Feedback Interference Alignment

Exact Alignment for Three Users in Two Time Slots

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Introduction

* Interference is a major bottleneck in today’s Wireless Networks...

  * Information Theoretic Challenges: Interference Channel

* Efficient Solutions?

![Diagram with TDMA and IA Schemes]
Introduction | IA Schemes

- Three User Single Antenna Interference Channel (no feedback)

\[
\begin{align*}
\mathbf{x}_1 & \quad \text{Tx1} & \quad \text{Rx1} & \quad \mathbf{y}_1 \\
\mathbf{x}_2 & \quad \text{Tx2} & \quad \text{Rx2} & \quad \mathbf{y}_2 \\
\mathbf{x}_3 & \quad \text{Tx3} & \quad \text{Rx3} & \quad \mathbf{y}_3 
\end{align*}
\]

- Time-Varying Channel Gains: 
  \[ y_k[t] = \sum_{\ell=1}^{3} h_{k\ell}[t] x_\ell[t] + z_k[t] \]

- Cadambe-Jafar Scheme: 
  \[ \text{DoF} = \frac{3n + 1}{2n + 1} \]

Three User Single Antenna Interference Channel (no feedback)

\[ y_k[t] = \sum_{\ell=1}^{3} h_{k\ell}[t] x_\ell[t] + z_k[t] \]

Infinite Channel Diversity: Symbol Extensions

Cadambe-Jafar Scheme: \[ \text{DoF} = \frac{3n + 1}{2n + 1} \]

Introduction | IA Schemes

- Three User Single Antenna Interference Channel (no feedback)

\[
\begin{align*}
    x_1 & \rightarrow \text{Rx1} \rightarrow y_1 \\
    x_2 & \rightarrow \text{Rx2} \rightarrow y_2 \\
    x_3 & \rightarrow \text{Rx3} \rightarrow y_3
\end{align*}
\]

- Flat fading Channel Gains: 
  \[ y_k[t] = \sum_{\ell=1}^{3} h_{k\ell} x_{\ell}[t] + z_k[t] \]

- Real Interference Alignment: 
  \[ \text{DoF} = \frac{3}{2} \]

Introduction | IA Schemes

- Three User Single Antenna Interference Channel (no feedback)

\[ x_1 \rightarrow \text{Tx1} \rightarrow \text{Rx1} \rightarrow y_1 \]
\[ x_2 \rightarrow \text{Tx2} \rightarrow \text{Rx2} \rightarrow y_2 \]
\[ x_3 \rightarrow \text{Tx3} \rightarrow \text{Rx3} \rightarrow y_3 \]

- Flat fading Channel Gains: \[ y_k[t] = \sum_{\ell=1}^{3} h_{k \ell} x_{\ell}[t] + z_k[t] \]

Infinite Channel Resolution: rational/irrational

- Real Interference Alignment: \[ \text{DoF} = \frac{3}{2} \]

Our Focus: Can Feedback Help?

- TDMA
- IA Schemes

In practice vs. in theory:
- Feedback?
Feedback Models

* Global Feedback

Each transmitter gets a vector feedback observation:

$$ b_k[t + 1] = \begin{bmatrix} y_1[t] \\ y_2[t] \\ y_3[t] \end{bmatrix} $$
Feedback Models

* “Backwards” Feedback

Transmitters get a **scalar** feedback observation through the backwards interference channel:

\[ b_k[t + 1] = h_{11}y_1[t] + h_{21}y_2[t] + h_{31}y_3[t] + z_k[t] \]
Feedback Models

* Local Feedback

Transmitters get **local** feedback information from their intended receivers:

\[ b_k[t + 1] = y_k[t] \]
C. Suh and D. Tse, “Feedback capacity of the Gaussian interference channel to within 2 bits,”

D. S. Papailiopoulos, C. Suh, and A. G. D. Dimakis, “Feedback in the K-user interference channel,”

Geng, Q., Kannan, S., & Viswanath, P. “Interactive Interference Alignment”.
Related Work


Our Work

**Local Feedback**

- Exact Alignment Proof
  - 3-users
  - 4-users

**Linear IA conditions**

- finite SNR optimization
  - IA + max-SINR

**Global Feedback**

**Backwards Feedback**

- K-users
  - 3-users
    - Exact DoF
  - 4-users
    - numerical evidence:

**Full-duplex Receivers**

- finite SNR
Main Result

Feedback Interference Alignment

• In the three-user (flat-fading) interference channel, 3/2 degrees-of-freedom are achievable over two time-slots with only local feedback information.
Feedback IA | The Achievable Scheme

