

# **A New Structural Approach to the Default Risk of Companies**

**By Pouyan Mashayekh Ahangarani<sup>1</sup>**

**June 2007**

---

<sup>1</sup> University of Southern California, Economics Department. Email: mashayek@usc.edu

The Merton model, which is used for modeling default probabilities, is based on the assumption that the equity value is an option on the asset of a company with the strike price equal to the company debts. In the Merton model, it is implicitly assumed that debts last for a fixed period of time, and determining the default point that is the strike price of the option has been controversial in the literature. In this paper, a new model is proposed that borrows the equity price determinant from the asset pricing literature where equity price is equal to present value of the expected dividends. The dividends should be paid from the asset surplus, which is the asset value minus the debts. Ultimately, the model will be proposed in which the parameters of asset surplus can be estimated from the equity prices without using the default point information. The empirical results show that the new model proposed in this paper has more information relative to the Merton model in explaining the default probabilities when the leverage of the company is fed to the model.

## Introduction

In this paper, I will propose a new model for corporate default risk. There are two types of models of default risk in the literature: structural models and reduced based models. In reduced based models, the default is modeled as a surprise. The probability of this surprise follows a jump diffusion process and therefore depends on an intensity parameter also called hazard rate. This hazard rate can be constant through time or allowed to be stochastic, thereby implying a term structure of probabilities of default. This hazard rate is either estimated to fit historical probability or fitted to current market data (calibration). The reduced form approaches are well documented by the contributions of Jarrow and Turnbull (1995); Jarrow, Lando, and Turnbull (1997); Duffie and Singleton (1999); Das and Tufano (1996); Lando (1998); Iben and Litterman (1991); Madan and Unal (1993); Shunbucher (1997); and Zhou (1997).

The structural approach relates the arrival of default to the dynamics of the underlying structure of the firm, thereby giving an economic significance to the establishment of the default rate. This approach was founded by Merton (1974, 1977) who used an application of option theory. In his theory, the value of the firm is supposed to be shared by two broad categories of claimants: the shareholders and the debt holders. Because of the limited liability of shareholders, they have a payoff that is positive whenever the face value owed to creditors can be reimbursed. Otherwise, it is zero. The shareholders' claim is then just a call on the value of the assets of the firm (also known as a European call). Thus, a bond is simply a right of a face amount to be reimbursed with the sale of a put to shareholders on the assets of the firm. A direct advantage of the structured approach is that credit default is not an unpredictable event here; there is a way to see the corporate conditions that affect the default rate. Relying on the evolution of the value of assets of the firm gives a continuity of the credit standing evolution of the firm, which makes the credit risk predictable. Unfortunately, since the value of the firm is not a tradable asset, the parameters of the structural model are difficult to estimate. Reduced form approaches appeared mainly because of this limitation. Black and Cox (1976) provided an

important extension of the Merton model. Their model has a first passage time structure where a default takes place whenever the value of the assets of a company drops below a barrier. The first passage models are more realistic in the sense that they let the default happens anytime before horizon. Other extensions of the Merton model are provided by Geske (1997); Kim, Ramaswamy, and Sundareson (1993); Leland (1994); Longstaff and Schwartz (1995); Leland and Toft (1996); and Zhou (2001).

Firms default when they cannot, or choose not to, meet their obligations. Credit events are triggered by movements of the firm's value relative to some (random or non-random) credit event triggering threshold. So if the firm's asset  $V_t$  is less than the face value of debt  $D_t$  at time  $t$ , then the firms will go bankrupt. Consequently, a major issue is the modeling of the evolution of the firm's value and of the firm's capital structure. In the structural approach, the default happens when  $V_t < D_t$ . In order to use the structural approach, we need to know the asset value  $V_t$  and the default point  $D_t$ . Asset values are not observable in the market, but since the equity price of firms can be found in the market, we are able to retrieve the asset values from the option theoretic approach of Merton if we know the default point.

Finding the default point has been one of the challenges of the structural approach in the credit risk modeling. In reality, firms often rearrange their liability structure when they have credit problems. Hence, there is no analytic method to derive the default point. Rather, it is estimated from a firm's liabilities information on its balance sheets. KMV Company has done empirical research based on large scale statistical studies of historical defaults, and they have found that the basic estimation for the default point is current liability plus half of long term liabilities (Demircubuk and Tse, 2001). Their approach has been based on defining the short term and long term liabilities in the balance sheet data. First, minority interests and deferred taxes are excluded from the total liabilities since those two items do not cause default stress. The problems arise when the firms do not break down their liabilities into current (due within one year) and long term on their balance sheets. There are no

regulations to stop firms from reporting their statements in this way, and there can be many reasons for them to do so. However, KMV has considered these adjustments for finding the default point estimation. Also, KMV has developed a model in which they have segmented the liabilities into six parts and have estimated the default point as a function of those liabilities<sup>2</sup>. All in all, finding the default point has been dependent on the precise analysis of accounting data. Since there are versatile accounting standards in different countries and since the firm level data are very noisy (because of moral hazard problem), relying on these data has been very risky.

The consensus in the finance literature is that the Merton model underpredicts the probability of defaults and the credit spreads. There are a couple of papers in the literature that test the Merton model. Jones, Mason, and Rosenfeld (1984) used the bond prices of companies during 1977-1981 and found that the prices implied from the Merton model overestimated the bond prices by 4.52%. Ogden (1987) found that the yield spread is underpredicted 104 bp if the Merton model is used. Eom, Helwege, and Huang (2004) found that the Merton model underestimates the credit yield spread. KMV Corporation is a successful company that used the Merton model to predict the default probabilities of individual firms. However, they found that the probabilities from the Merton model were too far from reality, and they calibrated a model in which the default probability is a function of distance of default found from the Merton model. For calibrating the model, they used a unique and proprietary database of corporate defaults of the last decades.

