What can we learn from the smile or frown of implied volatilities?

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This study is going to review the literature of implied volatility and we will try to use this theory to assess the price movement of two high-tech companies: Broadcom and Microsoft. We will try to get more information from their option prices which were traded on April 17, 2003.

Introduction

Under the complete market hypothesis, the derivative price and the price of an asset are supposed to satisfy a deterministic relationship. The Black_Scholes model is based on some assumptions which propose such a deterministic relationship between option prices and the prices of assets. Now assume that at date t we have a complete sample of observations on the price S_t of the asset , the risk free r_t, and two option prices with characteristics H_1, k_1 and H_2, k_2 respectively. From the Black_Scholes formula:

$$C(t; H_1, K_1) = S_t \Psi(s; k_1, H_1, r_t)$$
$$C(t; H_2, K_2) = S_t \Psi(s; k_2, H_2, r_t)$$

The true value of the volatility should arise as the solution of this bivariate nonlinear system. Yet, in practice the observed prices $S_t, r_t, C(t; H_1, K_1)$ and $C(t; H_2, K_2)$ are such that the system has no solution; as a consequence, the Black_Scholes model is immediately rejected by the data with probability 1.

The incompatibility stems from the unrealistic assumption of the Black_Scholes model on unique derivative prices, which implies a deterministic relationship between the asset price and option price. In reality such deterministic relationships do not exist. Statistical inference makes sense only in the presences of a random variation source or, more precisely of an error term. So there are other random processes other than the price of the stock which has impact on the price of the options. So, that is a violation of complete market hypothesis which has been assumed in Black_Scholes model.

Nevertheless, the Black_Scholes formula remains a valuable tool for comparing option prices. This leads to the so called implied volatility. Let us now assume an observed option price of a call with Maturity H and strike price K. Since the function y in the

Black_Scholes formula is a one-to-one function of volatility, there exists a unique volatility value such that:

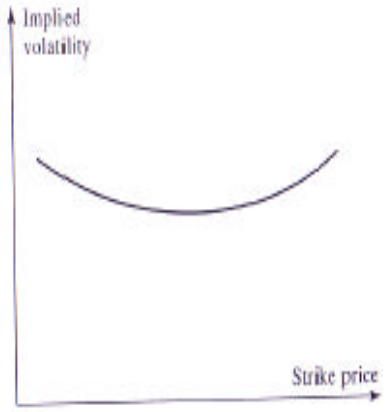
$$C(t;H,k) = S_t y \left[s_{BS}(t;H;K); K;H, r_t \right]$$

Where $s_{BS}(t;H;K)$ is the implied volatility associated with the derivative with an observed price. It can essentially be viewed as a corrected option price for which the correction pertains to the aforementioned effects. If the assumptions of the Black_Scholes model were all satisfied all implied volatilities would be equal and would coincide with the constant historical volatility. In fact, in the Black_Scholes world, all contingent claims are generated either by the risky and risk-free assets or by an option and risk-free asset.

Comparison of Volatilities

In the real world, implied volatilities vary depending on the data, maturity and strike. A plot of the implied volatility of an option as a function of its strike price is known as a volatility smile. Volatility smiles have been very useful in assessing the price distribution of the stock price at the time of maturity. If we assume that the agents are risk neutral, then the price of the option should be equal to the expected payoff of the option at the maturity date (Magill, M., and Quinzii, M. 1995 ,pp298-305). So with different patterns of volatility smile we will come up with different distributions of asset prices. Since the option prices emanates from the expectation of the agents about the asset prices in future, the implied distribution from volatility smiles gives us the information of the expected distribution of the asset price in the market. Implied volatilities have different features which are as below:

Smiling Volatility: This is the most famous pattern of volatilities and it is mostly common among the call options for exchange rates. The implied volatility increases as strike prices go far from the contemporary stock price (Figure 1). This kind of relationship between implied volatilities and strike price happens when the implied distribution has more weights at tails (Figure 2). But why are exchange rates not lognormally distributed? Two of the conditions for an asset price to have a lognormal distribution are: 1) The volatility of the asset is constant, 2) The price of the asset changes smoothly with no jumps. In practice, neither of these conditions is satisfied for an exchange rate. The volatility of an exchange rate is far from constant, and exchange rates frequently exhibits jumps because of the abrupt jumps in interest rates by central banks.