* Transmit one symbol to each receiver in two channel uses

\[
\begin{align*}
x_1^{[1]} &= s_1 \\
x_2^{[1]} &= s_2 \\
x_3^{[1]} &= s_3
\end{align*}
\]

1st slot

\[
\begin{align*}
x_1^{[2]} &= t_1 x_1^{[1]} + f_1 y_1^{[1]} \\
x_2^{[2]} &= t_2 x_2^{[1]} + f_2 y_2^{[1]} \\
x_3^{[2]} &= t_3 x_3^{[1]} + f_3 y_3^{[1]}
\end{align*}
\]

2nd slot
Feedback IA | The Achievable Scheme

* Transmit one symbol to each receiver in two channel uses

\[
\begin{align*}
x_1^{[1]} &= s_1 & \text{Tx1} & \rightarrow & \text{Rx1} & y_1^{[1]} \\
x_2^{[1]} &= s_2 & \text{Tx2} & \rightarrow & \text{Rx2} & y_2^{[1]} \\
x_3^{[1]} &= s_3 & \text{Tx3} & \rightarrow & \text{Rx3} & y_3^{[1]} \\
\end{align*}
\]

1st slot

Local Feedback

\[
\begin{align*}
x_1^{[2]} &= t_1 x_1^{[1]} + f_1 y_1^{[1]} & \text{Tx1} & \rightarrow & \text{Rx1} & y_1^{[2]} \\
x_2^{[2]} &= t_2 x_2^{[1]} + f_2 y_2^{[1]} & \text{Tx2} & \rightarrow & \text{Rx2} & y_2^{[2]} \\
x_3^{[2]} &= t_3 x_3^{[1]} + f_3 y_3^{[1]} & \text{Tx3} & \rightarrow & \text{Rx3} & y_3^{[2]} \\
\end{align*}
\]

2nd slot
Feedback IA | The Achievable Scheme

* Transmit one symbol to each receiver in two channel uses

\[
x_1^{[1]} = s_1 \quad \text{Tx1} \quad \text{Rx1} \quad y_1^{[1]}
\]
\[
x_2^{[1]} = s_2 \quad \text{Tx2} \quad \text{Rx2} \quad y_2^{[1]}
\]
\[
x_3^{[1]} = s_3 \quad \text{Tx3} \quad \text{Rx3} \quad y_3^{[1]}
\]

1st slot

Local Feedback

\[
x_1^{[2]} = t_1 x_1^{[1]} + f_1 y_1^{[1]} \quad \text{Tx1} \quad \text{Rx1} \quad y_1^{[2]}
\]
\[
x_2^{[2]} = t_2 x_2^{[1]} + f_2 y_2^{[1]} \quad \text{Tx2} \quad \text{Rx2} \quad y_2^{[2]}
\]
\[
x_3^{[2]} = t_3 x_3^{[1]} + f_3 y_3^{[1]} \quad \text{Tx3} \quad \text{Rx3} \quad y_3^{[2]}
\]

2nd slot

Transmit a Linear Combination
(of previous symbols and feedback)
Each receiver has a 2-dimensional observation:

\[
\begin{align*}
    x_1^{[1]} &= s_1 \\
    x_2^{[1]} &= s_2 \\
    x_3^{[1]} &= s_3 \\
    x_1^{[2]} &= t_1 x_1^{[1]} + f_1 y_1^{[1]} \\
    x_2^{[2]} &= t_2 x_2^{[1]} + f_2 y_2^{[1]} \\
    x_3^{[2]} &= t_3 x_3^{[1]} + f_3 y_3^{[1]} \\
    x_1^{[3]} &= t_1 x_1^{[2]} + f_1 y_1^{[2]} \\
    x_2^{[3]} &= t_2 x_2^{[2]} + f_2 y_2^{[2]} \\
    x_3^{[3]} &= t_3 x_3^{[2]} + f_3 y_3^{[2]} \\
\end{align*}
\]

\[
\begin{bmatrix}
    y_k^{[1]} \\
    y_k^{[2]}
\end{bmatrix} = \begin{bmatrix}
    s_1 \\
    s_2 \\
    s_3
\end{bmatrix} + \begin{bmatrix}
    w_k^{[1]} \\
    w_k^{[2]}
\end{bmatrix}
\]

Linear transformation of the input symbols plus colored noise
Feedback IA | The Achievable Scheme

* Each receiver has a 2-dimensional observation:

\[
\begin{bmatrix}
    y_{k1} \\
    y_{k2}
\end{bmatrix}
= 
G_k \in \mathbb{R}^{2 \times 3}
\begin{bmatrix}
    s_1 \\
    s_2 \\
    s_3
\end{bmatrix}
+ 
\begin{bmatrix}
    w_{k1} \\
    w_{k2}
\end{bmatrix}
\]