Since default is costly and violations to the absolute priority rule in bankruptcy proceedings are common, in practice shareholders have an incentive to put the firm into the receivership before the asset value of the firm hits the debt value (Hanson, Pesaran, and Schuermann 2005). In addition, the lending banks have the incentive to force the firm to default before the asset value hits the debt threshline (Garbade, 2001). Also, a borrower may be in default condition, e.g. a missed coupon payment, without going into bankruptcy, which usually happens in the banking-borrower relationship (Lawrence and Arshadi (1995)). Therefore, the default usually happens

---

<sup>2</sup> This model has not been disclosed for public.

when the asset value of the firm crosses the thresh line that is higher than the default point characterized by the liabilities of the firm. Since we have:  $V_i = D_i + E_i$ , where E is the equity value of the firm, we can deduct that the default happens when:  $0 < E_i < C_i$ . C is the positive threshold, which is time varying and is dependent on the firm's characteristics. The equity values are observable for the firms traded in the stock markets.

Estimation of threshold line C for each firm is the next step in modeling the default risk. As argued and elaborated in Pesaran, Hashem, Schuermann, Treutler, and Weiner (2005), accounting information is likely to be noisy and might not be all that reliable due to information asymmetries and agency problems between managers, shareholders, and debtholders. Moreover, the accounting based route presents additional challenges such as different accounting standards and bankruptcy rules in different countries. Also, other firm specific characteristics such as leverage, firm age, and management quality could be important in the determination of default thresholds that are quite difficult to observe. In view of these measurement problems, Pesaran et al. (2005) used the firm-specific credit ratings for finding the default thresholds. There are different rating agencies that rank the firms and assign them a credit rating. Moody, S&P, FITCH, CIBS, Nationsbank, and SBC are the companies that rate the firms, and they have their own terminology.

In this paper, I will propose a new model for the default risk that is unlike the Merton model. There is no need to use the default points, and it uses the equity prices as information for finding the default probabilities. Unlike the Pesaran et al. (2005) approach, it does not use just the equity prices. In the next section, I will propose the model. After that, I will show the empirical results of the model compare with other models. Finally, the paper concludes.

#### Thoretical Model

The main assumption of the Merton model is that the equity of a company is an option on the asset value. In that model, the strike price is the liabilities of the company, and time to maturity is usually assumed one year. However, these two assumptions might be far from reality since the liabilities of the company exit after a

year or a definite time period and are changing through time. Also, the asset value of the companies are not traded in the market, which is a key assumption in the option pricing.

Unlike the Merton model, the model I propose implements the main assumption of the asset pricing in which the equity price is the discounted dividend stream of the stock under the martingale measure:

$$\text{Equity Price} = E_Q[\int_0^\tau e^{-rt} \text{Div}(t) dt] \quad (2.1)$$

where  $\tau = \inf\{t \geq 0 : x_i(t) = 0\}$  is the time that company goes bankrupt.

$x_t$  is the asset surplus at time  $t$  and is equal to the assets minus the debts. In order to find the dividend stream, we assume that the manager of the company decides on paying the dividend in such a way that the equity price is maximized while he is taking into account that paying too many frequent and lavish dividends increases the probability of bankruptcy. I assume that the asset value minus the debt value of the company is the asset surplus in the company, which, if it hits zero, the company goes bankrupt. The question is what is the optimal dividend policy? I assume that the asset surplus is a process that satisfies the following stochastic differential equation:

$$dx(t) = \mu(x(t), t)dt + \sigma(x(t), t)dw(t)$$

and  $x$  will be absorbed at zero, which means the default happens as the asset value crosses the default point. Considering the dividend policy, the asset surplus can be written as:

$$x(t) = x_0 + \int_0^t \mu(x(t), t)dt + \int_0^t \sigma(x(t), t)dw - \text{div}(t), \quad (2.2)$$

where the manager of the company maximizes (2.1) with respect to the process defined in (2.2).

Shreve, Lehoczky, and Gaver (1984) showed that the solution to the above optimization problem is subtracting  $x-U$  if  $x > U$  where  $U$  is the optimal threshold. In other words, if asset surplus  $x$  is above the boundary  $U$ , then residuals should be paid as dividends and the value function will be:

$$EV_U(x) = \begin{cases} E_Q \left\{ \int_0^x e^{-rt} Div(t) dt \right\} & \text{if } 0 \leq x \leq U \\ x - U + EV_U(U) & \text{if } x \geq U \end{cases},$$

where  $r$  is the interest rate and the value function satisfies the following differential equation:

$$rEV_U(x) = rEV_U'(x) + \frac{1}{2}\sigma^2(x)EV_U''(x),$$

with the following boundary conditions:

$$EV_U(0) = 0, \quad EV_U'(U) = 1.$$

If  $\sigma^2(x) = \sigma^2$ , the general solution of the differential equation will be:

$$EV_U(x) = c_1 e^{m_1 x} + c_2 e^{m_2 x},$$

where  $m_1$  and  $m_2$  are the roots of the following characteristic function:

$$\sigma^2 m^2 + 2rm - 2r = 0,$$

And the answers are:

$$\begin{cases} m_1 = \frac{-r + \sqrt{r^2 + 2r\sigma^2}}{\sigma^2} \\ m_2 = \frac{-r - \sqrt{r^2 + 2r\sigma^2}}{\sigma^2} \end{cases}, \text{ in which } m_1 > 0 > m_2, \quad m_2^2 > m_1^2$$

With adding the condition:  $EV_U(0) = 0$ , the optimal  $U$  can be found and consequently the value function will be derived as:

$$U = \frac{1}{m_1 - m_2} \log \frac{m_2^2}{m_1^2},$$

$$EV(x) = \begin{cases} \frac{e^{m_1 x} - e^{m_2 x}}{m_1 e^{m_1 U} - m_2 e^{m_2 U}}, & 0 \leq x \leq U \\ EV(U) + x - U, & x > U \end{cases} \quad (2.3)$$

