The Importance of Simultaneous Jumps in Default Correlation

By Pouyan Mashayekh Ahangarani[1]

January 2007

[1] University of Southern California, Economics Department. Email: mashayek@usc.edu

Abstract

Correlated defaults have been an important area of research in credit risk analysis with the advent of a basket of credit derivatives. Even the simple credit derivatives should be considered a basket of two default risks since the bankruptcy risk of the derivative issuer is also a factor. Considering jumps in the asset value helps to model the surprise risk of default in a group of firms. Simultaneous jumps in the asset values of companies can explain the default correlation. The multivariate jump diffusion model is used for modeling the asset value in the structural approach to credit risk modeling. GMM implemented on the moments generated by empirical characteristic function is the method used for estimation of the parameters. The principal component method is used for reducing the hassle of moment conditions in the characteristic function estimation of the model. At the end, the empirical result of joint default credit risk of a basket of two firms, Ford and General Motors, are shown using two models: one without jump and the other one with the simultaneous jump. Model selection criterion proves that the model with jump is a better model. The model without simultaneous jump underestimates the joint default probability of two firms.

Introduction

This section provides a brief summary and explanation of default risk and the need for regulation in the banking system that was the main reason for the development of credit risk mitigation. Then the credit derivative will be briefly introduced. At the end, the summary of different approaches in credit modeling will be represented.

Credit Risk

Default risk is intrinsically linked to the payment obligation that the obligor should honor. An obligor, who does not have any payment obligations, does not have any default risk either. Therefore this definition only covers the default risk of a payment obligation but not the default risk of the obligor himself. In principal, an obligor could not pay one of his obligations but honor another. But the bankruptcy codes and contract laws have prevented the agents to default risk in one of their obligations. So we can speak of the default risk of an obligor without specifying a particular payment obligation because the obligor has to honor all his payment obligations as long as he is able to. If he is not able to do so, a workout procedure is entered. The obligor loses control of all his assets and an independent agent tries to find ways to pay off the creditors using the obligor's assets. The bankruptcy code ensures that all creditors of the obligor are treated fairly and in accordance with a determined procedure. In particular, it is ensured that a default on one obligation entails a default on all other obligations. The default only occurs if the obligor really cannot pay his obligations. The default almost always entails a loss to the creditor, but the obligors who are not bound by bankruptcy codes, e.g. sovereign borrowers in countries without a properly functioning legal system, frequently make use of the possibility to default only on selected obligations, sometimes without being in real financial distress.

The general properties of default risk that make their quantitative modeling difficult are below:

1) Default events are rare.

2) They may occur unexpectedly.
3) Default events involve significant losses.
4) The size of these losses is unknown before default.

The most important components of credit risk are:

1. Arrival risk: The measure of arrival risk is the probability of default during a given time horizon.
2. Timing risk: This is the uncertainty about the precise time of default. The information about the time of default covers the knowledge of arrival risk.
3. Recovery risk: The uncertain quantity of payoff to a creditor at the time of default is the recovery risk. This can be expressed by the conditional probability distribution of the recovery rate upon default.
4. Default correlation risk: If there are several obligors, the risk of a couple of them defaulting together is the default correlation risk. This risk can be explained by the joint arrival risk and joint timing risk.

The Need for Regulation

Risk taking is a normal behavior of financial institutions given that risk and expected return are so tightly interrelated. But the banking system is subject to "moral hazard;" enjoying profit sharing is an incentive for taking more risks because of the absence of penalties. Sometimes, adverse conditions are incentives to maximize risks. When banks face serious difficulties, the bankers do not care to limit risks. In such situations, and in the absence of aggregate behavior, failure becomes almost unavoidable. By taking additional risks, banks maximize the chances of survival. The higher the risk, the wider will be the range of possible outcomes, including favorable ones. At the same time, the losses of shareholders and managers do not increase because of the limited liability of shareholders. In the absence of real downside risk, it becomes rational for bankers to increase risk, so the manager gets the upside of the bet with a limited downside. The potential for upside gain without the downside risk encourages risk taking because it maximizes the expected gain.

However, the bankruptcy of a bank has a negative externality for other banks and financial institutions. Since banks work on the trust of people, if a bank goes bankrupt, other banks will be in trouble, too. There are a couple of bankruptcy cases in which other banks have set aside a lot of money in order to save a bankrupt bank. Therefore, the banking system has imposed some regulations on banks lest the banking system risk increases.

The first regulation in banking was the 1988 Basel Accord, which became the standard for capital requirement for internationally active banks, first in Group-of-Ten (G10) countries and Switzerland and subsequently in more than 100 countries. The basic idea of the accord is that banks must hold capital of at least 8% of the total risk-weighted assets. In order to calculate this total measure of assets, each asset is multiplied by a risk weighting factor that, in principal, represents the credit quality of that asset. As of June 2001, the risk weights for nontrading portion of bank portfolios set by 1988 BIS were:

1) 0% : Cash and claims as central government and central banks, denominated and funded in their national currency.
2) 20% : Claims on banks incorporated in OECD countries and cash items in the process of collection.
3) 50% : Loans fully secured by mortgage on residential property that is rented or occupied by the borrower.
4) 100% : Claims on the private sector, claims on banks outside the OECD with a residual maturity of more than one year, and real estate investments.

There were some imbalances in the 1988 BIS Accord. For example, corporate loans received 100% risk weighting even though they might have a AAA rating that was five times the cash requirement for a country bond of Turkey that was a speculatively rated OECD country.

In order to address the obvious imbalances created by the 1988 accord, a new capital accord was set in 2001 and implemented beginning in 2005. In the new Basel Accord, there are two approaches for assessing the credit risk: 1) standardized and 2) internal rating-based. Table 1 shows the proposed risk weights for the standardized approach.

The standardized approach will better recognize the benefits of credit risk mitigation compared to the 1988 Accord. A disadvantage of the standardized approach is that exposure to unrated obligors will receive 100% risk weights. As shown in the table, because certain low rated obligors carry an even higher risk of 150%, the standardized approach sets up a perverse incentive for obligors to become unrated or for banks to loan to poor-quality unrated borrowers over certain rated borrowers.

Under the internal rating-based approach, a bank could, subject to limits and approval, use its own internal credit ratings. The ratings must correspond to benchmarks for one year default probabilities. The internal ratings methodology must be recognized by the bank's regulator and have been in place for at least three years. Under this new accord, credit derivatives can be used for mitigating the credit risk of bank portfolios. That is the reason why credit derivatives have grown tremendously during the last few years.

Credit Derivatives

A credit derivative is a derivative security that has a payoff that is conditioned on the occurrence of a credit event. The credit event is defined with respect to a reference credit (or several reference credits), and the reference credit asset(s) issued by the reference credit. The reference credit is an issuer whose default triggers the credit event. A credit event is a precisely defined event, determined by negotiation between the parties at the outset of a credit derivative. Market standards typically specify the existence of publicly available information confirming the occurrence, with respect to the reference credit, of bankruptcy, insolvency, restructuring, failure to pay, and cross default. If the credit event occurs, the default payment has to be made by one of the counterparties. Besides the default payment, a credit derivative can have further payoffs that are not default contingent.

The credit derivative, probably one of the most important types of new financial products introduced during the last decade, was first publicly introduced in 1992 at the International Swaps and Derivative Association (ISDA) annual meeting in Paris. In addition to the credit derivative, there is a great variety of products: credit default swaps (CDS), credit options, credit spread products, total rate of return swaps and credit link notes (CLN), collaterized debt obligation (CDO), collaterized loan obligations (CLO), and collaterized mortgage obligations (CMO).

