times when dissipation dominates.

[15] Figure 3 [see also Figure 2 of Gary et al., 2012] shows how the wave vector anisotropy of the cascaded spectrum varies in time. These figures imply two important properties of the wave vector anisotropy. First, increasing $\beta_e$ from 0.10 to 1.0 makes the late-time wave vector distribution more nearly isotropic; Figure 3 from the 2-D PIC simulations of Saito and Gary [2012] shows a similar trend. Second, increasing $\beta_e$ from 0.01 to 1.0 increases the early-time rate of wave vector anisotropy development. Comparison of Figures 1 and 3 shows the clear correlation between the electron scattering (i.e., dissipation) time and the wave vector anisotropy development time. In other words, both wave-wave and wave-particle interactions contribute to the development of the wave vector anisotropy here.

[16] Figure 4a illustrates the electron kinetic energy density as a function of time for each run; as with the field dissipation, the electron heating rate increases with $\beta_e$, although the asymptotic late-time electron kinetic energy density is approximately the same in all three cases. Figure 4b demonstrates the electron kinetic energy anisotropy as a function of time for the three runs; here the late-time anisotropy decreases with increasing $\beta_e$, the same general trend as exhibited by the late-time wave vector anisotropy. An electron anisotropy which decreases with increasing $\beta_e$ was also demonstrated in the Saito and Gary [2012] 2-D PIC simulations of whistler turbulence (see Figure 5 of that reference).

[17] Figure 5 illustrates the late-time reduced electron velocity distributions $f_e(v_{||})$ and $f_e(v_{\perp})$. As in Figure 4b, $T_{\parallel} > T_{\perp}$ for each value of $\beta_e$, implying that the primary channel of energy transfer from whistler fluctuations to the electrons is via Landau damping. The same $T_{\parallel} > T_{\perp}$ anisotropy was demonstrated by the 2-D whistler turbulence simulations of Saito and Gary [2012] (see their Figure 4).

[18] Our interpretation of Figure 5 is based on our solutions of the full kinetic linear dispersion equation, as illustrated in the linear theory plots of section 3 below. At $\beta_e = 0.01$, the Landau resonance factor $\xi_e \equiv \omega/\sqrt{2k_{||}v_e}$ is much greater than unity over $0.3 \leq k_{||}c/\omega_e$, and whistler waves resonate only with the few particles on the tail of the electron velocity distribution (e.g., the nonresonant phase speed of Figure 2.1 in Gary [1993]). Thus, our $\beta_e = 0.01$ simulation shows that it is these fast electrons which exhibit the strongest heating, forming a broad tail on both the $v_{\perp}$ and especially on the $v_{||}$ reduced distributions. The whistler anisotropy instability driven by an electron temperature anisotropy $T_{\perp} > T_{\parallel}$ yields similar enhanced tails on $f_e(v_{||})$ at very low $\beta_e$ [Gary et al., 2011].

[19] As $\beta_e$ increases from 0.01 to 1.0, our solutions of the dispersion equation show a relatively weak change in the electron thermal speed, so that $\xi_e$ typically becomes of order of or less than unity, resonating with the large number of electrons in the thermal and subthermal parts of $f_e(v_{||})$ (e.g., the resonant phase speed of Figure 2.1 in Gary [1993]). Thus, our $\beta_e = 0.10$ and 1.0 simulations show electron heating concentrated in the thermal part of the electron velocity distribution, so that especially at $\beta_e = 0.10$, wave-particle interactions lead to a flattening of the subthermal part of $f_e(v_{||})$, as pointed out by Mithaiwala et al. [2012]. Note, however, our results differ from those of Mithaiwala et al. [2012] in two potentially important ways. First, Figure 5 shows that the flattening of $f_e(v_{||})$ is relatively strong only at relatively large values of $\beta_e$. Second, although Figure 1 of Mithaiwala et al. [2012] shows no significant electron heating at $v \geq v_e$, our simulations demonstrate substantial electron energy gain at thermal and suprathermal electron energies.

