The Frequency And Damping Of Soil-Structure Systems With Embedded Foundation

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Abstract: The effect of foundation embedment on fundamental period and damping of buildings has been the title of several researches in three past decades. A review of the literature reveals some discrepancies between proposed formulations for dynamic characteristics of soil-embedded foundation-structure systems that raise the necessity of more investigation on this issue. Here, first a set of approximate polynomial equations for soil impedances, based on numerical data calculated from well known cone models, are presented. Then a simplified approach is suggested to calculate period and damping of the whole system considering soil medium as a viscoelastic half space. The procedure includes both material and radiation damping while frequency dependency of soil impedance functions is not ignored. Results show that soil-structure interaction can highly affect dynamic properties of system. Finally the results are compared with one of the commonly referred researches. The methods in most parts lead to same results.

Keywords: soil-structure interaction, embedment, impedance functions, equivalent damping

INTRODUCTION

Soil-structure interaction has significant effect on dynamic behavior of the super structure. This effect is usually investigated in two separate sections of kinematic Interaction (KI) and Inertial Interaction (II). To account for II, it is common to calculate effective period and effective damping of the system as the dynamic properties of replacement oscillator [1-5]. Yet, many researches have been done to make rational estimations of effective dynamic parameters. Some of the applied methods for this purpose were, eigen value analysis[5], numerical calculation of transfer function[4] and equivalent single degree of freedom approach[2,3]. It is note worthy to mention that in every method for effective parameter calculation, soil impedance functions play important role. These functions may be calculated from analytical[6] or numerical[7] methods or some times combination of both[8]. The estimated effective period and damping of soil-structure system may take different values with various soil impedance functions and different methods of dynamic parameter extraction. In this study, dynamic parameters of soil-structure systems, with embedded foundation, are investigated. The impedance functions were calculated through approximate analytical cone model solutions. This method has an attractive scene of being used in future works on embedded foundation in layered stratum[9]. ImPLYing the above impedances, equations of motion are formulated and target dynamic parameters are calculated in an eigen value extraction procedure. Then a comparison with a commonly referred previous works is presented.

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The interacting system investigated here constituted of SDOF structure mounted on cylindrical rigid foundation embedded in viscoelastic half space.

**BASIC SOIL-STRUCTURE MODEL**

Fig1. shows the model used in this research. The superstructure is considered as a single mass, \( m_{\text{str}} \), with moment of inertia, \( I_{\text{str}} \), mounted on a spring having stiffness, \( K_{\text{str}} \), at an elevation of, \( \bar{h} \). These parameters can be representatives for first modal mass and inertia, effective stiffness and effective height of a MDOF structure.

The foundation is represented by a rigid cylindrical mass, \( m_f \), with moment of inertia, \( I_f \), radius \( r \) and depth of embedment \( e \) and is in full contact with surrounding soil. The flexibility of the surrounding soil is modeled by coupled sway-rocking springs. \( S_{hh}, S_{rr}, S_{hr}, \text{and} S_{rh} \) in Fig2., are representatives of soil stiffness matrix components.

![Figure1: Basic soil-structure model](image)

The equilibrium equation can then be written as follows:

\[
M \ddot{X} + KX = F
\]  

(1)

Substituting M, K and F leads to:

\[
\begin{bmatrix}
  m_{\text{str}} & 0 & m_{\text{str}}(\bar{h} + e) \\
  0 & m_f & m_f(e/2) \\
  m_{\text{str}}(\bar{h} + e) & m_f(e/2) & I_{\text{str}} + I_f + m_{\text{str}}(\bar{h} + e)^2
\end{bmatrix}
\begin{bmatrix}
  \ddot{x}_s \\
  \ddot{x}_f \\
  \ddot{\phi}_f
\end{bmatrix}
= 
\begin{bmatrix}
  K_{\text{str}} & -K_{\text{str}} & 0 \\
  -K_{\text{str}} & K_{\text{str}} + S_{hh} & S_{hr} \\
  0 & S_{rh} & S_{rr}
\end{bmatrix}
\begin{bmatrix}
  x_s \\
  x_f \\
  \phi_f
\end{bmatrix} + 
\begin{bmatrix}
  m_{\text{str}}(\bar{h} + e) \\
  m_f(e/2) \\
  m_{\text{str}}(\bar{h} + e) + m_f(e/2) + m_{\text{str}}(\bar{h} + e)^2 + I_{\text{str}} + I_f
\end{bmatrix}
\begin{bmatrix}
  \ddot{x}_g \\
  \ddot{\phi}_g
\end{bmatrix}
\]  