The market for the credit derivative was created in the early 1990s in London and New York, and the growth of the credit derivative market has been extraordinary over the last decade. British Bankers' Association (BBA) indicated that the notional volume of credit derivatives increased globally from $180 bn in 1997 to $0.95 tn in 2002. The credit derivatives market grew by approximately 80% in 2004. The notional of outstanding contracts rose from $3.5 trillion (Source: British Bankers Association) in 2003 to $6.3 trillion in 2004 (Source: Bank for International Settlements). Tavakoli Structured Finance Company estimated outstanding credit derivative contracts would reach $8.0 trillion by the end of 2005, and $9.7 trillion by the end of 2006. According to Deutsche Bank estimates, just the Credit Default Swap (CDS) market notional value will top 30 trillion dollars in 2007.

The most famous credit derivative is the Credit Default Swap (CDS). The seller of CDS agrees to pay the default payment to the buyer of CDS if default occurs. The default payment is structured to replace the loss that a typical lender would incur upon a credit event of the reference entity. Instead, the buyer pays a regular fee to the seller of the CDS before the default happens, so the CDS is a kind of insurance contract. A First to Default (FtD) is an extension of a CDS to portfolio credit risk. Instead of referencing just a single credit, an FtD is specified with respect to a basket of N reference credits. The protection buyer pays a regular fee to the protection seller until any default event occurs or the FtD matures. The default event is the first default of any of the reference credits. The basket for a typical FtD can contain between two to twelve reference credits. The FtDs remove most of the default risk of a basket of defaults, so the protection buyer, instead of buying N CDSs, can buy one FtD and still remove a great portion of his credit risk. The most important parameter of a basket of default is its correlation, which is the topic of this paper.

Credit Risk Models

There are two types of models of default risk in the literature: structural models and reduced based models. In reduced based models, the default is modeled as a surprise. The probability of this surprise follows a jump diffusion process and therefore depends on an intensity parameter also called the hazard rate. This hazard rate can be constant through time or allowed to be stochastic, thereby implying a term structure of probabilities of default. This hazard rate is either estimated to fit historical probability or fitted to current market data (calibration). The reduced form approaches are well documented by contributions of Jarrow and Turnbull (1995); Jarrow, Lando, and Turnbull (1997); Duffie and Singleton (1999); Das and Tufano (1996); Lando (1998); Iben and Litterman (1991); Madan and Unal (1993); Schunbucher (1997); and Zhou (1998).

The reduced form models can incorporate correlations between defaults by allowing hazard rates to be stochastic and correlated with macro economic variables. Duffie and Singleton (1999) modeled and simulated the correlated default times using reduced form models. They emphasized the impact of correlated jumps in credit quality on the performance of a large portfolio of positions. Lando (1998) also used the Cox process for modeling the correlated default rates. The reduced form models have a mathematical advantage, but their main disadvantage is that the range of default correlations that can be achieved is limited as studied by Andreasen (2001). Even when there is a perfect correlation between two hazard rates, the corresponding correlation between defaults in any chosen period of time is usually very low. We expect that when two companies are in the same industry, they should have high default correlation that cannot be attained by reduced form models. However, Jarrow and Yu (2005) showed that if a large jump is allowed in the default intensity of a firm when there is a default by another company, then reduced form models behave better in maintaining the correlated defaults. However, the method used by them is a kind of contagion effect that should be differentiated from default correlation.

The structural approach relates to the arrival of default to the dynamics of the underlying structure of the firm, thereby giving an economic significance to the establishment of the default rate. This approach was founded by Merton (1974, 1977) who used an application of option theory. In his theory, the value of the firm is supposed to be shared by two broad categories of claimants: the shareholders and the debt holders. Because of the limited liability of shareholders, they have a payoff that is positive whenever the face value owed to creditors can be reimbursed; otherwise, it is zero. The shareholders' claim is then just a call on the value of the assets of the firm (also known as a European call). Thus, a bond is simply a right of a face amount to be reimbursed with the sale of a put to shareholders on the assets of the firm. A direct advantage of the structured approach is that credit default is not an unpredictable event here; there is a way to see the corporate conditions that affect the default rate. Relying on the evolution of the value of assets of the firm gives a continuity of the credit standing evolution of the firm that makes the credit risk predictable. Unfortunately, since the value of the firm is not a tradable asset, the parameters of the structural model are difficult to estimate. Reduced form approaches appeared mainly because of this limitation. Black and Cox (1976) provided an important extension of the Merton model. Their model has a first passage time structure where a default takes place whenever the value of the assets of a company drops below a barrier. The first passage models are more realistic in the sense that they let the default happen anytime before horizon. Other extensions of the Merton model are provided by Geske (1997); Kim, Ramaswamy, and Sundareson (1993); Leland (1994); Longstaff and Schwartz (1995); Leland and Toft (1996); and Zhou (2001a).

According to the continuous diffusion structural model, firms never default by surprise. As Zhou (1997) argued, in reality, default can occur in both ways: firms can default either gradually or by surprise due to unforeseen external shocks. The philosophies behind the structural and reduced form approaches have been combined in Zhou (2001a) using a jump diffusion model that allows both gradual and sudden defaults. The jump diffusion approach overcomes some difficulties encountered in traditional diffusion based pricing approach. In particular, a CDS pricing approach based on the diffusion process produces zero credit spreads for very short maturities. This happens because, if there is a finite distance to the default point, a continuous process cannot reach that point in a very short period of time. This is problematic because in reality the credit spreads would not go to zero even for contracts with very short maturities.

Zhou (2001b) and Hull and White (2001) were the first to incorporate default correlation into the Black and Cox first passage structural model. Zhou (2001b) found a closed form formula for the joint default probability of two issuers, but his results cannot easily be extended to more than two issuers. In addition, he did not included the jump in the model.

Another line of research in modeling joint default probability has been survival time involving copulas. This was suggested for two obligors by Li (2000) and extended to many obligors by Laurent and Gregory (2003). Their model has been used in industry due to its computational capabilities. But since they have assumed a fixed correlation between each pair of obligors, the model is very restricted and also there is no known economic rationale for the model.

In the literature, correlation number has been used for capturing the comovements of assets. The correlation number can explain all the dependency of two random variables if they are normally distributed while the distribution of random variables could be more complex to be captured by normally distributed assumption. In this paper, comovements between firms will be explained by a multivariate jump diffusion model. The approach is based on the structural modeling. I assume that the correlation between firms is not only between the diffusion components but also simultaneous jumps in firm assets can contribute to the correlation. In order to fit the companies' harmonies, we need to have a model that can be estimated from market data.

In the next section, the theory of firm default will be explained. After that, correlated defaults will be discussed. Next, the multivariate jump diffusion model used in this research will be explained. After that, the econometric approach that is used for parameter estimation will be shown. Finally, an empirical example of two firms will be shown.

Firm Default

Firms default when they cannot, or choose not to, meet their obligations. Credit events are triggered by movements of the firm's value relative to some (random or non-random) credit event triggering threshold. So if the firm's asset V_t is less than the face value of debt D_t at time t, then the firms will go bankrupt. Consequently, a major issue is the modeling of the evolution of the firm's value and of the firm's capital structure. This idea is the basis of the structural approach to default risk that was founded by Merton (1974, 1977).

In the structural approach, the default happens when $V_t < D_t$. In order to use the structural approach, we need to know the asset value V_t and the default point D_t. Asset values cannot be observable in the market, but since the equity price of firms can be found in the market, we are able to retrieve the asset values from option theoretic approach of Merton if we know the default point.

Finding the default point has been one of the challenges of the structural approach in the credit risk modeling. In reality, firms often rearrange their liability structure when they have credit problems. Hence, there is no analytic method to derive the default point. Rather, it is estimated from a firm's liabilities information on its balance sheets. KMV Company has done the empirical research based on large scale statistical studies of historical defaults, and they have found that the basic estimation for the default point is current liability plus half of long term liabilities (Demircubuk and Tse, 2001). Their approach has been based on defining the short term and long term liabilities in the balance sheet data. First, minority interests and deferred taxes are excluded from the total liabilities since those two items do not cause default stress. But problems arise when the firms do not break down their liabilities into current (due within one year) and long term on their balance sheets. There are no regulations to stop firms from reporting their statements in this way, and there can be many reasons for them to do so. However, KMV has considered these adjustments for finding the default point estimation. Also, KMV has developed a model in which they have segmented the liabilities into six parts and have estimated the default point as a function of those liabilities[2]. All in all, finding the default point has depended on the precise analysis of accounting data. Since there are versatile accounting standards in different countries and since the firm level data are very noisy (because of the moral hazard problem), relying on these data has been very risky.