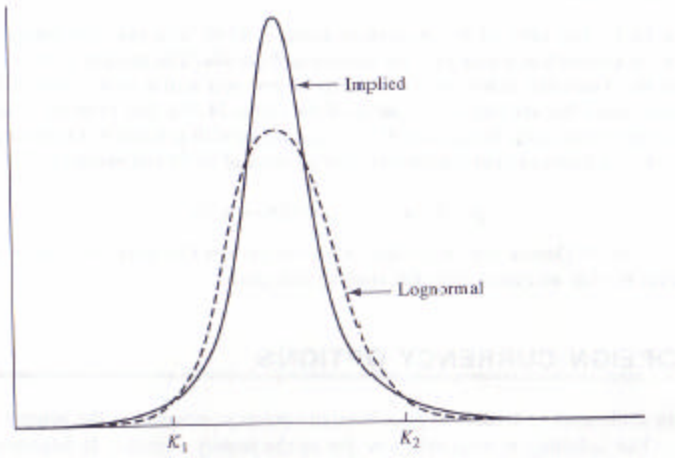
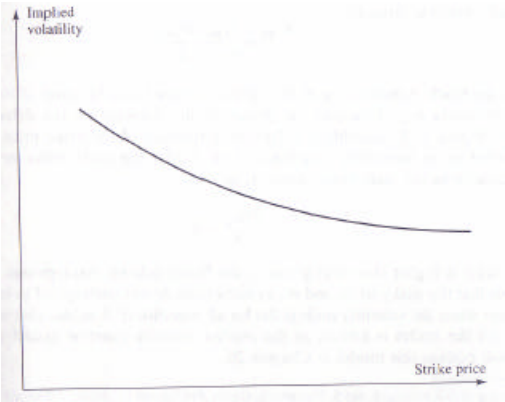


Figure 1. Smiling volatility

Figure 2. Implied distribution of a smiling volatility

Volatility Skew: In this case, the volatility decreases as the strike price increases (Figure_3). The volatility used to price a low-strike-price option (i.e a deep-out-of-the-money put or deep-in-the-money call) is significantly higher than that used to price a high-strike-price option (i.e a deep-in-the-money put or a deep-out-of-the-money call). The implied volatility resulted from volatility skew has more skewness at the left tail relative to the right tail (Figure 4),



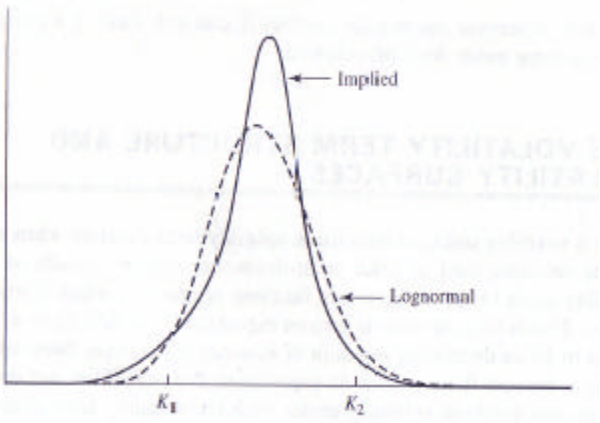


Figure 3. Volatility skew

Figure 4. Implied distribution for volatility skew

One possible explanation for "implied skew" in equity options concerns leverages. As a company's equity declines in value, the company's leverage increases. As a result the volatility of its equity increases, making even lower stock prices more likely. As a company's equity increases in value, leverage decreases. As a result the volatility of its equity declines, making higher stock prices less likely. This argument shows that we can expect the volatility of equity to be a decreasing function of price. It is an interesting observation that the skew pattern for equities has existed only since the stock market

crash of October 1987. Prior to October 1987, implied volatilities were much less dependent on strike price. This is called as " Crash phobia" (Rubinstein 1994).

Volatility frowning: When volatility has a hump-shape pattern, it is said that volatility is frowning (figure 5). This usually happens when an important new announcement is expected which will either increase or decrease greatly the stock price. Then the probability distribution of the stock price consists of a mixture of two lognormal distributions (figure 6), the first corresponds to favorable news and the second to unfavorable news.