(2)

Where \( x_s, x_f, \text{and} \phi_f \) are deformation of structure plus foundation translation, foundation translation with respect to ground, foundation rotation with respect to ground respectively. \( x_g \) and \( \phi_g \) are effective translation input motion and effective rotational input motion respectively. By introducing five dimensionless parameters, \( a_0 = \omega r/V_s, \) \( \bar{h}/r, \) \( e/r, \) \( \alpha = m_f/m_{\text{str}}, \) \( \gamma = m_{\text{str}}/(\pi r^2 \bar{h}) \), Eq.(4) can be interpreted in new form that provides deeper insight into formulation and improves its generality. Parameters \( \varphi \) and \( V_s \)
are vibration frequency and soil shear wave velocity respectively. It should be added that the material damping in the soil and structure may be included in the formulations as hysteretic form of damping by using the correspondence principle, i.e. just by multiplying the stiffness of the structure and soil by \((1+2i\xi_{str})\) and \((1+2i\xi_{soil})\) respectively. Where, \(\xi_{str}\) and \(\xi_{soil}\) are the material damping ratios in the structure and soil.

**IMPEDANCES OF SOIL-Foundation SYSTEM**

In this section, the impedances of rigid cylindrical foundation embedded in a homogeneous half space are investigated. For this case soil stiffness matrix includes \(S_{hh}\), \(S_{rr}\), \(S_{hr}\) and \(S_{rh}\) as horizontal, rocking and coupling constituents. Definition of each term can be stated as follows:

\[
\begin{align*}
S_{hh} &= K_{HH}[k_{hh} + ia_0c_{hh}] \\
S_{rr} &= K_{RR}[k_{rr} + ia_0c_{rr}] \\
S_{hr} &= S_{rh} = K_{HR}[k_{hr} + ia_0c_{hr}]
\end{align*}
\]

(3)

Where \(K_{HH}\), \(K_{RR}\) and \(K_{HR}\) are horizontal, rocking and coupling terms of static stiffness matrix and \(k_{hh}\), \(k_{rr}\), \(k_{hr}\), \(c_{hh}\), \(c_{rr}\) and \(c_{hr}\) are impedance coefficients of soil-foundation system. Different components of static stiffness matrix can be expressed as follows \[8\]:

\[
\begin{align*}
K_{HH} &= \frac{8Gr}{2 - \nu} (1 + e/r) \\
K_{RR} &= \frac{8Gr^3}{3(1 - \nu)} \left(1 + 2.3e/r + 0.58(e/r)^3\right) \\
K_{HR} &= \frac{e}{3} K_{HH}
\end{align*}
\]

(4)

In which \(G\) is soil shear modulus. The equivalent radius may be used in the case of cubic foundation by matching the area or moment of inertia of the foundation with a replacement cylindrical foundation for the sway and rocking DOF’s respectively. The method to derive soil impedance coefficients is approximate analytical formulation using cone models. Such type of formulation can be applied to more complex problems including layered soil texture and foundation irregularities in depth\[9\]. In this method the foundation is divided into several disks. Calculating related stiffness functions and implying geometric constraint leads to final form of soil-foundation stiffness matrix. The impedance coefficients, calculated by this method, are all frequency dependent and differ with variation of embedment ratio.

Here for wide ranges of embedment ratio and vibration frequency, the impedance coefficients are numerically calculated. The results are used to complete stiffness matrix in Eq.(2). As an alternative, using two dimensional regressions for same data, a set of approximate formulations for impedance coefficients is introduced. It can be used as substitutive for the above multi-stage procedure. The general form of represented formulation is described in (3).

\[
(k_{ij} or c_{ij}) = C_1a_0^2 + C_2a_0\left(\frac{\varepsilon}{r}\right) + C_3\left(\frac{\varepsilon}{r}\right)^2 + C_4a_0 + C_5\left(\frac{\varepsilon}{r}\right) + C_6 \quad i, j = (r or h)
\]

(5)

Where \(C_1 \) to \(C_6\) are the coefficients calculated from regression. Fig(2) shows fitted surfaces and original data for different impedance coefficients. Typical results are sketched for von Mises’s ratio of 0.4 as functions of \(a_0\) and \(e/r\).
Similarly for posision’s ratios of 0.25 and 0.33, the coefficients $C_1$ to $C_6$ are estimated and final results for different terms are tabulated in table (1). The last column in this table shows an index of how represented curves match numerical data. As it is seen the agreement is good enough.