Since default is costly and violations to the absolute priority rule in bankruptcy proceedings are common, in practice shareholders have an incentive to put the firm into the receivership before the asset value of the firm hits the debt value (Hanson, Pesaran, and Schuermann,2005). In addition, the lending banks have the incentive to force the firm to default before the asset value hits the debt threshline (Garbade, 2001). Also, a borrower may be in a default condition, e.g. a missed coupon payment, without going into bankruptcy. This usually happens in the banking-borrower relationship (Lawrence and Arshadi, 1995). Therefore, the default usually happens when the asset value of the firm crosses the threshline that is higher than the default point characterized by the liabilities of the firm. Since we have $V_i = D_i + E_i$, where E is the equity value of the firm, we can deduct that the default happens when: $0 < E_i < C_i$. C is the positive threshold which is time varying and is dependent on the firm's characteristics. The equity values are observable for the firms traded in the stock markets. We assume that the equity prices satisfy the following process:

$$LogE_{i,t+1} - LogE_{i,t} = G_{i,t} ,$$

$G_{i,t}$ is the return of equity prices for firm i at time t, so the equity returns for a time period H will be:

$$LogE_{i,t+H} - LogE_{i,t} = Log(\frac{E_{i,t+H}}{E_{i,t}}) = Log(\frac{E_{i,t+H}}{E_{i,t+H-1}} \times \frac{E_{i,t+H-1}}{E_{i,t+H-2}} \times ... \times \frac{E_{i,t+1}}{E_{i,t}}) = \sum_{i=1}^{H} G_{i,t+i}$$

The equity value distribution is dependent on the distribution function of G functions. If we assume a functional form for G functions, the distribution function can be estimated from the historical data of equity prices.

Estimation of threshold line C for each firm is the next step in modeling the default risk. As argued and elaborated in Pesaran, Hashem, Schurermann, Treutler, and Weiner (2005), accounting information is likely to be noisy and might not be all

[2] This model has not been disclosed to the public.

that reliable due to information asymmetries and agency problems between managers, shareholders. and debtholders. Moreover, the accounting-based route presents additional challenges such as different accounting standards and bankruptcy rules in different countries. Also, other firm specific characteristics such as leverage, firm age, and management quality could also be important in the determination of default thresholds that are quite difficult to observe. In view of these measurement problems, Pesaran et al. (2005) have used the firm-specific credit ratings for finding the default thresholds. There are different rating agencies that rank the firms and assign them a credit rating. Moody, S&P, FITCH, CIBS, Nationsbank, and SBC rate the firms, and each agency has its own terminology. Table 2 shows the correspondences between the default scheme and rating classes of different rating companies. The rating companies use all the accounting data and information including the economic sector and the geographical region where the firms are working. They use inside information for the rating. So the rating data are reliable for finding the firm probability of default. If we have the distribution function of the equity returns, we can find the treshline that corresponds to the default rate set by the rating agency. The probability for each rating comes as a range. For example, the probability of default for Ba3 rating is in the range 72-101 bp. The mean of the default rate for that range will be used for finding the thresh line.

Default Correlation

Historically, defaults tended to cluster as the following examples from the USA show (Schunbucher, 2003):

- Oil industry, 22 companies defaulted in 1982-1986.
- Railroad conglomerates: 1 default each year in 1970-1977.
- Airlines: 3 defaults in 1970-1971, 5 defaults in 1989-1990.
- Thrifts (savings and loan crisis): 19 defaults in 1989-1990.
- Casinos/hotel chains: 10 defaults in 1990.
- Retailers: more than 20 defaults in 1990-1992.
- Construction/ real estate: 4 defaults in 1992.

Also, the times series of defaulted companies shows that the historical time series exhibits much higher variation than the simulated time series that are based on independent default rates. The majority of historical default rates are far from the average default rate of the companies. Therefore, the correlation between default rates is an important factor in explaining the historical bankruptcies in different industries. For example, if we have two firms A and B with probability of default P_a and P_b, then the default correlation can be defined as:

$$\rho_{ab} = \frac{P_{ab} - P_a P_b}{\sqrt{P_a(1-P_a)P_b(1-P_b)}} ,$$

where P_{ab} is the probability of joint default. Now we can write P_{ab} as:

$$P_{ab} = P_a P_b + \rho_{ab}\sqrt{P_a(1-P_a)P_b(1-P_b)} ,$$

if we assume $\mathrm{P_a} = P_b = p << 1$ then we have:

$$\mathrm{P_{ab}} \approx p^2 + \rho_{ab} p \approx \rho_{ab} p .$$

This shows that the correlation between defaults is an important factor in the probability of joint default.

Many banks and financial institutions are at the credit risk exposure of many reference identities. Clearly, if they want to hedge their credit risk, they should buy the protection for the individual exposures, which is very costly and inefficient for them. Therefore, the new generation of credit derivatives has been developed in order to hedge the credit risk of a portfolio. Many new credit derivatives are now associated with a portfolio of credit risk. A typical example is the product with payment contingent upon the time and identity of the first or second-to-default in a given risk portfolio. Other derivatives are instruments that their payments are contingent on the cumulative loss before a given time in the future. Collaterized loan obligation (CLO) and collaterized bond obligation (CBO) are the common credit derivatives for portfolios.

Modeling correlated default has been one of the challenges for researchers in credit risk. Vasicek (1987, 1991) had the first works in this area. He used a single factor model in order to obtain the correlation in asset values of firms. In his model, the firms default when their asset value drops down a threshold while the asset values are correlated between different firms that make the defaults correlated. Reduced form models have not been successful in modeling the correlated defaults. The reduced form models can capture high correlated defaults unless some kind of contagion effect is considered in the model. Even when there is a perfect correlation between two hazard rates, the corresponding correlation between defaults in any chosen period of time is usually very low. We expect that when two companies are in the same industry, they should have high default correlation ; this cannot be attained by reduced form models. However, Jarrow and Yu (1999) showed that if a large jump is allowed in the default intensity of a firm when there is a default by another company, then reduced form models behave better in maintaining the correlated defaults. But the method used by them has a kind of contagion effect that should be differentiated from correlation between default rates.

Another line of research in modeling joint default probability has been survival time involving copulas. This was suggested for two obligors by Li (2000) and extended to many obligors by Laurent and Gregory (2003). Their model has been used in industry due to its computational capabilities. But since they have assumed a fixed correlation between each pair of obligors, the model is very restricted and also there is no known economic rationale for the model.

Zhou (2001b) and Hull and White (2001) were the first to incorporate default correlation into the Black and Cox first passage structural model. Zhou (2001b) found a closed form formula for the joint default probability of two issuers, but his results cannot easily be extended to more than two issuers. Neither Zhou (2001b) nor Hull and White (2005) included the jump in their models.