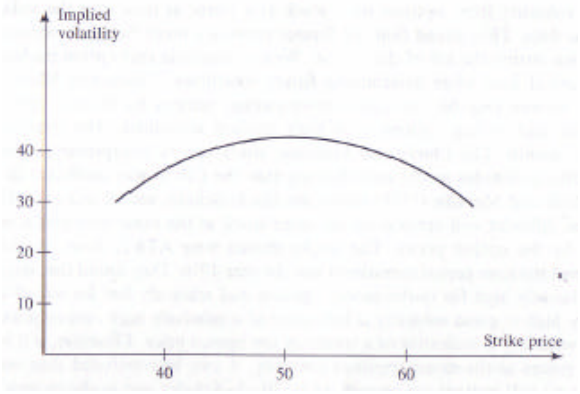


Figure 5 . Volatility frowning

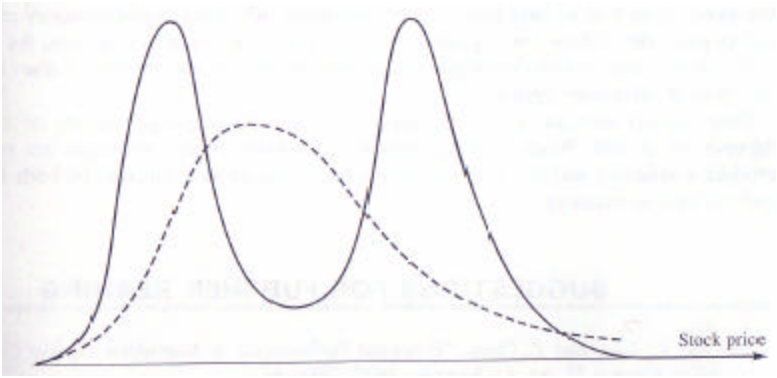


Figure 6. Implied volatility (dashed line) which is approximately the sum of two lognormal distributions.

Empirical results

The call options of two companies: Broadcom and Microsoft have been chosen for assessing their implied volatilities. The call option prices of two companies on April 17th 2003 have been used for this study. The implied volatilities have been found from Black-Shules formula (figure 7 & 8).