[20] Whistler turbulence not only undergoes a strong cascade toward shorter wavelengths and anisotropic wave vectors but it is also subject to the strong dispersion of the whistler mode. The combination of these three properties can lead to the following: if wave energy is injected at intermediate frequencies, some is transferred to higher frequencies, but some moves toward lower frequencies near...
Figure 5. Reduced electron velocity distributions as (top) functions of $v_k$ and (bottom) one component of $v_\perp$ at late times during the three simulations with initial $\beta_e = 0.01$, 0.10, and 1.0.

the lower hybrid frequency at quasi-perpendicular propagation [e.g., Ganguli et al., 2010, Figure 2].

[21] This is illustrated in Figure 6, which compares magnetic fluctuation spectra as functions of frequency computed over the full temporal extent of each simulation for all three values of $\beta_e$. The initial frequency spectra for each run are approximately constant between the two dashed lines and are at the thermal noise level for fluctuations at frequencies greater than those at the right-hand dashed lines. Thus, this figure indicates a transfer of wave energy to both higher and lower frequencies. In particular, the $\beta_e = 0.01$ and 0.10 spectra show decided peaks near the lower hybrid frequency $\Omega_e$. [22] Figure 7 illustrates $\omega$ versus wave number dispersion plots for the simulations at three different values of $\beta_e$ computed over the same time intervals as those used in Figure 1. Here a specific perpendicular wave number is chosen as $k_\perp c/\omega_e = 0.61$. We chose this value because it is primarily in the cascaded wave vector range, while the damping rates are still small enough to clearly show dispersion. Dispersion curves (black dashed lines) of plasma waves from the numerical solution of the full linear kinetic dispersion equation are superimposed on each of the three panels. The similarity of the fluctuation energy in the simulation and linear dispersion curve not only confirms the whistler wave range of turbulence but also more importantly shows the nonlinear cascaded frequency and wave vector properties that linear dispersion theory alone could not predict. Dispersion of lower-frequency magnetosonic waves in PIC simulations of turbulence has been demonstrated earlier by Svidzinski et al. [2009] and Karimabadi et al. [2013].

3. Wave-Wave Coupling and the Forward Cascade

[23] In this section, we develop a simple test to examine whether wave-wave coupling implies a wave vector anisotropy for the forward cascade of whistler turbulence. This test has also been applied recently to lower-frequency turbulence consisting of Alfvén, magnetosonic, and magnetosonic-whistler fluctuations [Gary, 2013].

[24] The transfer of energy by three-wave interactions is a nonlinear process which must satisfy the frequency and wave vector matching conditions which represent conservation of energy and momentum among the three interacting modes [Shebalin et al., 1983]:

$$\omega_1(k_1) + \omega_2(k_2) = \omega_3(k_3)$$ (1)

and

$$k_1 + k_2 = k_3,$$ (2)

where $\omega(k)$ is the real frequency part of the solution to the linear dispersion equation. Although the rates of energy transfer must be obtained from the full nonlinear dynamical equations of the plasma, equations (1) and (2) represent necessary (but not sufficient) conditions for wave-wave coupling to proceed.

[25] In the simplest case where the dispersion relation is

$$\omega = \nu_{\text{phase}} k$$ (3)

equations (1) and (2) are trivially satisfied, and turbulent cascade processes proceed directly for all frequencies and wave numbers. The collisionless magnetized plasmas of space, however, are the basis for many different normal modes, each of which exhibit anisotropic propagation (due to the presence of the background magnetic field $B_0$), strong dispersion [i.e., violation of equation (3)], and wave-particle dissipation [i.e., Landau and/or cyclotron damping] [Gary, 1993]. Then, the matching conditions are satisfied for a limited or perhaps empty range of $\omega$ and $k$.

[26] We may gain insight into wave-wave cascade processes in plasma turbulence by combining linear dispersion
Figure 6. Magnetic fluctuation energy spectra computed over the full temporal evolution of each simulation as functions of frequency. The two vertical black dashed lines bound the frequency range of the initially imposed whistler fluctuations, and the blue lines represent the spectra.