Jumps have been a major factor in credit risk analysis. With jump risk, a firm can default instantaneously because of a sudden drop in its value. Under a diffusion process, because a sudden drop in the firm value is impossible, firms never default by surprise. Thus, the large credit spreads of corporate bonds, especially those with short maturities, are unexplained in the structural approach. Some recent papers have documented the large discrepancy between the predictions of structural models and the observed credit spreads, which is also known as the credit risk premium puzzle

(Amato and Remolona, 2003). Huang and Huang (2003) calibrated a wide range of structural models to be consistent with the data on historical default and loss experience. They showed that in all models credit risk only explains a small fraction of the historically observed corporate-treasury yield spreads. Similarly, Collin-Dufresne, Goldstein, and Helwege (2001) suggested that default risk factors have rather limited explanatory power on variation in credit spreads, even after the liquidity consideration is taken into account. As a result, a credit model with the jump risk is able to explain the credit spread of bonds and CDS in a short time horizon to maturity. The jump diffusion model is consistent with the fact that bond prices often drop in a surprising manner at or around the time of default (Beneish and Press, 1995). Duffie and Lando (1997) attributed this phenomenon to incomplete accounting information. The information is revealed to the market. Because of a jump in market information, bond prices jump accordingly. Zhou (2001a) was the pioneer who showed the importance of jump risk in credit risk analysis of an obligor. He implemented the simulation method to show the effect of jump risk in the credit spread of defaultable bonds and showed that the misspecification of stochastic processes governing the dynamics of firm value, i.e., falsely specifying a jump-diffusion process as a continuous Brownian motion process, can substantially understate the credit spreads of corporate bonds. Most of the papers in credit risk have come up with almost zero credit spreads in a short time horizon; this is not realistic since the researchers have not considered the jump surprise in their models. Jumps have more significant importance in the default correlation. Simultaneous negative jumps enhance the chance of simultaneous defaults that increases the correlation defaults.

This paper takes simultaneous jumps into consideration via a multivariate jump diffusion model. The next section is devoted to explaining the theoretical part of this model.

Multivariate Jump Diffusion Model

Researchers who have studied financial markets have long noted that financial time series exhibit unusual behavior relative to what would be expected from the

Gaussian distribution. There are too many small changes and some outliers (although not that frequent), but they are very important for participants in the financial market. Modeling financial price returns in a way that the returns are a realization of continuous time diffusion process plays a central role in modern financial economics. This model simplifies the hedging calculations and derivative pricing. Although these models are very important in finance literature, it is not clear that with extreme violent movements in financial data, they can be reliable enough for modeling the reality of data. Jump diffusion models have considered the surprise jumps in the financial returns in addition to the Gaussian increments from the diffusion part.

This section defines the affine jump diffusion model that will be the focus of this analysis. For a given complete probability space (Ω, F, P) and the augmented filtration $\{F_t : t \geq 0\}$ generated by a standard Brownian motion W in R^n and satisfying the stochastic differential equation,

$$dY_t = \mu(Y_t, t)dt + \sigma(Y_t, t)dW_t + dZ_t ,$$

where $\mu : D \to R^n$ (the drift function) and $\sigma : D \to R^{n \times n}$ (diffusion function) and Z is a pure jump process with intensity $\{\lambda(Y_t) : t \geq 0\}$ and jump amplitude distribution ν on R^n. Intuitively, the drift term $\mu(.)$ represents an instantaneous deterministic time trend of the process, the diffusion term shows the stochastic small increments in the process and jumps are surprise changes in the model. The above model is called an affine model if:

$$\mu(Y_t, t) = K_0 + K_1 Y_t$$

$$[\sigma(Y_t, t)\sigma(Y_t, t)']_{ij} = [H_0]_{ij} + [H_1]_{ij} Y_j \qquad (4.1)$$

$$\lambda(Y_t) = l_0 + l_1' Y_t$$

where $K = (K_0, K_1) \in R^n \times R^{n \times n}$, $H = (H_0, H_1) \in R^{n \times n} \times R^{n \times n \times n}$, $l = (l_0, l_1) \in R \times R^n$ and the jump transform $\psi(c) = \int_{R^n} \exp(c.z) d\nu(z)$, for $c \in C^n$, is known in closed form whenever the integral is well defined (Appendix 1).

In order to find the probability distribution of the random variable Y, we need to know the transition density function which, under regularity conditions, satisfy both the Kolmogorov forward and backward equations of the Markov process. However, the transition density functions are not analytically derivable except for a few very simple functional forms like the Ornstein-Uhlenbeck process. So the estimation of the parameters is not possible when those analytical transition density functions are not available. An alternative approach is finding the Conditional Characteristic Function (CCF), which is defined as:

$\phi_t(\tau,u) = E(e^{iu'Y_T} \mid Y_t)$, $u \in R^N$, where $\tau = T - t$, $i = \sqrt{-1}$.

There is a one-to-one relationship between characteristic functions and the density functions. So if the CCF of a stochastic process is available, we have all the information about the conditional density function of that variable.

Duffie, Pan, and Singleton (2000) showed that the CCF of an affine jump diffusion model has a closed form, which can be written as:

$\phi(u;Y_{t+\tau} \mid Y_t) = e^{C(\tau,u)+D(\tau,u)'Y_t}$.

$C(.)$ and $D(.)$ are functions satisfying the complex-valued Riccati equations:

$$\frac{\partial D(\tau,u)}{\partial \tau} = K_1' D(\tau,u) + \frac{1}{2} D(\tau,u)' H_1 D(\tau,u) + l_1(\psi(D(\tau,u)) - 1),$$

$$\frac{\partial C(\tau,u)}{\partial \tau} = K_0' D(\tau,u) + \frac{1}{2} D(\tau,u)' H_0 D(\tau,u) + l_0(\psi(D(\tau,u)) - 1), \tag{4.2}$$

with boundary conditions: $D(0,u) = iu$, $C(0,u) = 0$. If the parameters of the jump diffusion model are given, we can solve for functions C (.) and D (.) explicitly.

Econometric Approach

The estimation of continuous-time stochastic processes has been a challenge for econometricians and statisticians. As mentioned in the previous section, there are a few specific models for which the maximum likelihood is possible as there exist explicit closed form transition density functions. However, these models have not proved to be popular in finance due to unrealistically simplistic model specifications. In the multivariate framework, the estimation problem becomes even more difficult. Chan, Kariyli, Longstaff, and Sanders (1992) have used the Generalized Method of Moments (GMM) for estimation of the parameters for a univariate diffusion model after discretizing the model. Due to the bias from discretizing the model, simulation based methods have been developed in order to have consistent estimates. However, since the Euler discretization is an approximation, the model is misspecified, causing an asymptotic bias of its estimators, which may be arbitrarily large. The indirect inference uses simulations performed under the initial model to correct for the asymptotic bias of which is Maximum Likelihood (ML) estimator of the discretized method (Gourieroux, Monfort, and Renault, 1993). Besides the intensive computation involved, these methods can compound the estimation error and consequently may lead to poor finite sample properties (Knight and Yu, 2000). Jump models are more difficult to estimate, at least by simulation-based methods. The discontinuous sample paths create discontinuities in the econometric objective function that have to be accommodated by rounding out the corners as in Andersen, Bollerslev, and Diebold (2002) and Chernov, Gallant, Ghysels, and Tauchen (2003). Recent work on bipower variation measures, which are developed in a series of papers by Barndorff-Nielsen and Shephard (2003a, 2003b, 2004), has developed a method to disentangle realized volatility into continuous and jump components, as in Andersen et al. (2004), Huang and Tauchen (2004), Zhang, Zhou, and Zhu (2005). They have used the bipower variation method to estimate the jump component. The problem with the bipower method is that jumps are defined as a surprise in a short time range (like intradaily data). Therefore, small changes in a day will be detected as jumps in tranquil days. In this paper, another approach that is based on the GMM estimation of characteristic function will be used.

The use of characteristic function for parameter estimation was developed by Feuerverger and McDunnough (1981) and Feuerverger (1990). Das and Foresi (1996) and Bates (1996) used characteristic function for estimation by inverting them before estimation. Independent works by Singleton (2001) and Jiang and Knight (2001) show that this inversion is not necessary, and we can use the characteristic function directly for estimation. The justification for the Empirical Characteristic Function (CCF) is that the CF is the Fourier transform of the cumulative distribution function (CDF), and hence there is a one-to-one correspondence between the CF and CDF.