**Table (1): Dynamic stiffness coefficients of embedded foundation**

<table>
<thead>
<tr>
<th></th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C_5$</th>
<th>$C_6$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu = 0.25$</td>
<td>$k_{bb}$</td>
<td>-0.0086</td>
<td>-0.0116</td>
<td>0.0657</td>
<td>-0.0108</td>
<td>-0.1384</td>
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<td></td>
<td>$k_{rr}$</td>
<td>0.0282</td>
<td>-0.0383</td>
<td>0.0119</td>
<td>-0.1721</td>
<td>-0.0607</td>
<td>1.0480</td>
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<td></td>
<td>$k_{hr}$</td>
<td>0.0190</td>
<td>0.1168</td>
<td>0.3022</td>
<td>-0.3876</td>
<td>-0.8710</td>
<td>1.5955</td>
</tr>
<tr>
<td></td>
<td>$c_{hh}$</td>
<td>-0.0016</td>
<td>0.0151</td>
<td>-0.1117</td>
<td>0.0237</td>
<td>0.7273</td>
<td>0.6728</td>
</tr>
<tr>
<td></td>
<td>$c_{rr}$</td>
<td>-0.0336</td>
<td>0.0401</td>
<td>0.1422</td>
<td>0.1878</td>
<td>-0.1306</td>
<td>0.0676</td>
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<tr>
<td></td>
<td>$c_{hr}$</td>
<td>-0.0213</td>
<td>0.0551</td>
<td>-0.2070</td>
<td>0.1563</td>
<td>1.0491</td>
<td>0.4096</td>
</tr>
<tr>
<td>$\nu = 0.33$</td>
<td>$k_{bb}$</td>
<td>-0.0079</td>
<td>-0.0079</td>
<td>0.0641</td>
<td>-0.0131</td>
<td>-0.1355</td>
<td>1.0606</td>
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<tr>
<td></td>
<td>$k_{rr}$</td>
<td>0.0285</td>
<td>-0.0394</td>
<td>0.0133</td>
<td>-0.1729</td>
<td>-0.0608</td>
<td>1.0476</td>
</tr>
<tr>
<td></td>
<td>$k_{hr}$</td>
<td>0.0198</td>
<td>0.1194</td>
<td>0.2912</td>
<td>-0.3831</td>
<td>-0.8710</td>
<td>1.5955</td>
</tr>
<tr>
<td></td>
<td>$c_{hh}$</td>
<td>-0.0015</td>
<td>0.0132</td>
<td>-0.1119</td>
<td>0.0240</td>
<td>0.7273</td>
<td>0.6728</td>
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<tr>
<td></td>
<td>$c_{rr}$</td>
<td>-0.0336</td>
<td>0.0401</td>
<td>0.1422</td>
<td>0.1878</td>
<td>-0.1306</td>
<td>0.0676</td>
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<tr>
<td></td>
<td>$c_{hr}$</td>
<td>-0.0209</td>
<td>0.0508</td>
<td>-0.2055</td>
<td>0.1552</td>
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<td>0.4096</td>
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<tr>
<td>$\nu = 0.45$</td>
<td>$k_{bb}$</td>
<td>-0.0070</td>
<td>-0.0025</td>
<td>0.0616</td>
<td>-0.0162</td>
<td>-0.1307</td>
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<tr>
<td></td>
<td>$k_{rr}$</td>
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<td>-0.0101</td>
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<tr>
<td></td>
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<td>0.1231</td>
<td>0.2751</td>
<td>-0.3761</td>
<td>-0.8003</td>
<td>1.5503</td>
</tr>
<tr>
<td></td>
<td>$c_{hh}$</td>
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<td>0.0103</td>
<td>-0.1122</td>
<td>0.0245</td>
<td>0.7125</td>
<td>0.5945</td>
</tr>
<tr>
<td></td>
<td>$c_{rr}$</td>
<td>-0.0313</td>
<td>0.0370</td>
<td>0.1292</td>
<td>0.1786</td>
<td>-0.1031</td>
<td>0.0263</td>
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<tr>
<td></td>
<td>$c_{hr}$</td>
<td>-0.0204</td>
<td>0.0445</td>
<td>-0.2031</td>
<td>0.1535</td>
<td>1.0188</td>
<td>0.3432</td>
</tr>
</tbody>
</table>
**DYNAMIC PARAMETERS OF SYSTEM**