Mithaiwala et al. [2012] examined the coupling between two relatively high-frequency whistlers and a low-frequency kinetic Alfvén wave. Our PIC simulations of whistler turbulence run for only a few proton cyclotron periods and do not provide a long-time representation of kinetic Alfvén waves. So we here consider wave-wave coupling among three separate whistler modes. Although whistler dispersion at relatively long wavelengths exhibits $\omega \sim k k_{||}$ dependence [e.g., Gary, 1993, equation (6.2.6)], there is a wave number regime ($0.20 \lesssim k c / \omega_e \lesssim 1.0$) on which whistler dispersion approximately satisfies the dimensionless equation

$$\omega = a + b k,$$

where $a$ and $b$ are the fitting parameters to the solution of the dispersion equation and are functions of $\beta_e$ and $\theta$ [e.g., Gary, 1993, Figure (6.7)]. Using equation (6) in the real part of equation (4) yields the dimensionless result

$$2a + b(k_1 + k_2) = a + bk_3 + \Delta \omega$$

so that (returning to dimensional variables) $\Delta \omega / |\Omega_e| = a$. In other words, equation (1) is more likely to be satisfied if the quantity $a$ is as small as possible.

Some recent studies have utilized the cold plasma approximation to yield analytic expressions for whistler dispersion in turbulence applications [Mithaiwala et al., 2011, 2012; Crabtree et al., 2012]. However, thermal plasma effects produce substantial differences in whistler dispersion properties for the three different values of $\beta_e$ which we consider here. Therefore, we here use numerical solutions of the full linear kinetic dispersion equation for Maxwellian
velocity distributions [Gary, 1993, Chapter 6] to provide whistler dispersion properties in the following analysis.

[29] Figure 8a illustrates whistler dispersion at $k_{\perp} = 0$ for $\beta_e = 0.01, 0.10,$ and 1.0. The solid lines indicate the full dispersion equation solution for $\omega(k_{\parallel})$, and the dotted lines indicate the damping rates. There is no Landau damping for electromagnetic waves at $k \times B_0 = 0$, so the increase in damping at $\beta_e = 1.0$ is due to the whistler fluctuations moving into cyclotron resonance with suprathermal and thermal electrons. Although $\omega \sim k^2$ at $kc/\omega_e << 1$, the dispersion

**Figure 7.** Fluctuation dispersion (real frequency $\omega$ as a function of parallel wave number $k_{\parallel}$) computed over the full temporal evolution of each simulation at three different values of $\beta_e$ as labeled. The colors represent simulation results, and the black dashed lines represent numerical solutions of the full kinetic dispersion equation for whistler fluctuations. Here $k_{\perp}c/\omega_e = 0.61$ for all three values of $\beta_e$.

![Figure 7](image1.png)

**Figure 8.** Linear theory results for whistler dispersion properties as functions of wave number. (a) $\theta = 0^\circ$. (b) $\theta = 75^\circ$. Solid lines represent the real frequency $\omega$ and dotted lines show the damping rate $\gamma$. Blue denotes $\beta_e = 0.01$ and $\omega_e^2/\Omega_e^2 = 0.5$, green denotes $\beta_e = 0.10$ and $\omega_e^2/\Omega_e^2 = 5.0$, and red denotes $\beta_e = 1.00$ and $\omega_e^2/\Omega_e^2 = 50$. 

![Figure 8](image2.png)
approximately satisfies equation (6) at shorter wavelengths. The figure further suggests that the frequency mismatch decreases with increasing $\beta_e$.

For the more general case of nonparallel propagation, the real frequencies of the whistler mode also approximately satisfy equation (6) at sufficiently large wave numbers for a broad range of propagation directions. For example, Figure 8b shows whistler dispersion at $\theta = 75^\circ$ for the same three values of $\beta_e$. Here, however, the frequency mismatch increases with increasing $\beta_e$.