Another important property of the characteristic function is its uniqueness and the fact that it carries all the same information as the likelihood function. Since ECF uses all the information, it should be as efficient as the maximum likelihood method. There are many cases in econometric analysis in which the likelihood function can not be derived analytically while the characteristic function has an explicit functional form. Switching regression introduced by Quandt (1958), regime switching models introduced by Hamilton (1989), the variance gamma distribution proposed by Madan and Senata (1990), stable distribution proposed by Mandelbrot (1963), discrete time stochastic volatility model used in the modeling exchange rate, proposed by Ghysels, Harvey, Renault (1996) are all examples of the privilege of using characteristic functions. While the likelihood function could be unbounded, the characteristic functions are always bounded (Grimmett and Stirzaker, 1992, Theorem 5.7.3). If two random variables have the same characteristic function, then they have the same probability distribution. Also, all the moments of the random variable can be derived from the characteristic function:

$$E[Y_{t+1}{}^{n} \mid Y_t] = \frac{1}{i^n}\frac{d^n}{du^n}\phi_t(\tau,u)\mid_{u=0}$$

We can construct the moment conditions as:

$$h(u;\theta) = e^{iu'Y_{t+1}} - \phi(u, y_t;\theta)$$

where

$$E[h(u;\theta) \mid Y_t] = E[e^{iu'Y_{t+1}} - \phi(u, y_t;\theta) \mid Y_t] = 0.$$

Obviously, h satisfies

$E^{\theta}[h(u;\theta)] = 0$ for all u in R^N .

ECF estimator can be assumed as the GMM estimator of Hansen (1982), and the parameters can be estimated from the following optimization problem:

$$\underset{\theta}{Min}\ \frac{1}{n}\sum_{t=1}^{T} H(Y_t;\theta)' \times W_T \times \sum_{t=1}^{T} H(Y_t;\theta) \qquad (5.1)$$

where H is a K by 1 vector, in which each row of H corresponds to the moment condition of one of the elements of U vector. W_T is a positive semidefinite weighting matrix that converges to a positive definite matrix W_0 almost surely. Under some regularity conditions, the GMM estimator is consistent and asymptotically normally distributed for arbitrary weighting matrices. When the system is identified, the GMM estimator does not depend on the choice of W_T. When the system is over identified, Hansen (1982) shows that if $W_T = \Sigma^{-1}$, the GMM estimator is asymptotically efficient in the sense that the covariance matrix of the GMM estimator is minimized, where Σ is the long run covariance matrix of $H(Y_t;\theta)$.

The most difficult part of the ECF method is that choices of moments should be made before estimation. Since for each realization of u we can have one moment, an infinite number of moments can be generated for the ECF method. Feuerverger and McDunnough (1981) showed that the asymptotic variance of the GMM estimator can be made arbitrarily close to the Cramer-Rao bound by selecting the grid sufficiently fine and extended. This led them to conclude that their estimator was asymptotically efficient. This is not true, since when the grid is too fine, the covariance matrix becomes singular and the GMM objective function is not bounded; hence, the efficient GMM estimator can not be computed. Singleton (2001) proposed to use a couple of u vectors on the axis, which is not necessarily optimal. Carrasco, Chernov, Florens, and Ghysels (2005) proposed an estimation method based on a continuum of moment conditions. They introduced the continuous counterpart of moment conditions defined in (5.1) as:

$$\hat{\theta}_T = \arg\min_{\theta} \| K_T^{\frac{-1}{2}} \hat{h}_T(\theta) \| \quad (5.2)$$

where $\| . \|$ is the Euclidean norm that in infinite case converts to:

$$\|f\|^2 = \int_R f(\tau)\overline{f(\tau)}\pi(\tau)d\tau \quad (5.3)$$

where $\overline{f}$ denotes the complex conjugate of f and π denotes a probability density function (pdf) that they assumed to be Gaussian. In order to estimate parameters from (5.2), we need to have an estimate of $K_T^{\frac{-1}{2}}$. Carrasco et. al (2005) suggested a two-stage estimation that is first to estimate parameters $\hat{\theta}_T$ from (5.2) considering $\hat{K}_T$ = 1 and retrieving $\hat{K}_T$ matrix from $\hat{h}_T(\tau,\hat{\theta}_T^1)$. Then the estimated parameter will be:

$$\hat{\theta}_T = \arg\min_{\theta} \| K_T^{\frac{-1}{2}} \hat{h}_T(\tau;\theta) \| = \arg\min_{\theta} \sum_{j=1}^{T} \frac{1}{\hat{\lambda}_j} \left| \left\langle \hat{h}_T(\tau;\theta), \varphi_j \right\rangle \right|^2$$

where $\hat{\varphi}_j$ and $\hat{\lambda}_j$ are the eigenvector and eigenvalue of $\hat{K}_T$ matrix estimated from the first stage. The authors confronted by close to zero eigenvalues that is the counterpart of near singular covariance matrix. Therefore, they have considered a penalizing term α_T in order to solve the near zero eigenvalue problem.The C-GMM estimator is defined as:

$$\hat{\theta}_T = \arg\min_{\theta} \sum_{j=1}^{T} \frac{\hat{\lambda}_j}{\hat{\lambda}_j^2 + \alpha_T} \left| \left\langle \hat{h}_T(\tau;\theta), \hat{\varphi}_j \right\rangle \right|^2$$

In this paper, I use the idea of Tikhonov approximation for the estimation of an infinite moment conditions. Carrasco et. al (2005) have tried to use all the moment information in a C-GMM estimator but finally came up with eliminating some of the moments out of a finite set via using Tikhonov approximation. In their approach, different moments that correspond to different values of the U variable in the characteristic function are weighted by a Gaussian function defined in (5.3). The choice of π function is very critical in the estimation since it affects the estimated parameters. The problem is how we can choose different moments out of an infinite set of moments in such a way that more information is incorporated in the estimation. I return to the GMM theory first and explain what the infinite moments mean in that context.

The inversion of covariance matrix is not possible when a lot of moment conditions are implemented. If we assume L is the T by K matrix of observed moments, where T is the number of observations and K is the number of moments, then we can write the covariance matrix of L as Σ. Since Σ is a positive semidefinite symmetric matrix we can decompose this matrix to its diagonalized form as:

$\Sigma = M \Lambda M^{-1}$,

where the columns of the M are the eigenvectors of covariance matrix Σ and Λ is a diagonalized matrix in which its diagonal elements are eigenvalues and the nondiagoanl elements are zero. Then if we have the moment conditions in a vector G, we can write the objective function of GMM method as:

$$G'\Sigma^{-1}G = G'(M\Lambda M^{-1})^{-1}G = G'(M\Lambda^{-1}M^{-1})G = (M'G)'\Lambda^{-1}(M'G)$$

P = M'G is the new vector of moments in the new projected space and are orthogonal to each other. The elements of the diagonalized matrix Λ^{-1} is the inverse of the eigenvalues. So the new objective function can be written as:

$$P'\Lambda^{-1}P = \sum_{i=1}^{K} \frac{p_i^2}{\lambda_i}, \quad (5.1)$$

The inversion problem of the first objective function now is transformed to having close to zero eigenvalues in the objective function defined in (5.1). If we include more moments then the chance of having nonzero or close to zero eigenvalues will be increased. This decreases the stability of the estimation. Eliminating those moments causes the loss of more information, which makes the ECF method inefficient. In order to circumvent this problem, the regularization parameters will be used. This method is called Tikhonov approximation (Groetsch, 1993). The Tikhonov parameter is an α that changes the objective function as:

$$T(\alpha) = \sum_{i=1}^{K} \frac{\lambda_i}{\lambda_i^2 + \alpha} p_i^2,$$

As α goes to zero, $T(\alpha)$ converges to the true value of the objective function. This approximation is a trade off between stability and the biasness of estimator. Carrasco and Florens (2002), Carrasco et al. (2005) used the idea of a continuum of moments in order to circumvent the inversion problem of covariance matrix while they tried to use all the information of all moments lest they lose the efficiency of the estimator. They also came up with the idea of using Tikhonov approximation to generate stable estimators.

The asymptotic distribution of the estimated parameters is:

$$T^{1/2}(\hat{\theta} - \theta_0) \xrightarrow{d} N(0, (M'\Sigma^{-1}M)^{-1})$$

where M is the jacobian of the moments or $M = \frac{\partial H(Y_t;\theta)}{\partial \theta'}$.