In the above notation,  $x$  is the asset surplus of the company and  $EV$  is the equity price observed in the market. When the asset surplus is greater than the threshold  $U$ ,

the company pays the difference as dividend, and that is the reason the equity value of the company at the time of paying dividend is equal to the equity value at U plus the dividend. That is what is found in (2.3). When asset surplus is less than the threshold U, the equity value is a nonlinearly increasing function of the asset surplus since  $m_1 > 0 > m_2$ . In other words, as the asset surplus increases, the equity value of the company increases as well. Also, as  $\sigma^2$  increases, the difference between  $m_1$  and  $m_2$  increases, which makes the equity value more sensitive to the asset surplus changes. This makes sense since, as the volatility of the asset surplus increases, the probability of default in the worst case and the probability of paying dividends increases.

The above formula does not need to consider the default point or the capital structure of the company to determine the relationship between asset surplus value and the equity value of the company. Also, there is no need to consider a fixed time to maturity that is necessary for using the option pricing theory. In order to find the probability of default or the distance to default, the parameters of the asset surplus defined in (2.1) should be estimated from the market data since the asset surplus value is not observable and just the equity values are observable in the market. The likelihood function is:

$$L(\mu, \sigma^2) = -\frac{n}{2} \ln(2\pi\sigma^2) - \frac{1}{2} \sum_{t=2}^n \frac{(EV(t+1) - EV(t) - \mu)^2}{\sigma^2} - \sum_{t=2}^n \ln EV(t) - \sum_{t=2}^n \ln \left( \frac{\partial EV(x_t)}{\partial x_t} \right) \quad (2.4)$$

in which the derivative of the equity relative to the asset surplus is:

$$\frac{\partial EV(x_t)}{\partial x_t} = \frac{m_1 e^{m_1 x_t} - m_2 e^{m_2 x_t}}{m_1 e^{m_1 U} - m_2 e^{m_2 U}}$$

When the parameters of the asset surplus are estimated, the distance to default can be found as:

$$\text{Distance to Default for a one year horizon} = \frac{x + 252 \times \mu}{\sqrt{252} \times \sigma} \quad (2.5)$$

## Empirical Results

The data of 666 companies in the year 2004 have been selected from CRSP database (WRDS database at Wharton school) for empirical comparison of the Merton model with the model proposed in this paper. Table 1 shows the distribution of the companies based on their industry names. These are all the companies that are traded for the 252 days of the year, and whose fiscal year ends by the end of the December. All the firms in the financial industry are excluded from the database. Finding the default point for the Merton model is not straightforward for financial firms. The default point for other firms is assumed as the current liabilities plus half of the long term liabilities. In the COMPUSTAT database of WRDS, variable data9 is the long term liability and data34 is the short term liability.  $\text{Data34} + 0.5\text{data9}$  has been used as the default point for the firm in the Merton model. For each company, the distance to default from the Merton model and the new model and finally the distance to default from equity price are calculated.

The distance to default from the Merton model is widely used in industry. In the Merton model, it is assumed that the asset value is equal to the equity value plus the debt, and the equity value can be derived from the option pricing formula as:

$$E_t = V_t \Phi(d_t) - F e^{-r(T-t)} \Phi(d_t - \sigma \sqrt{T-t}) \quad (3.1)$$

where  $\Phi$  is the cumulative distribution of standard Normal,  $F$  is the default point of the firm,  $T$  is equal to 252 days,  $r$  is  $0.04/252$ , and  $d_t$  is defined as:

$$d_t = \frac{\ln(F/V_t) + (\mu + \frac{1}{2}\sigma^2)(T-t)}{\sigma \sqrt{T-t}} \quad (3.2)$$

From (3.1) and (3.2), I can solve for  $\sigma$ . Consequently, the time series of asset values can be found, and finally  $\mu$  can be estimated from the asset values. After finding  $\mu$  and  $\sigma$ , the distance to default is derived using the following formula:

$$DD = \frac{\ln(\frac{V_t}{F}) + (\mu - \frac{\sigma^2}{2})T}{\sigma T} \quad (3.3)$$

The value found in (3.3) is the distance to default implied from the Merton model and will be called DDMerton hereafter.

The distance to defaults of the firms implied from the model proposed in this paper is found from (2.5) after maximum likelihood estimation of the parameters from (2.4) and will be called DDnew hereafter. Also, I found another distance to default derived from (2.5), but I simply assumed that the equity prices are following a simple diffusion model derived from:

$$dEV(t) = \mu dt + \sigma dw(t) \quad (3.4)$$

The parameters of the model defined in (3.4) can be estimated by finding the average and the standard deviation of the first difference of the equity value. I will name the distance to default found from (3.5) from the parameters of (3.4) simply as the DDEq.

The probability of default can be implied from the credit ratings of companies from S&P Company. The distribution of the default probabilities and their ratings are shown in Table 2 and Figure 1.

Table 3 shows the statistics of the three distances to defaults from the three different models: DDMerton, DDNew, and DDEq. On average, the distance to default from the new model presented in this paper has a higher value relative to the Merton model and DDEq.