Finally, Figure 9 quantifies the frequency mismatch, showing $\Delta \omega [\Omega_e]$ calculated from linear theory as a function of $\theta$ for each of the three simulations. At $\beta_e = 0.01$ and 0.10, the frequency mismatch is relatively large at quasi-parallel propagation but decreases toward zero as $\theta$ approaches the perpendicular. This suggests that the forward cascade of whistler fluctuations is favored at quasi-perpendicular propagation, consistent with our simulation results of a strong $k_\perp >> k_\parallel$ wave vector anisotropy at $\beta_e << 1$. For parameters corresponding to the $\beta_e = 1.0$ simulation, linear dispersion of whistlers at $\theta \geq 75^\circ$ does not well satisfy equation (6), and at smaller angles of propagation, the frequency mismatch is relatively large. This suggests there is less difference between the forward cascade rates at quasi-parallel and quasi-perpendicular propagation and is therefore consistent with our simulation results of much weaker wave vector anisotropies at $\beta_e = 1.0$.

4. Conclusions

We have carried out three-dimensional particle-in-cell simulations of the initial value problem for whistler turbulence in a homogeneous, magnetized, collisionless plasma with three initial values of $\beta_e$, namely, 0.01, 0.10, and 1.0. The $\beta_e$ is varied by changes in the background magnetic field magnitude $B_0$, whereas the initial electron temperature is the same for all three runs. The total energy in the initial magnetic fluctuations is also held constant, so the consequences of electron wave-particle heating can be compared directly.

The simulations follow the temporal evolution of the fluctuations; at early times, wave-wave interactions dominate and yield a forward cascade into a broadband, anisotropic, turbulent spectrum at shorter wavelengths. In addition, wave-particle interactions yield fluctuating field damping and electron heating; at sufficiently late times, these dissipative processes dominate. Larger values of $\beta_e$ correspond to a faster forward cascade in wave number and to a faster rate of electron heating, as well as to a less anisotropic wave vector distribution and to a less anisotropic electron velocity distribution. Although the electron heating trend with increasing $\beta_e$ is different from that in Saito and Gary [2012] because of a different choice of initial fluctuation amplitudes, the trends to less anisotropy in the wave vector distribution and the electron velocity distribution as well as the $T_\parallel > T_\perp$ character of the electron anisotropy are all qualitatively similar to those obtained by Saito and Gary [2012] in their two-dimensional PIC simulations of whistler turbulence.

Figures 4 and 5 demonstrate that the overall electron heating yields $T_\parallel > T_\perp$. This implies that the primary wave-particle interaction is through the Landau resonance (although the cyclotron resonance which should yield a $T_\parallel > T_\perp$ plays a larger role as $\beta_e$ increases). The Landau resonance for whistlers is ineffective at $k \times B = 0$, so Landau damping alone should lead to whistler spectra which are anisotropic in the sense of $k_\parallel > k_\perp$. But Figures 2 and 3 show the simulated spectra have the opposite wave vector anisotropy. Therefore, we conclude that wave-wave interactions are overall stronger and faster than the summed consequences of wave-particle interactions here, even at the relatively large value of $\beta_e = 1.0$.

We have used linear dispersion theory for whistler fluctuations in conjunction with the frequency and wave vector matching conditions [equations (1) and (2)] to determine the conditions favoring a forward cascade of whistler turbulence on the range $0.20 \leq k_e / \omega_e \leq 1.0$. At $\beta_e = 0.01$ and 0.10, the matching conditions imply that the forward cascade rate at quasi-perpendicular propagation should be much faster than at quasi-parallel propagation. This is consistent with the simulation results at low $\beta_e$ showing strong wave vector anisotropies with $k_\perp >> k_\parallel$. At $\beta_e = 1.0$, the matching conditions imply that the forward cascade rate for both quasi-parallel and quasi-perpendicular propagating may be similar. This is consistent with the early-time high $\beta_e$ simulation results showing a relatively isotropic wave vector distribution.