Newey and West (1987) developed a theory for testing any null hypothesis. They proposed the Wald test for testing the null hypothesis where r(.) is an s-element vector of continuous differential functions denoted by , s is the number of restrictions that cannot exceed the number of parameters. The Wald test statistic is:

$$W_T = Tr(\hat{\theta})'[R(\theta)(M'\Sigma^{-1}M)^{-1}R'(\theta)]^{-1}r(\hat{\theta})$$

Under the null hypothesis, it is asymptotically distributed as χ_s^2.

Empirical Results

In this study, I selected two companies, General Motors and Ford. Both companies are in the same industrial sector. The five year daily price data of both companies from August 5, 2001 to August 5, 2005 was used for the analysis (Figure 1).

General Motors Corporation engages in the design, manufacture, and marketing of cars and light trucks worldwide. It operates through automotive and financing and insurance operations (FIO) segments. The automotive segment designs, manufactures, and markets passenger cars, trucks, and locomotives, as well as related parts and accessories. The company offers its vehicles primarily under the brands Chevrolet, Pontiac, GMC, Oldsmobile, Buick, Cadillac, Saturn, and HUMMER. The FIO segment provides a range of financial services, including: consumer vehicle financing, full service leasing and fleet leasing, dealer financing, car and truck extended service contracts, residential and commercial mortgage services, vehicle and homeowners' insurance, and asset-based lending. The company’s automotive-related products are marketed through retail dealers and distributors primarily in the United States, Canada, and Mexico.

Ford Motor Company manufacturers and distributes automobiles and finances and rents vehicles and equipment. It operates in two sectors, Automotive and Financial Services. Automotive sector sells cars and trucks. Ford primarily sells Ford, Lincoln, and Mercury brand vehicles and related service parts in North America, including the United States, Canada, and Mexico; it also sells Ford-brand vehicles and related service parts in South America. The automotive sector also sells Ford-brand vehicles and related service parts in Europe and Turkey, as well as in Asia Pacific and Africa.

In addition to producing and selling cars and trucks, Ford Motor provides retail customers with a range of after-the-sale vehicle services and products. The financial services sector primarily includes vehicle-related financing, leasing, and insurance.

The credit rating for General Motors was downgraded to BB+ on May 24, 2005. Ford's rating was downgraded to BBB- on July 20, 2005. All ratings were announced by FITCH Company. The probability of default for the FITCH rating is shown in Table 3.

The multivariate jump diffusion is used for modeling the equity price data of Ford and General Motors (Appendix 2). The ECCF method has been used to estimate the parameters of the models. The grid points have been selected from two axis of u vector. Twenty points on each axis from 0.1 to 2 with 0.1 steps were generated for each component of the u elements (the dimension of u is two). Since the objective function is a complex number, each point on the axis produced two moments: one for the real part and the other for the imaginary part. Eighty total moments were generated for estimation, and the principal component method explained earlier was used for the estimation.

The Nelder-Mead simplex (direct search) optimization method was used in the MATLAB program for estimating the parameters. The algorithm belongs to the class of direct search methods, a class of optimization algorithms that neither compute nor approximate any derivatives of the objective function. In fact, the method was inspired by the simplex method of Spendley, Hext, and Himsworth (1962) and the simplex method of Nelder and Mead. The multi-directional search algorithm is inherently parallel. The basic idea of the algorithm is to perform concurrent searches in multiple directions. These searches are free of any interdependencies, so the information required can be computed in parallel.

I assumed the jumps are normally distributed with mean μ and standard deviation σ. The estimated parameters are as below (the standard deviations are in parentheses):

$$K = \begin{pmatrix} 0.000024(0.000001) \\ 0.000017(0.000001) \end{pmatrix}, \; H_0 = \begin{pmatrix} 0.000181(0.000051) & -0.002477(0.000152) \\ -0.00058(0.000097) & -0.002271(0.000165) \end{pmatrix},$$

$\lambda_0 = 0.0240(0.00233)$, $\mu =$ -0.000126(0.000008), $\sigma = 0.0238(0.000077)$.

The parameters estimated for the restricted model is as below:

$$K = \begin{pmatrix} 0.000220(0.000011) \\ 0.000160(0.000009) \end{pmatrix}, \; H_0 = \begin{pmatrix} 0.000403(0.000030) & -0.000545(0.0000402) \\ -0.00078(0.000019) & -0.000311(0.000345) \end{pmatrix}$$

The estimated parameters show that Ford and GM are growing with a very slow rate of 0.000024 and 0.000017 returns in each day, respectively. The parameters of the diffusion part exhibit the correlation in tiny changes from two independent Brownian motions. The jump part explains that both companies experience a jump every 42 days with the expected magnitude of close to zero percent but with a relatively huge variance of two percent on that day. The very high variance of the jump makes the jump dominant. These jumps are common for both firms that are captured by considering them in the model and they let the model accommodate the surprise default risk for both companies. The significant coefficient for the hump component shows that the unrestricted model is better than the restricted one. However, the Wald test has been used for comparing two models. The Wald statistics is 10.7 with a p-value of 0.01. Therefore, the model with simultaneous jump has a better explaining power relative to the model that does not consider the jump.

With 100,000 simulations, the default threshold has been found for each of the 10 year probability of default. Based on the simulations, the joint probability of default has been found for different years. The results are in Table 4. The comparison of joint default between two models shows that the joint default probability of Ford and GM is underestimated by the model thta does not consider the simultaneous jumps. For all different time horizons, the joint default probabilities of two firms are higher with simultaneous jump present in the model. Banks and financial institution need to hold capital for 3 bp probability for keeping their AA rating (under Basel II regulation). For a one year horizon, the probability of joint default is 5 bp, which is more than 3 bp and much more than the 0.3 bp resulting from the independence assumption.

Conclusions

In this paper, a more general approach than using correlation number has been used for finding the comovements of firm asset values. Jumps as the surprise changes were shown to be significant as a default factor. A jump diffusion model was the basis of taking both Brownian motion and jump into the default modeling. In order to estimate the model, principal component analysis with Tikhonov approximation was used. The empirical result from two well-known firms, Ford and General Motors shows that common jumps are important default factors. If they are not estimated and incorporated in the model, the joint probability of default will be underestimated.

More work can be done as the extension of this study. The MATLAB code can be extended to cover more than two firms. More firms can be considered in the model in order to find the various comovement patterns in different firms. The econometrics approach should be elaborated more in order to find the most informative moments since some information may be lost in the Tikhonov approximation. Unfortunately, the FtD data are not commonly available for researchers. Upon the availability of the data, the implied correlation of firms can be found from data and their comparison with the findings can validate the model.

Appendix 1- Jump Transform for Multivariate Normal Amplitudes

The jump transform is defined as: $\psi(c) = \int_{R^n} \exp(c.z) d\nu(z)$, for $c \in C^n$,

where ν is a random variable whose distribution is dependent on the state variable z. If we assume ν is normally distributed as $N(\mu, \Sigma)$, then we will have:

$$\psi(c) = \int_{R^n} \exp(c.z) d\nu(z) =$$

$$= |2\pi\Sigma|^{\frac{-1}{2}} \int \exp(c.z) \exp\left\{-\frac{1}{2}(z-\mu)'\Sigma^{-1}(z-\mu)\right\} dz$$

$$= |2\pi\Sigma|^{\frac{-1}{2}} \int \exp(c.z) \exp\left\{-\frac{1}{2}(z'\Sigma^{-1}z - z'\Sigma^{-1}\mu - \mu'\Sigma^{-1}z + \mu'\Sigma^{-1}\mu)\right\} dz$$

$$= |2\pi\Sigma|^{\frac{-1}{2}} \int \exp\left\{-\frac{1}{2}(z'\Sigma^{-1}z - z'\Sigma^{-1}\mu - \mu'\Sigma^{-1}z + \mu'\Sigma^{-1}\mu - 2c.z)\right\} dz$$