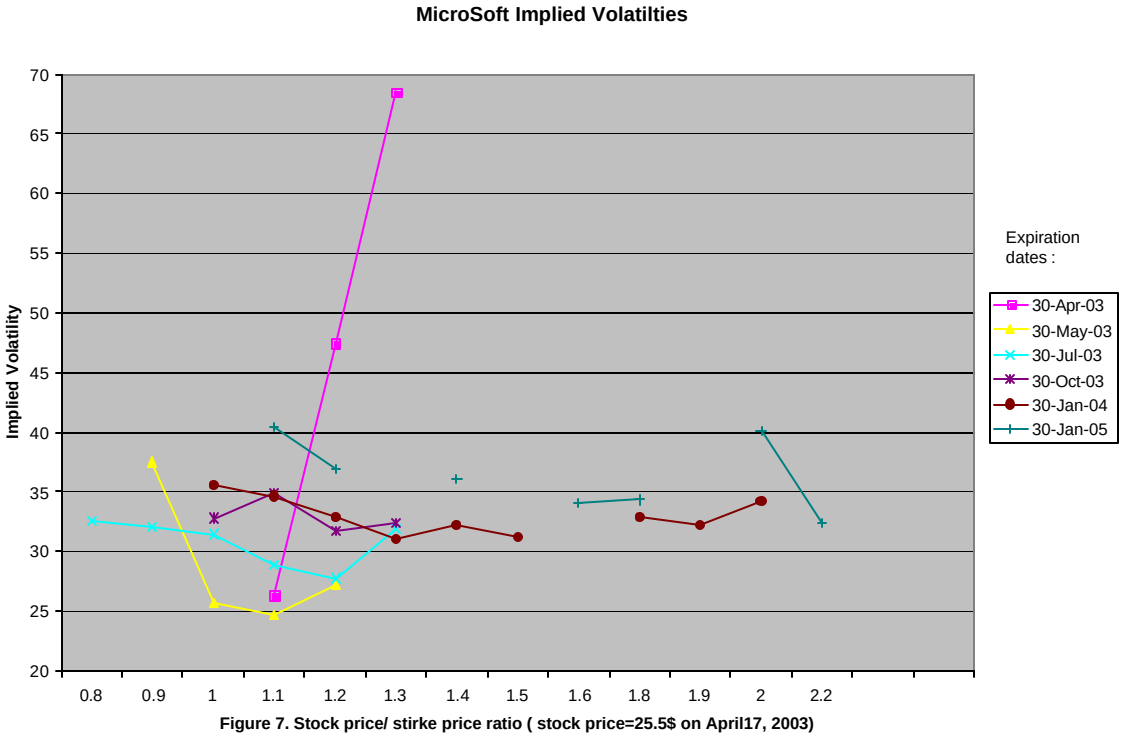


Figure 7. Stock price/ stirke price ratio (stock price=25.5$ on April17, 2003)

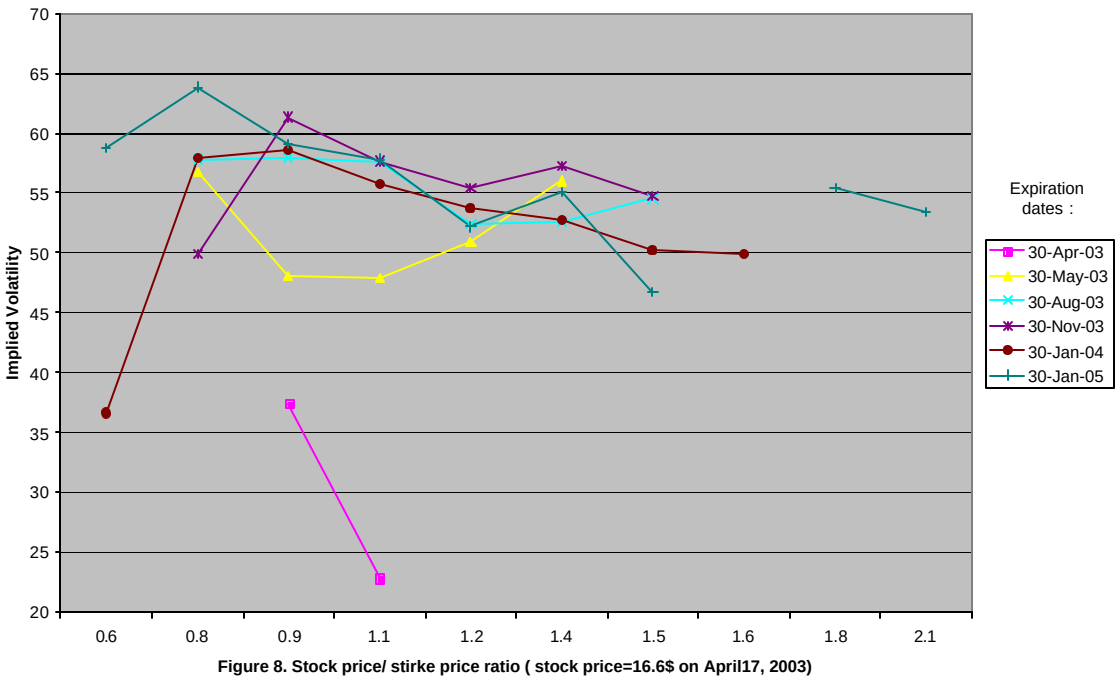


Figure 8. Stock price/ stirke price ratio (stock price=16.6$ on April17, 2003)

The regression of implied volatilities (impv) over strike/price ratio (s/k) and maturity (d) for those two company shows that for Broadcom there is no significant relationship between implied volatilities and strike/price ratios. So the expected implied distribution of the stock price is lognormally distributed. The implied volatility is almost constant for Broadcom in all different strike prices and maturities.