We further note the peculiar “star-shaped” spectra shown at late times and $\beta_e = 0.10$ and 1.00 in Figure 2. We believe there are two possible sources of these spectra. First, Figure 9 shows that, at the two larger values of $\beta_e$, the frequency mismatch has its largest values at intermediate values of $\theta$, suggesting that the wave-wave interactions which lead to the forward cascade are weakest for such oblique wave vector directions. Second, Figure 7 of Saito et al. [2008] shows that linear dispersion theory predicts that the strongest damping of whistlers at $\beta_e = 0.10$ is at oblique propagation ($40^\circ \leq \theta \leq 70^\circ$); the damping of such waves should also contribute to such star-shaped spectra.

Our interpretations of the simulation results have been based upon the linear theory of wave-particle interactions [Gary, 1993] in conjunction with the nonlinear theory of whistler wave-wave interactions. There are other possible
nonlinear processes, e.g., nonlinear Landau damping
[Crabtree et al., 2012], which may also play a role here, but are beyond the purview of this manuscript.

[37] Current observations of solar wind turbulence at electron-scale lengths are not sufficient to determine whether the constituent fluctuations are predominantly whistler or kinetic Alfvén waves. The PIC simulation results presented here may help to resolve this issue when more complete measurements of short wavelength solar wind turbulence become available.

Appendix A: Notation and Definitions

[38] We denote the jth species plasma frequency as \( \omega_j = \sqrt{4\pi n_j e^2/m_j} \), the jth species cyclotron frequency as \( \Omega_j = eB_z/m_jc \), the jth species thermal speed as \( v_j = \sqrt{k_B T_j/m_j} \), and \( \beta_j = 8\pi n_j k_B T_j/m_j c^2 \). We define \( \theta_j \), the angle of mode propagation, by \( k \cdot B_z = k B_z \cos(\theta_j) \). The uniform background magnetic field is \( B_0 = \bar{B}_0 \), so that the subscripts \( z \) and \( \parallel \) represent the same direction. Thus, \( k = \bar{k}_x + \bar{k}_y + \bar{k}_z \parallel \) and \( k_\perp = \sqrt{k_x^2 + k_y^2} \). The complex frequency is \( \omega + i\gamma \); \( \gamma \approx 0 \) corresponds to a damped fluctuation. We consider an electron-proton plasma where subscript \( e \) denotes electrons and \( p \) stands for protons.

[39] In three dimensions, the reduced \( k_\perp \) spectrum, which we here denote as \( \mathbf{k}(k_\perp) \), must be summed over both \( k_\parallel \) and \( \phi \), the azimuthal angle of the perpendicular vector wave. Thus, in the continuum limit, the total fluctuating magnetic energy density is

\[
\int d^3k |\delta B(k)|^2/8\pi = \int dk_\perp k_\perp E(k_\perp).
\]

The average wave vector anisotropy angle \( \theta_\parallel \) is defined through what we call the wave vector anisotropy factor [Shebalin et al., 1983]:

\[
\tan^2 \theta_\parallel = \frac{\Sigma_k k_\perp^2 |\delta B(k)|^2}{\Sigma_k k_\perp^2 |\delta B(k)|^2}.
\]

[40] Our concern here is the anisotropy of the forward cascaded fluctuations, so this quantity is evaluated by summations over wave vectors which satisfy \( 0.65 < k c/\omega_\parallel < 3.0 \). Figure 5 of the text shows that the heated electron velocity distributions become strongly non-Maxwellian at very low values of initial \( \beta_e \), so instead of using the electron temperature as a diagnostic of electron heating, we use the electron kinetic energy density which we define as

\[
K_e = \frac{m_e}{2} \int d^3v v^2 f_e(v)
\]

with parallel and perpendicular components

\[
K_{\parallel e} = \frac{m_e}{2} \int d^3v v_{\parallel}^2 f_e(v)
\]

and

\[
K_{\perp e} = \frac{m_e}{2} \int d^3v v_{\perp}^2 f_e(v)
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