Distance to default has been used for finding the probability of default of companies. The probability implied from the distance to default has been smaller than the historical probabilities. However, distance to default has been used as an informative variable in industry. The implied probabilities from the credit ratings of the firms have been used as a dependent variable in order to see how the three different distances to defaults can explain the default probabilities. For all three models, the distance to default was a significant variable in explaining the default probability. The regression results are shown in Table 4. The square of the distance has been significantly nonzero, so it is added to regressions while it was not significant for DDEq. Comparing the adjusted R-square shows that the DDNew has a higher explanatory power relative to DDMerton and DDEq since the R-squares are 18, 23, and 17% for DDMerton, DDNew, and DDEq, respectively. Comparison of regressions for DDEq and DDNew show that the transformation done in the model presented in this paper increases the explanatory power of the new distance to default

since DDNew and DDEq just use the equity price information and do not use the accounting information of the companies. While the Merton model uses the default point as an input to the model, in order to compare the models better, the ratio of default point to the equity price also has been added to the regressions. The results are shown in Table 5. This variable, which is a proxy for the leverage of a company, has been significant for the all three models. The coefficient of variables for DDNew and DDEq model is positive, which makes sense, since as the leverage of the company increases, the probability of default increases. Interestingly the R-Square of the DDNew model is 25% while for DDMerton is 20% this shows the distance to default found from the new model has a better explanatory. Therefore, the model proposed in this paper that drops the option theoretic assumption of the Merton model carries more information compared to the distance to default implied from the Merton model.

Industry equity prices have been used for finding the correlation between different firms. The transformation derived from the model proposed in this paper explains the distance to default better, so the asset surplus defined in this paper will be a better variable to explain the default correlation between companies.

### Conclusion

In this study, a new model for corporate default has been proposed. The Merton model that has been widely used in industry is dependent on the assumption that the equity value is an option on the asset value of the company. This assumption is far from reality. First, the asset value of a company is not tradable, and second, using an option approach, we need to assume an expiration date for the option that implicitly assumes that the debts of company have a fixed age while in practice the debts are changing through time.

In the model proposed in this paper, the asset pricing approach has been used, and the dividend policy assumed follows an optimal strategy. In this model, there is no need to use the default point that brings difficulties in the Merton model. Finally, the empirical test for two models shows that the new model has a better explaining power for default probabilities when the leverage ratios are added to the model.

More work should be done. By using CDS data, the explanatory power of the distances to default can be tested by using CDS spreads. Also, by assuming the geometric Brownian motion for the asset surplus, distance to default can be derived, which may explain the CDS spreads better.

Table 1 The Industry Names of the Companies Used in This Study and Their Distribution

Industry Name	Number of firms	Percentage
ABRASIVE,ASBESTOS,MISC MINRL	1	0.15
ACCIDENT & HEALTH INSURANCE	4	0.6
ACCOUNT,AUDIT,BOOKKEEP SVCS	2	0.3
ADVERTISING	2	0.3
ADVERTISING AGENCIES	2	0.3
AGRICULTURAL SERVICES	2	0.3
AGRICULTURE CHEMICALS	4	0.6
AGRICULTURE PRODUCTION-CROPS	2	0.3
AIR COND,HEATING,REFRIG EQ	2	0.3
AIR TRANSPORT, SCHEDULED	4	0.6
AIRCRAFT	2	0.3
AIRCRAFT AND PARTS	1	0.15
AIRCRAFT PARTS, AUX EQ, NEC	2	0.3
APPAREL & OTHER FINISHED PDS	1	0.15
ARRANGE TRANS-FREIGHT, CARGO	1	0.15
AUTO AND HOME SUPPLY STORES	1	0.15
AUTO DEALERS, GAS STATIONS	5	0.75
AUTO RENT & LEASE,NO DRIVERS	1	0.15
BAKERY PRODUCTS	1	0.15
BALL AND ROLLER BEARINGS	1	0.15
BEER,WINE,ALCOHOLIC BEV-WHSL	1	0.15
BEVERAGES	1	0.15
BIOLOGICAL PDS,EX DIAGNSTICS	8	1.2
BITMNS COAL,LIGNITE SURF MNG	1	0.15
BITUMINOUS COAL, LIGNITE MNG	3	0.45
BLANKBOOKS,BINDERS,BOOKBIND	1	0.15
BLAST FURNACES & STEEL WORKS	1	0.15
BOOKS: PUBG, PUBG & PRINTING	1	0.15
BTLD & CAN SOFT DRINKS,WATER	6	0.9
BUSINESS SERVICES, NEC	3	0.45
CABLE AND OTHER PAY TV SVCS	4	0.6
CALCULATE,ACCT MACH,EX COMP	2	0.3
CARPETS AND RUGS	1	0.15
CATALOG, MAIL-ORDER HOUSES	2	0.3
CEMENT, HYDRAULIC	1	0.15
CHEMICALS & ALLIED PRODS	2	0.3
CIGARETTES	2	0.3
CMP INTEGRATED SYS DESIGN	4	0.6
CMP PROCESSING,DATA PREP SVC	3	0.45
CMP PROGRAMMING,DATA PROCESS	3	0.45
COGENERATN-SM POWER PRODUCER	1	0.15
COML PHYSICAL, BIOLOGCL RESH	2	0.3
COMMERCIAL PRINTING	2	0.3