The method that is chosen here to extract dynamic parameters of system is eigen value analysis using matrices M and K in an iterative process. Components of dynamic stiffness matrix, $S_{hh}$, $S_{rr}$, $S_{hr}$ and $S_{rh}$, are updated in primary frequency of soil-structure system in each loop. Using this procedure the damped frequency and damping ratio of the system are evaluated from real and imaginary parts of the principal eigen value respectively. The method is efficient and just a few iterations are required for convergence into the results. Among the dimensionless parameters, $a_0$, $\bar{h}/r$ and $e/r$ have been selected as the key parameters here. Parameter $\alpha$ is evaluated as a function of the previous three. The next parameter, $\gamma$, is set to a typical value of (0.15) for ordinary buildings[3,4]. Material dampings are set to common values of (0.0) and (0.05).

**REPRESENTATIVE RESULTS**

In this section, the effect foundation embedment on dynamic properties of soil-structure systems is investigated. First the graphs of effective period and damping of soil-structure systems for extensive variation in non-dimensional parameters are presented. Then the results are compared with one of the commonly used procedures of dynamic parameter extraction.

Fig.(3) shows effective period ratios for three embedment ratios $e/r=0$, 0.5, 1 and three aspect ratios $h/r=1$, 2, 4 as a function of fixed base non-dimensional frequency $(a_0)_{fix}$.

![Figure(3): Effective system-to-structure period ratio with respect to system, aspect ratio, embedment ratio and $\nu=0.4$, $\xi_{soil}=0$.](image)

As it can be seen, supposing a specific non-dimensional frequency, increasing in aspect ratio lead to substantial increase in effective period ratios. Also increase in embedment ratio slightly decreases effective period ratios.

Focusing on damping of the system, $\beta_h$, Fig.(4) shows the results for a wide range of $\tilde{T}/T$ covering systems with no SSI, $\tilde{T}/T = 1$, to systems with severe SSI effect, $\tilde{T}/T = 2$, where $\tilde{T}$ is the main period of the total system and $T$ is the period of the super structure. Also a range of short squat buildings to slender ones are considered by varying $h/r$ from 1...
to 4 while the embedment ratios, $e/r$, have taken values of 0, 0.5 and 1. Soil material dampings of 0 and 0.05 are tried.

It can be seen the higher the aspect ratio, the lower the system damping. A reverse trend is observed for embedment ratio, increasing in which lead to higher system dampings. Though possessing increasing effect, material dampings don’t affect total damping severely.

As last a comparison of results of this paper with one of the commonly referred researches, Veletsos[4], has been made. Formulations of that reference were originally derived for surface foundation; But has been recently implemented in the case of embedded foundations [10] which will be sited as “Modified Veletsos” here. Fig.(5) shows system dampings derived in this study besides “Modified Veletsos” related data. As it can be seen for high aspect ratios, i.e. $h/r=2, 4$, both methods propose essentially same results. But in low aspect ratios, i.e. $h/r=1$, results of this study are up to 20% higher than the other method.
CONCLUSION

The issues discussed in this paper include two major parts. At first a set of approximate formulations for impedance coefficients for soil-embedded foundation systems are presented. The equations were derived based on numerical data, calculated from well known cone models. Next, using the same impedances, a general equilibrium equation for soil-structure system is represented. Then using the method of eigen value analysis, main dynamic parameters of the system are calculated for different values of system-to-structure period ratio, aspect ratio, embedment ratio and material damping of the soil. Results show sharp increase in effective period ratio as the aspect ratio increases and slight decrease in effective period ratio when embedment ratio rises. On the other hand any increase in aspect ratio has decreasing effects on damping while increase in embedment ratio results in higher dampings. Finally the results are compared with one of the commonly referred researches. For high aspect ratios both methods show good agreement. But for low aspect ratios proposed method of this study are up to 20% higher than the other method.

Figure(5): Effective damping of soil-structure systems calculated from two different methods with respect to system-to-structure period, aspect ratio, embedment ratio and material damping of soil.
REFERENCES