$$= |2\pi\Sigma|^{\frac{-1}{2}} \int \exp\left\{-\frac{1}{2}(z'\Sigma^{-1}z - z'\Sigma^{-1}\mu - \mu'\Sigma^{-1}z + \mu'\Sigma^{-1}\mu - 2c.z)\right\} dz$$

$$= |2\pi\Sigma|^{\frac{-1}{2}} \int \exp\left\{-\frac{1}{2}(z'\Sigma^{-1}z - z'\Sigma^{-1}\mu - \mu'\Sigma^{-1}z + \mu'\Sigma^{-1}\mu - 2c.z + c'\Sigma c + 2\mu'c - c'\Sigma c - 2\mu'c)\right\} dz$$

$$= |2\pi\Sigma|^{\frac{-1}{2}} \int \exp\left\{-\frac{1}{2}[z - (\Sigma c + \mu)]'\Sigma^{-1}[z - (\Sigma c + \mu)] + \mu'c + \frac{1}{2}c'\Sigma c\right\} dz$$

$$= \exp(\mu'c + \frac{1}{2}c'\Sigma c) \times |2\pi\Sigma|^{\frac{-1}{2}} \int \exp\left\{-\frac{1}{2}[z - (\Sigma c + \mu)]'\Sigma^{-1}[z - (\Sigma c + \mu)]\right\} dz$$

$$= \exp(\mu'c + \frac{1}{2}c'\Sigma c)$$

Appendix 2- Characteristic Function Assumptions

In this study, the following assumptions have been used for the parameters of the jump diffusion model defined in (4.1):

1. $K_1 = 0$,
2. $H_1 = 0$,
3. $l_1 = 0$,
4. The time intervals are equally spaced,
5. Jumps are normally distributed and are i.i.d.

Therefore, the complex-valued Ricatti equations will be:

$$\frac{\partial D(\tau,U)}{\partial \tau} = 0,$$

$$\frac{\partial C(\tau,U)}{\partial \tau} = K_0{}' D(\tau,U)' H_0 D(\tau,U) + l_0[\psi(D(\tau,U)) - 1].$$

$$D(0,U) = iU,\ C(0,U) = 0.$$

From the assumptions, we can solve the above differential equation as:

$$D(\tau,U) = iU\ ,$$

$$C(\tau,U) = K_0{}' \times iU + \frac{1}{2}(iU)' H_0 (iU) + l_0[\exp(\mu' \times iU' + \frac{1}{2}(iU)' \Sigma (iU)) - 1]$$

Table 1 The Capital Requirement for Different Ratings

Security type	AAA to AA-	A+ to A-	BBB+ to BBB-	BB+ to B-	Below B-	Unrated
Asset backed	20	50	100	150	Deducted	Deducted
Banks	20	50	100	100	150	100
Corporates	20	100	100	100	150	100
Sovereigns	0	20	50	100	150	100

Source: BIS reports

Table 2 Correspondence Between the Default Rate and Public Rating Classes

Default (bps)	S&P	Moody's	CIBC	Nationsbank	SBC
2-4	>AA	>Aa2	1	AAA	C1
4-10	AA-A	A1	2	AA	C2
10-19	A to BBB+	Baa1	3	A	C3
19-40	BBB+ to BBB-	Baa3	4	A to BB	C4
40-72	BBB- to BB	Ba1	4.5	BBB to BB	C5
72-101	BB to BB-	Ba3	5	BB	C6
101-143	BB- to B+	B1	5.5	BB	C7
143-202	B+ to B	B2	6	BB to B	C8
202-345	B to B-	B3	6.5	B	C9

Source: Crouhy et. Al (2000)

Table 3- Cumulative Probability of Default for Different Ratings of FITCH

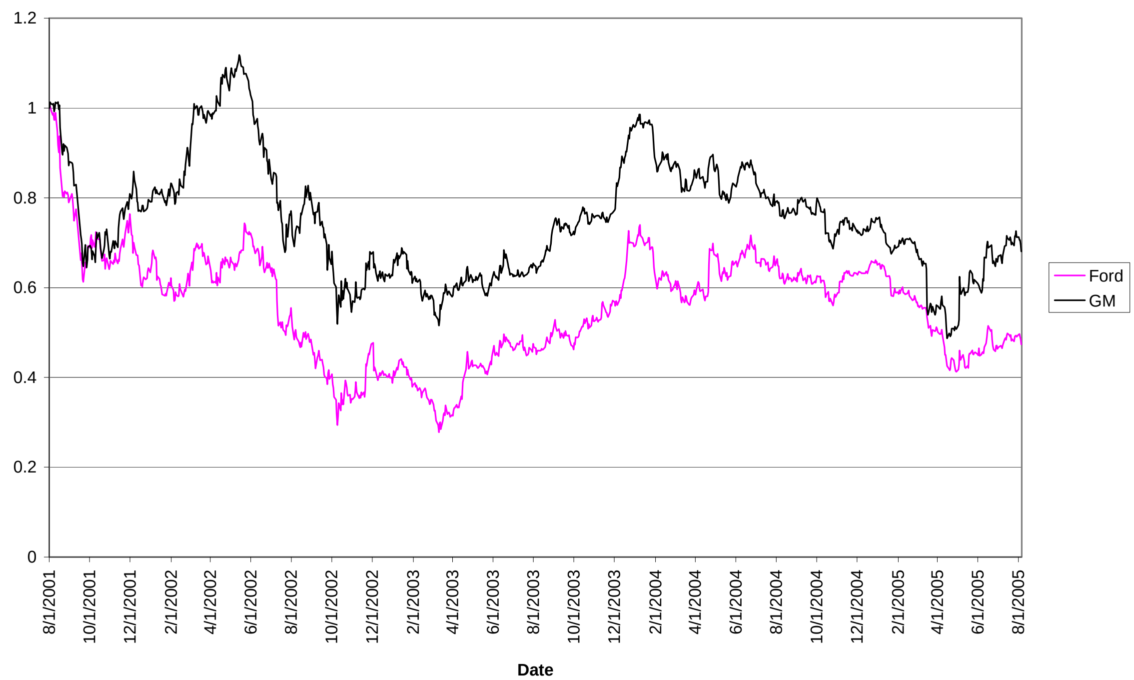
Rating	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10
AAA	0.00	0.00	0.02	0.03	0.05	0.08	0.10	0.13	0.16	0.19
AA+	0.00	0.02	0.05	0.13	0.19	0.26	0.33	0.40	0.48	0.57
AA	0.01	0.02	0.07	0.16	0.26	0.38	0.49	0.62	0.75	0.89
AA-	0.01	0.05	0.13	0.23	0.36	0.51	0.66	0.82	0.98	1.15
A+	0.03	0.11	0.22	0.37	0.56	0.76	0.98	1.20	1.43	1.65
A	0.04	0.13	0.26	0.43	0.62	0.84	1.07	1.32	1.58	1.85
A-	0.08	0.23	0.42	0.66	0.92	1.20	1.49	1.80	2.12	2.44
BBB+	0.12	0.32	0.57	0.87	1.20	1.55	1.93	2.32	2.72	3.13
BBB	0.21	0.54	0.91	1.32	1.89	2.30	2.67	2.97	3.34	3.74
BBB-	0.42	1.07	1.87	2.74	3.63	4.48	5.27	6.00	6.66	7.26
BB+	0.72	1.89	3.20	4.52	5.74	6.85	7.84	8.75	9.47	10.18
BB	1.46	3.08	4.79	6.51	8.11	9.48	10.69	11.78	12.71	13.53
BB-	2.80	5.19	7.48	10.63	12.50	14.06	15.36	16.44	17.46	18.46
B+	4.15	8.81	12.54	15.02	17.09	18.86	20.05	21.51	22.22	22.84
B	5.71	11.75	16.29	19.12	21.36	23.36	24.51	26.26	26.98	27.67
B-	10.55	16.81	20.89	24.60	27.08	29.20	29.99	32.12	33.50	34.98
CCC+	15.93	22.52	26.14	30.86	33.64	35.90	37.38	38.87	41.00	43.36
CCC	17.83	25.20	29.25	34.53	37.64	40.16	41.82	43.50	45.87	48.52

Table 4 The Joint Default Probability (in bp) for Ford and General Motors in Different Years

Year	**Ford**	**General Motors**	**Joint default if they are independent**	**Joint Default Probability from the model with jump**	**Joint Default Probability from the model without jump**
1	42	72	0.3	34	30
2	107	189	2	90	83
3	187	320	6	164	150
4	274	452	12	243	224
5	363	574	21	321	296
6	448	685	31	393	366
7	527	784	41	466	429
8	600	875	52	527	493
9	666	947	63	589	550
10	726	1018	74	642	596

Figure 1 - General Motors and Ford Price data

Figure 2- Ford and GM Normalized price data



References

Amato, Jeffrey and Eli Remolona (2003), "The Credit Premium Puzzle", *BIS Quarterly Review*, Vol. 18, 351-416.