COMMUNICATIONS SERVICES, NEC	3	0.45
COMPUTER & OFFICE EQUIPMENT	1	0.15
COMPUTER PERIPHERAL EQ, NEC	3	0.45
COMPUTER STORAGE DEVICES	1	0.15
COMPUTERS & SOFTWARE-WHSL	1	0.15
CONGLOMERATES	3	0.45
CONSTR,MINING,MATL HANDLE EQ	1	0.15
CONSTRUCTION MACHINERY & EQ	2	0.3
CONSTRUCTION-SPECIAL TRADE	1	0.15
CONVENIENCE STORES	1	0.15
CONVRT PAPR,PAPRBRD,EX BOXES	5	0.75
CREDIT REPORTING AGENCIES	1	0.15
CRUDE PETROLEUM & NATURAL GS	16	2.4
CUTLERY,HAND TOOLS,GEN HRDWR	2	0.3
DAIRY PRODUCTS	2	0.3
DEEP SEA FRN TRANS-FREIGHT	6	0.9
DENTAL EQUIPMENT & SUPPLIES	1	0.15
DEPARTMENT STORES	1	0.15
DETECT,GUARD,ARMOR CAR SVCS	1	0.15
DOLLS AND STUFFED TOYS	1	0.15
DRAWNG,INSULATNG NONFER WIRE	1	0.15
DRILLING OIL AND GAS WELLS	8	1.2
DRUG & PROPRIETARY STORES	3	0.45
DRUGS AND PROPRIETARY-WHSL	1	0.15
EATING PLACES	5	0.75
ELEC APPARATUS & EQUIP-WHSL	1	0.15
ELECTR, OTH ELEC EQ, EX CMP	1	0.15
ELECTRIC & OTHER SERV COMB	18	2.7
ELECTRIC HOUSEWARES AND FANS	1	0.15
ELECTRIC LIGHTING,WIRING EQ	2	0.3
ELECTRIC SERVICES	21	3.15
ELECTRICAL INDL APPARATUS	1	0.15
ELECTRICAL WORK	2	0.3
ELECTROMEDICAL APPARATUS	1	0.15
ELECTRONIC COMP, ACCESSORIES	1	0.15
ELECTRONIC COMPONENTS, NEC	2	0.3
ELECTRONIC CONNECTORS	1	0.15
ELECTRONIC PARTS,EQ-WHSL,NEC	1	0.15
ENGINES AND TURBINES	2	0.3
EQUIP RENTAL & LEASING, NEC	3	0.45
FABRICATED STRUCTURAL METAL	1	0.15
FACILITIES SUPPORT MGMT SVCS	1	0.15
FARM MACHINERY AND EQUIPMENT	1	0.15
FEDERAL CREDIT AGENCIES	1	0.15
FIRE, MARINE, CASUALTY INS	15	2.25
FOOD AND KINDRED PRODUCTS	2	0.3
GAMES,TOYS,CHLD VEH,EX DOLLS	1	0.15
GAS & OTHER SERV COMBINED	2	0.3
GEN BLDG CONTRACTORS-NONRES	1	0.15

GEN MED & SURGICAL HOSPITALS	6	0.9
GENERAL INDUSTRIAL MACH & EQ	2	0.3
GLASS CONTAINERS	2	0.3
GLASS, GLASSWR-PRESSED, BLOWN	1	0.15
GOLD AND SILVER ORES	3	0.45
GRAIN MILL PRODUCTS	2	0.3
GROCERIES, GENERAL LINE-WHSL	1	0.15
GROCERY STORES	1	0.15
GUIDED MISSILES & SPACE VEHC	1	0.15
HAZARDOUS WASTE MANAGEMENT	2	0.3
HEAVY CONSTR-NOT BLDG CONSTR	2	0.3
HELP SUPPLY SERVICES	1	0.15
HOME HEALTH CARE SERVICES	2	0.3
HOSPITAL & MEDICAL SVC PLANS	4	0.6
HOTELS, MOTELS, TOURIST COURTS	4	0.6
HOUSEHOLD APPLIANCES	1	0.15
HOUSEHOLD FURNITURE	2	0.3
IN VITRO, IN VIVO DIAGNOSTICS	2	0.3
INDL INORGANIC CHEMICALS	3	0.45
INDUSTRIAL MEASUREMENT INSTR	3	0.45
INDUSTRIAL ORGANIC CHEMICALS	5	0.75
INS AGENTS, BROKERS & SERVICE	5	0.75
INVESTMENT ADVICE	2	0.3
KNIT OUTERWEAR MILLS	1	0.15
LAB ANALYTICAL INSTRUMENTS	3	0.45
LIFE INSURANCE	15	2.25
LUMBER AND WOOD PDS, EX FURN	3	0.45
MACHINERY AND EQUIPMENT-WHSL	2	0.3
MALT BEVERAGES	1	0.15
MANAGEMENT CONSULTING SVCS	1	0.15
MANAGEMENT SERVICES	1	0.15
MED, DENTAL, HOSP EQ-WHSL	1	0.15
MEDICAL LABORATORIES	3	0.45
METAL CANS	3	0.45
METAL MINING	3	0.45
METALWORKING MACHINERY & EQ	2	0.3
MILLWORK, VENEER, PLYWOOD	1	0.15
MISC AMUSEMENT & REC SERVICE	12	1.8
MISC BUSINESS CREDIT INSTN	1	0.15
MISC BUSINESS SERVICES	1	0.15
MISC CHEMICAL PRODUCTS	5	0.75
MISC ELEC MACHY, EQ, SUPPLIES	1	0.15
MISC EQUIP RENTAL & LEASING	1	0.15
MISC FABRICATED METAL PRODS	3	0.45
MISC HEALTH & ALLIED SVC, NEC	2	0.3
MISC MANUFACTURNG INDUSTRIES	1	0.15
MISC PDS OF PETROLEUM & COAL	1	0.15
MISC PRIMARY METAL PRODUCTS	1	0.15
MISC SHOPPING GOODS STORES	1	0.15