Table 1- The regression output for Broadcom

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**************************************************************************

. reg  impv  d sk sk2

      Source |       SS       df       MS     Number of obs =      36
-------------+------------------------------  F(  3,    32) =    0.27
       Model |  71.0970462     3  23.6990154  Prob > F      =  0.8499
    Residual |  2858.49909    32  89.3280966  R-squared     =  0.0243
-------------+------------------------------  Adj R-squared = -0.0672
       Total |  2929.59614    35  83.7027468  Root MSE      =  9.4514

------------------------------------------------------------------------------
 impv |      Coef.   Std. Err.   t    P>|t|     [95% Conf. Interval]
------+---------------------------------------------------------------------
    d |    .004054    .007297    0.56   0.582    -.0108096    .0189176
   sk |  -10.91913   27.00642    -0.40   0.689     -65.9294    44.09114
  sk2 |   3.295794    10.6977     0.31   0.760     -18.4947    25.08629
 cons |   58.61274   16.85463     3.48   0.001     24.28099    92.94449
------------------------------------------------------------------------------

. reg  impv  d sk

Source |       SS       df       MS            Number of obs =      36
-------------+------------------------------   F(  2,    33) =    0.36
       Model |  62.6183737     2  31.3091868   Prob > F      =  0.7001
    Residual |  2866.97776    33  86.8781141   R-squared     =  0.0214
-------------+------------------------------   Adj R-squared = -0.0379
       Total |  2929.59614    35  83.7027468   Root MSE      =  9.3208

------------------------------------------------------------------------------
 impv |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    d |   .0048248   .0067601     0.71   0.480    -.0089287    .0185783
   sk |  -2.722066   4.566274    -0.60   0.555    -12.01222    6.568089
_cons |   53.70755   5.453723     9.85   0.000     42.61187    64.80323
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The result of regression for Microsoft Company shows that the implied volatility is frowning (Table 2)! The following equation has been found for Microsoft implied volatility:

$$\mathrm{Im}\, pv = 34.3\, SK - 22.5\, SK^2$$

Thus implied volatility has a minimum at sk=0.76. So we can assume that the market expects important news is going to change the price of Microsoft share.

Table 2- The regression output for Microsoft

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**************************************************************************
. reg impv d sk sk2

   Source |       SS       df       MS     Number of obs =      35
----------+------------------------------  F(  3,    31) =    1.87
    Model |  459.055755     3  153.018585  Prob > F      =  0.1557
 Residual |  2540.27144    31    81.94424  R-squared     =  0.1531
----------+------------------------------  Adj R-squared =  0.0711
    Total |  2999.32719    34  88.2155057  Root MSE      =  9.0523

------------------------------------------------------------------------------
impv |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   d |   .0012842   .0087371     0.15   0.884    -.0165352    .0191036
  sk |   70.16767   34.36426     2.04   0.050     .0812979     140.254
 sk2 |  -22.56956   11.79029    -1.91   0.065    -46.61601    1.476893
cons |  -17.21865   23.29984    -0.74   0.465    -64.73899    30.30168
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On April 16th the quarterly results of Broadcom had been announced, which caused an 18% increase in stock price on April 17th. So there was not any uncertainty for near future. But for Microsoft the Quarterly results had not been announced on April 17th, so we can expect that the uncertainty of the dividends has been able to distort the implied volatility to a frowning one!

Conclusion:

In this study, the literature of volatility smile has been reviewed. For two high-tech companies we found different implied volatility figures which could give us more information about the risk-neutral implied distributions for two asset prices. While for Microsoft we found a frowning figure which hints to a big shift in the asset price, for Broadcom we did not distinguish any significant reason for rejecting a lognormal distribution of the asset price. If we had more data of option prices we could have been able to extract more information about the dynamic of market expectation of the asset prices. One of the main assumptions of this literature is the risk neutrality of agents which resulted in the implied distribution. This assumption is far from reality since most of the people are risk averse and they give more weights to the probability of unfavorable outcomes (Magill, M., and Quinzii, M. 1995 pp305-320). Thus, that could be a good reason for "Crashphobia" which was explained for volatility skew. So the implied distributions are somehow distorted by risk aversion of people and we should explain the results as expected distribution rather than objective distributions.

References :

1) Magill,M. , Quinzii, M(1995). *The incomplete market theory*, The MIT press.

2) Rubinstein, M., " Implied Binomial trees," *Journal of Finance*, 49,3 (July 1994), 771-818.