Linear transformation of the input symbols plus colored noise

- The matrix \( G_k \) and the covariance of \( \begin{bmatrix} w_{k1} \\ w_{k2} \end{bmatrix} \) depend on the choice of the linear combination coefficients \( t = [t_1, t_2, t_3] \) and \( f = [f_1, f_2, f_3] \) for all \( k \).
Each receiver has a 2-dimensional **observation**: 

![Diagram of signal processing](image)

For a given choice of \( \mathbf{t} = [t_1, t_2, t_3] \) and \( \mathbf{f} = [f_1, f_2, f_3] \) the optimal linear receiver will project \( \mathbf{y}_k \) in order to obtain an (MMSE) estimate for \( s_k \).
Feedback IA | Degrees of Freedom

- **Interference Alignment:**

\[
\begin{bmatrix}
y_k^{[1]} \\
y_k^{[2]}
\end{bmatrix} = \begin{bmatrix} G_k \in \mathbb{R}^{2 \times 3} \end{bmatrix} \begin{bmatrix} s_1 \\
 s_2 \\
 s_3
\end{bmatrix} + \begin{bmatrix} w_k^{[1]} \\
 w_k^{[2]}
\end{bmatrix}
\]

- **We can choose** \( t = [t_1, t_2, t_3] \) and \( f = [f_1, f_2, f_3] \) **such that interference aligns at all receivers:** 3/2 DoF are achievable.

- **Main result extends to K=4 users** as well... *(alignment for 4-users over two time slots)*
Feedback IA | Optimization at Finite SNR

* System Parameters (9-dimensional space)

\[ a = \begin{bmatrix} \text{linear-combination coefficients and receiver projections} \end{bmatrix} \]

IA Conditions

\[
\begin{align*}
Ba &= 0 & & \text{cross-channel gains must be zero} \\
Qa &= [\lambda_1 \lambda_2 \lambda_3]^T & & \text{direct-channel gains non-zero} \\
\lambda_1 \lambda_2 \lambda_3 &\neq 0
\end{align*}
\]

IA Optimization Problem:

\[
\begin{align*}
\text{maximize} \quad & ||Qa||^2 \\
\text{s.t.:} \quad & Ba = 0 \\
& a \in P
\end{align*}
\]
System Parameters (9-dimensional space)

\[ \mathbf{a} = \begin{bmatrix} \text{linear-combination coefficients and receiver projections} \end{bmatrix} \]

The space of perfect IA solutions
Numerical Results

Fig. 1. Performance of the proposed feedback scheme. We assume real right balance between useful signal and interference power.

In our simulations, we plot the average achievable sum rates for all of the above schemes versus SNR measured in dB. Figure 1 shows the performance comparison in the interference channel with feedback. Transmitters can use the two bit gap scheme proposed in [9] for the two-user 2-user Feedback.

Table 1 shows the comparison of ergodic IA and feedback IA schemes in a weak interference regime, where the cross channel gains are scaled such that the max-SINR heuristic is able to provide considerable gains in the low to medium SNR range while it maintains the right balance between useful signal and interference power.

We consider a time sharing version of the two-user feedback scheme proposed in [9] for the two-user 2-user Feedback IC / 2-user Feedback IC.

**References**


Numerical Results

![Numerical Results Graph]

- **Channel Coefficients**: Assumption of real right balance between useful signal and interference power.

- **Performance**: The proposed feedback scheme achieves performance gains, especially for the max-SINR design. The max-SINR heuristic provides considerable gains in two time realizations (if the channel were time-varying). Notice that the average achievable sum rates for all schemes versus SNR measured in dB. Figure 1 shows the performance comparison in the standard setting where the two user feedback scheme proposed in [9].

**Fig. 1**: Performance of the proposed feedback scheme. We assume real right balance between useful signal and interference power. Especially for the max-SINR design that seems to achieve the performance gains obtained by feedback become more significant, such that the max-SINR heuristic is able to provide considerable gains in two time slots – rates that are not too far from what ergodic IA proposed max-SINR feedback scheme gets – in two time standard setting where the channel in pairs and transmit with power interference channel with feedback. Transmitters can use the two bit gap scheme proposed in [9].

**Numerical Results References**


[16] N. Lee and R. W. Heath Jr, “Not too delayed CSIT achieves the optimal average sum rate (bits/sec/Hz) 10 log EFERENCES

**Figure 2** shows the same comparison in a weaker interference and interference channel with feedback. Transmitters can use the two bit gap scheme proposed in [9].
Thank You