MISCELLANEOUS METAL ORES	2	0.3
MNG, QUARRY NONMTL MINERALS	3	0.45
MOBILE HOMES	1	0.15
MOTION PIC, VIDEOTAPE PRODTN	1	0.15
MOTION PICTURE THEATERS	2	0.3
MOTOR VEH PARTS, SUPPLY-WHSL	1	0.15
MOTOR VEHICLE PART,ACCESSORY	5	0.75
MOTOR VEHICLES & CAR BODIES	2	0.3
MOTORCYCLES,BICYCLES & PARTS	1	0.15
MUSICAL INSTRUMENTS	1	0.15
NATURAL GAS DISTRIBUTION	6	0.9
NATURAL GAS TRANSMIS & DISTR	2	0.3
NATURAL GAS TRANSMISSION	4	0.6
NEWSPAPER:PUBG, PUBG & PRINT	3	0.45
NURSING & PERSONAL CARE FAC	1	0.15
OIL & GAS FIELD MACHY, EQUIP	6	0.9
OIL, GAS FIELD SERVICES, NEC	2	0.3
OPERATIVE BUILDERS	7	1.05
ORTHO,PROSTH,SURG APPL,SUPLY	5	0.75
PAINTS, VARNISHES, LACQUERS	3	0.45
PAPER AND ALLIED PRODUCTS	1	0.15
PAPER MILLS	5	0.75
PAPERBOARD CONTAINERS, BOXES	2	0.3
PAPERBOARD MILLS	6	0.9
PATENT OWNERS AND LESSORS	1	0.15
PERFUME,COSMETIC,TOILET PREP	2	0.3
PERIODICAL:PUBG,PUBG & PRINT	1	0.15
PERSONAL SERVICES	4	0.6
PETROLEUM REFINING	15	2.25
PHARMACEUTICAL PREPARATIONS	20	3
PHONE COMM EX RADIOTELEPHONE	14	2.1
PHOTOGRAPHIC EQUIP & SUPPL	1	0.15
PIPE LINES, EX NATURAL GAS	1	0.15
PLASTIC MATL,SYNTHETIC RESIN	1	0.15
PLASTICS PRODUCTS, NEC	3	0.45
PLASTICS,RESINS,ELASTOMERS	5	0.75
PREPACKAGED SOFTWARE	3	0.45
PRIM SMELT,REFIN NONFER METL	3	0.45
PROF & COML EQ & SUPPLY-WHSL	2	0.3
PUBLIC BLDG & REL FURNITURE	2	0.3
PUBLIC WAREHOUSING	1	0.15
PULP MILLS	3	0.45
PUMPS AND PUMPING EQUIPMENT	2	0.3
PWR,DISTR,SPECL TRANSFORMERS	1	0.15
RACING,INCL TRACK OPERATIONS	1	0.15
RADIO BROADCASTING STATIONS	9	1.35
RADIO,TV BROADCAST, COMM EQ	5	0.75
RADIO,TV,CONS ELECTR STORES	1	0.15
RADIOTELEPHONE COMMUNICATION	10	1.5

RAILROAD EQUIPMENT	1	0.15
RAILROADS,LINE-HAUL OPERATNG	7	1.05
REAL ESTATE	1	0.15
REAL ESTATE AGENTS & MGRS	2	0.3
REFUSE SYSTEMS	3	0.45
ROLLING & DRAW NONFER METAL	6	0.9
RR SWITCHING, TERMINAL ESTAB	1	0.15
SAVINGS INSTN, NOT FED CHART	2	0.3
SAVINGS INSTN,FED CHARTERED	4	0.6
SCRAP & WASTE MATERIALS-WHSL	1	0.15
SEC SMELT,REFIN NONFER METAL	1	0.15
SECURITY BROKERS & DEALERS	5	0.75
SEMICONDUCTOR,RELATED DEVICE	11	1.65
SHIP & BOAT BLDG & REPAIRING	2	0.3
SILVER ORES	1	0.15
SKILLED NURSING CARE FAC	2	0.3
SOAP,DETERGENT,TOILET PREPS	1	0.15
SOCIAL SERVICES	2	0.3
SPEC OUTPATIENT FACILITY,NEC	1	0.15
SPECIAL INDUSTRY MACHINERY	1	0.15
SPECIAL INDUSTRY MACHY, NEC	3	0.45
SPORTING & ATHLETIC GDS,NEC	1	0.15
SRCH,DET,NAV,GUID,AERO SYS	2	0.3
STEEL PIPE AND TUBES	1	0.15
STEEL WORKS & BLAST FURNACES	5	0.75
SUBDIVID,DEVELOP,EX CEMETERY	1	0.15
SUGAR & CONFECTIONERY PRODS	1	0.15
SURETY INSURANCE	1	0.15
SURGICAL,MED INSTR,APPARATUS	3	0.45
TELE & TELEGRAPH APPARATUS	2	0.3
TELEVISION BROADCAST STATION	9	1.35
TIRES AND INNER TUBES	1	0.15
TITLE INSURANCE	2	0.3
TOBACCO PRODUCTS	1	0.15
TRANSPORTATION SERVICES	2	0.3
TRUCKING, EXCEPT LOCAL	3	0.45
TRUCKING,COURIER SVC,EX AIR	1	0.15
UNSUPP PLASTICS FILM & SHEET	1	0.15
WATER SUPPLY	5	0.75
WATER TRANSPORTATION	3	0.45
WOMENS,MISSES,JRS OUTERWEAR	3	0.45

Table 2 The Distribution of the Companies Based on Their Credit Ratings

S&P Credit Rating	Corresponding Probability of Default in One Year in bp	Number of Companies	Probability Distribution
AAA	0	5	0.75
AA+	0	2	0.3
AA	0.01	11	1.65
AA-	0.01	8	1.2
A+	0.03	23	3.45
A	0.04	46	6.91
A-	0.08	58	8.71
BBB+	0.12	65	9.76
BBB	0.21	83	12.46
BBB-	0.42	68	10.21
BB+	0.72	40	6.01
BB	1.46	59	8.86
BB-	2.8	83	12.46
B+	4.15	57	8.56
B	5.71	33	4.95
B-	10.55	15	2.25
CCC+	15.93	6	0.9
CCC	17.83	3	0.45

Table 3 The Statistics of the Three Different Distances to Default

<b>Variable</b>	<b>Obs</b>	<b>Mean</b>	<b>Std. Dev.</b>	<b>Min</b>	<b>Max</b>
DDmerton	666	8.93	4.23	0.91	28.14
DDnew	666	10.60	8.41	0.10	41.74
DDeq	666	4.59	2.13	0.04	10.71

Table 4 Regression Results for Different Distances to Default

Model 1: Heteroskedasticity-Corrected Estimates Using the 666 Observations 1-666  
 Dependent Variable: defprob

<i>Variable</i>	<i>Coefficient</i>	<i>Std. Error</i>	<i>t-statistic</i>	<i>p-value</i>	
const	4.95234	0.414919	11.9357	<0.00001	***
ddmerton	-0.554884	0.0566501	-9.7949	<0.00001	***
ddmerton2	0.0158674	0.00190659	8.3224	<0.00001	***

Statistics based on the weighted data:

Unadjusted  $R^2 = 0.187614$

Adjusted  $R^2 = 0.185164$

Model 2: Heteroskedasticity-Corrected Estimates Using the 666 Observations 1-666  
 Dependent Variable: defprob

<i>Variable</i>	<i>Coefficient</i>	<i>Std. Error</i>	<i>t-statistic</i>	<i>p-value</i>	
const	3.27636	0.233502	14.0314	<0.00001	***
ddnew	-0.23326	0.0205545	-11.3484	<0.00001	***
ddnew2	0.00426019	0.000455815	9.3463	<0.00001	***

Statistics based on the weighted data:

Unadjusted  $R^2 = 0.236763$

Adjusted  $R^2 = 0.23446$

Model 3: Heteroskedasticity-Corrected Estimates Using the 666 Observations 1-666  
 Dependent Variable: defprob

<i>Variable</i>	<i>Coefficient</i>	<i>Std. Error</i>	<i>t-statistic</i>	<i>p-value</i>	
const	4.51487	0.431177	10.4710	<0.00001	***
ddneweq	-0.850019	0.126953	-6.6955	<0.00001	***
ddneweq2	0.041599	0.00920547	4.5189	<0.00001	***

Statistics based on the weighted data:

Unadjusted  $R^2 = 0.174565$

Adjusted  $R^2 = 0.172075$

Table 5 Regression Results for Different Distances to Default and Considering the Leverage Ratio

Model 4: Heteroskedasticity-Corrected Estimates Using the 666 Observations 1-666  
Dependent Variable: defprob

<i>Variable</i>	<i>Coefficient</i>	<i>Std. Error</i>	<i>t-statistic</i>	<i>p-value</i>	
const	5.35262	0.43858	12.2044	<0.00001	***
ddmerton	-0.571522	0.0552708	-10.3404	<0.00001	***
ddmerton2	0.0156459	0.00179456	8.7185	<0.00001	***
ratio	-0.810494	0.30609	-2.6479	0.00829	***

Statistics based on the weighted data:

Unadjusted  $R^2 = 0.206049$

Adjusted  $R^2 = 0.202451$

Model 5: Heteroskedasticity-Corrected Estimates Using the 666 Observations 1-666  
Dependent Variable: defprob

<i>Variable</i>	<i>Coefficient</i>	<i>Std. Error</i>	<i>t-statistic</i>	<i>p-value</i>	
const	3.04878	0.211678	14.4029	<0.00001	***
ddnew	-0.231707	0.0220659	-10.5007	<0.00001	***
ddnew2	0.0043569	0.00057132	7.6260	<0.00001	***
ratio	0.486152	0.170649	2.8488	0.00452	***

Statistics based on the weighted data:

Unadjusted  $R^2 = 0.259682$

Adjusted  $R^2 = 0.256327$

Model 6: Heteroskedasticity-Corrected Estimates Using the 666 Observations 1-666  
Dependent Variable: defprob

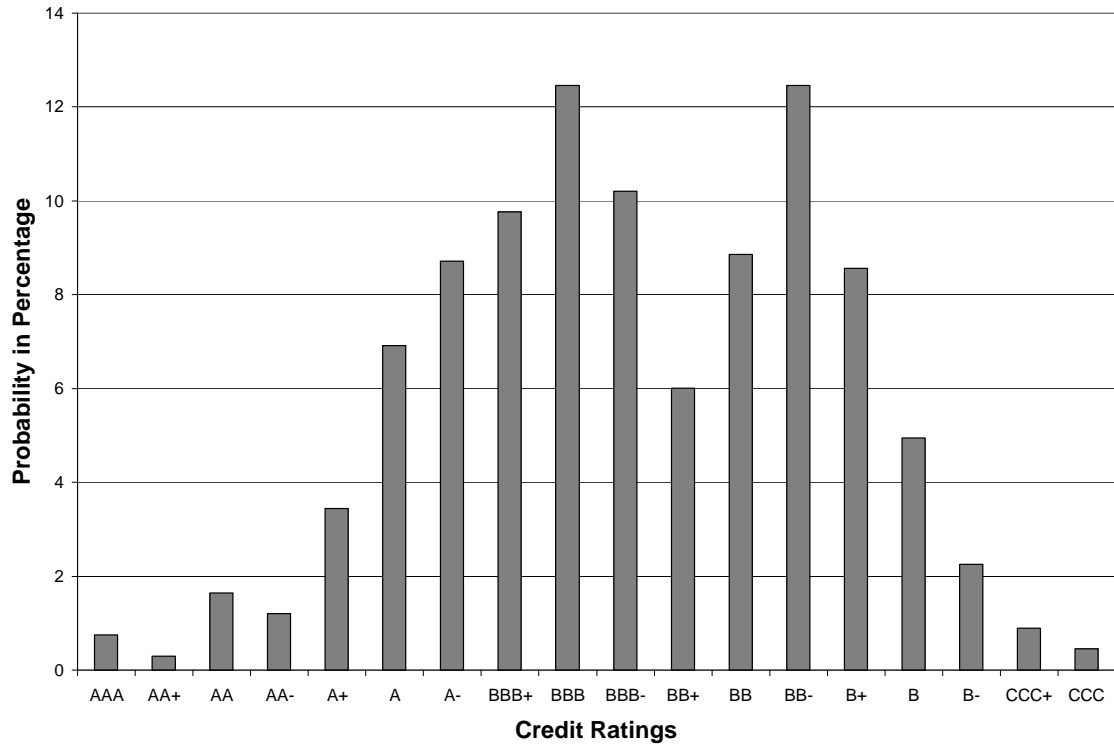
<i>Variable</i>	<i>Coefficient</i>	<i>Std. Error</i>	<i>t-statistic</i>	<i>p-value</i>	
const	3.52534	0.440168	8.0091	<0.00001	***
ddneweq	-0.653803	0.142902	-4.5752	<0.00001	***
ddneweq2	0.0291605	0.0115951	2.5149	0.01214	**
ratio	0.950986	0.258266	3.6822	0.00025	***

Statistics based on the weighted data:

Unadjusted  $R^2 = 0.14189$

Adjusted  $R^2 = 0.138002$

Figure 1- The Probability Distribution of the Companies Based on Their Credit Ratings



## References

- Andreasen, (2001), Credit Explosives, *working paper*, Bank of America. Black, Fisher and John C. Cox, (1976), “Valuing Corporate Securities: Some Effects of Bond Indenture Provisions”, *The Journal of Finance*, 31 (2), 351-67. Das, Sanjiv and Peter Tufano, (1996),”Pricing Credit Sensitive Debt when Interest Rates, Credit Ratings and Credit Spreads are Stochastic”, *Journal of Financial Engineering*, 5 (2), June.
- Demircubuk, N. and Tse, T., (2001), “Default Point Estimation”, *KMV company*. Duffie, D., Lando, D., (1997), “Term Structure of Credit Spreads with Incomplete Accounting Information. *Working Paper*, Stanford University, Stanford, CA. Duffie, Darrel and Ken Singleton, (1999), “Modeling Term Structure of Defaultable Bonds”, *Review of Financial Studies*, Special 1999, 12 (4), 687-720.
- Eom, Y., J. Helwege and J. Huang, (2004), “ Structural Models of Corporate Bond Pricing: An Empirical Analysis”, *Review of Financial Studies* 17,499-544.
- Garbade, Kenneth (2001). *Pricing Corporate Securities as Contingent Claims*, Cambridge, MA: MIT Press.
- Geske, Robert, (1977), “The Valuation of Corporate Liabilities as Compound Options”, *Journal of Financial and Quantitative Analysis*, pp. 541-52.
- Hanson, S. , Pesaran, M.H and Schuermann, T., (2005), “The Scope for Credit Risk Diversification”, *Working Paper*, University of Cambridge.
- Iben, Th. And R. Litterman, (1991), “Corporate Bond Valuation and the Term Structure of Credit Spreads”, *Journal of Portfolio Management*, 17 (3), Spring, 1991, 52-64.
- Jarrow, R. and Stuart M.Turnbull, (2000),”Pricing Derivatives on Financial Securities Subject to Credit Risk” , *Journal of Finance*,50 (1), March,53-85.
- Jarrow, Robert, David Lando and Stuart Turnbull,(1997) , “A Markov Model of Term Structure of Credit Spread”, *Review of Financial Studies*, 10 (2), Summer.

Jones, E., Mason, S. and E. Rosenfeld, (1984), "Contingent Claims Analysis of Corporate Capital Structures: An Empirical Investigation", *Journal of Finance* 39, 611-627.

Kim, I. J., Ramaswamy, K., and Sundaresan, S. M.,(1993) "Valuation of Corporate Fixed-Income Securities", *Financial Management*, p.117-131.

Lando, D., (1998), "On Cox Processes and Credit Risk Bonds", *Review of Derivatives Research*, 2, 99-120.

Lawrence, Edward C. and Nasser Arshadi (1995), "A Multinomial Logit Analysis of Problem Loan Resolution Choices in Banking", *Journal of Money, Credit and banking* 27, 202-216.

Leland, Hayne E., (1994), "Corporate Debt Value, Bond Covenants and Optimal Capital Structure", *Journal of Finance*, 49 (4), September, 1213-52.

Leland, H. E. and K. B. Toft, (1996), "Optimal Capital Structure, Endogenous Bankruptcy and the Term Structure of Credit Spreads", *Journal of Finance*, 51 (3), July, pp. 987-1019.

Longstaff, F. and E. Schwartz, (1995), "A Simple Approach to Valuing Risky Fixed and Floating Rate Debt", *Journal of Finance*, Vol 50, No. 3., 789-819.

Madan, D. and H. Unal, (1993), "Pricing of risks of Default", *Working paper*, College of Business, University of Maryland.

Merton, Robert C., (1974), "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates", *The Journal of Finance*, 29, May, 449-70.

Merton, Robert C., (1977), "On the Pricing of Contingent Claims and the Modigliani-Miller Theorem", *Journal of Financial Economics*, 5, 241-9.

Ogden, J. (1987), "Determinants of Ratings and Yields on Corporate Bonds: Tests of the Contingent Claims Model", *Journal of Financial Research*, 10,329-339.

Pesaran, M.Hashem, Til Schuermann, Bjorn-Jakob Treutler and Scott M. Weiner (2005), "Macroeconomic Dynamics and Credit Risk: A Global Perspective." Forthcoming, *Journal of Money, Credit and Banking*; available at Wharton Financial Institution Centre Working Paper #03-13B.

Schunbucher, P., (1997), "The Term Structure of Defaultable Bond Prices", *Working Paper*, University of Bonn.

Shreve, S. E., J. P. Lehoczky and D. P. Gaver (1984), "Optimal Consumption for General Diffusions with Absorbing and Reflecting Barriers", *Siam J. Control and Optimization*, Vol 22, No.1, January 1984.

Zhou, Chunsheng, (1997), "A Jump-Diffusion Approach to Modeling Credit Risk and Valuing Defaultable Securities", *Working Paper*, Federal Reserve Board, Washington, 47pp.

Zhou, C., (2001), "The Term Structure of Credit Spreads with Jump Risk", *Journal of Banking and Finance*, Vol.25, 2015-2040.