Andersen, T.G., T. Bollerslev, and F. Diebold (2004), "Some like it Smooth, and Some Like it Rough: Untangling Continuous and Jump Components in Measuring, Modeling and Forecasting Asset Return Volatilities", Unpublished Paper: Economics Department, Duke University.

Andreasen, (2001), Credit Explosives, *working paper,* Bank of America.

Barndorf-Nilsen, Ole and Neil Shephard (2003a), "Econometrics of Testing for Jumps in Financial Economics using Bipower Variation", *Working Paper*, Oxford University.

Barndorf-Nilsen, Ole and Neil Shephard (2003b), "Realized Power Variation and Stochastic Volatility", *Bernoulli*, vol. 9, 243-165.

Barndorf-Nilsen, Ole and Neil Shephard (2004), "Power and Bipower Variation with Stochastic Volatility and Jumps", *Journal of Financial Econometrics*, vol. 2, 1-48.

Bates, D.,(1996), "Jumps and Stochastic Volatility: The Exchange Rate Process Implicit in Deutschmark Options." *Review of Financial Studies*, 9, 69-107.

Beneish, M., Press, E., (1995). Interrelation among events of default. *Working Paper*, Duke University, Durham, NC.

Black, Fisher and John C. Cox, (1976), "Valuing Corporate Securities: Some Effects of Bond Indenture Provisions", *The Journal of Finance*, 31 (2), 351-67.

Carrasco, M. and J.P. Florens (2000), "Efficient GMM Estimation Using the Empirical Characteristic Function", *Working Paper*, CREST, Paris.

Carrasco, M., Chernov, M., Florens, J.P. and Ghysels, E. , (2005), "Efficient Estimation of Jump Diffusion and General Dynamic Models with a Continuum of Moment Conditions", *Working Paper.*

Chan K.C.; G. Andrew Karolyi; Francis A. Longstaff; Anthony B. Sanders, (1992), An empirical comparison of alternative models of the short term interest rate, *Journal of Finance*, 1209-1227.

Chernov, M., A. R. Gallant, E. Ghysels, and G. Tauchen (2003), "Alternative Models for Stock Price Dynamics", *Journal of Econometrics* 116, 225-258.

Collin-Dufresne, Pierre, Robert Goldstein, and Jean Helwege (2003), "Is Credit Event Risk Priced? Modleing Contagion via Updating of Beliefs", Working Paper, Carnegie Mellon University.

Das, Sanjiv and Silverio Foresi, (1996), " Exact Solutions for Bond and Option Prices with Systematic Jump", *Review of Derivative Research*, Vol1, Number 1.

Das, Sanjiv and Peter Tufano, (1996),"Pricing Credit Sensitive Debt when Interest Rates, Credit Ratings and Credit Spreads are Stochastic", *Journal of Financial Engineering*, 5 (2), June.

Demircubuk, N. and Tse, T., (2001), "Default Point Estimation", *KMV company*.

Duffie, D., Lando, D., (1997), "Term Structure of Credit Spreads with Incomplete Accounting Information. *Working Paper*, Stanford University, Stanford, CA.

Duffie, Darrel and Ken Singleton, (1999), "Modeling Term Structure of Defaultable Bonds", *Review of Financial Studies*, Special 1999, 12 (4), 687-720.

Duffie, Darrel, J. Pan and Ken Singleton, (2000), "Transform Analysis and Option Pricing for Affine Jump-Diffusions", *Econometrica*, 68, 1343-1376.

Feuerverger, A. (1990), "An efficiency result for the empirical characteristic function in stationary time-series models", *The Canadian Journal of Statistics*, 18,155-161.

Feuerverger, A. and P. McDunnough (1981), "On the Efficiency of Empirical Characteristic Function Procedures", *J. R. Statis*. Soc. B, 43, 20-27.

Garbade, Kenneth (2001). *Pricing Corporate Securities as Contingent Claims*, Cambridge, MA: MIT Press.

Geske, Robert, (1977), "The Valuation of Corporate Liabilites as Compound Options", *Journal of Financial and Quantitative Analysis*, pp. 541-52.

Ghysels, E., Harvey, A. C., Renault, E. (1996), "Stochastic Volatility. In: Maddala G. S. , Rao, C. R., eds. *Handbook of Statistics, Vol 14, Statistical Methods in Finance*. Amsterdam: North-Holland, pp 119-191.

Gourieroux, C., A. Monfort and E. Renault (1993), "Indirect Inference", *Journal of Applied Econometrics* , 8, 85-118.

Groetsch, C. (1993), *Inverse Problems in the Mathematical Sciences*, Viewweg, Wiesbaden.

Grimmet, GR. and David Stirzaker (2001), "Probability and Random Processes", Oxford University Press.

Hamilton, J. D. (1990), "A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle". *Econmetrica* 57:357-384.

Hansen, L.P., (1982), "Large Sample Properties of Generalized method of Moments Estimator", *Econometrica* 50:1029-1054.

Hanson, S. , Pesaran, M.H and Schuermann, T., (2005), "The Scope for Credit Risk Diversification", *Working Paper*, University of Cambridge.

Huang, Xin and Ming Huang (2003), "How Much of the Corporate-treasury Yield Spread is Due to Credit Risk?", *Working Paper*, Penn State University.

Huang, Xin and George Tauchen (2004), " The Relative Contribution of Jumps to Total Price Variance", *Working Paper,* Duke University.

Hull, J., and White, A., (2001), "Valuing Credit Default Swaps II: Modeling Default Correlations", *Journal of Derivatives*, Vol. 8, No. 3, 12-22.

Hull, J., and White, A., (2005), "The Valuation of Correlation-Dependent Credit Derivatives Using a Structural Model", *Working Paper*, Joseph L. Rotman School of Management, Vol. 8, No. 3, 12-2

Iben, Th. And R. Litterman, (1991), "Corporate Bond Valuation and the Term Structure of Credit

Spreads", *Journal of Portfolio Management*, 17 (3), Spring, 1991, 52-64.

Jarrow, R. and Stuart M. Turnbull,(2000),"Princing Derivatives on Financial Securities Subject to Credit Risk" , *Journal of Finance*,50 (1), March,53-85.

Jarrow, Robert, David Lando and Stuart Turnbull,(1997) , "A Markov Model of Term Structure of Credit Spread", *Review of Financial Studies*, 10 (2), Summer.

Jarrow, Robert, David Lando and Fan Yu, (2005), "Default Risk and Diversification: Theory and Empirical Impilications", *Mathematical Finance*, 15(1), 1-26.

Jiang, George and John Knight (2001), " An Nonparametric Approach to the Estimation of Diffusion Process, with an Application to a Short-Term Interest Rate Model", *Econometric Theory,* 13,615-645.

Kim, I. J., Ramaswamy, K., and Sundaresan, S. M.,(1993) "Valuation of Corporate Fixed-Income Securities", *Financial Management*, p.117-131.

Knight, J.L. and J. Yu (2000), "Empirical Characteristic Function Estimation in Time Series", *Working Paper*, University of Western Ontario.

Lando, D., (1998), "On Cox Processes and Credit Risk Bonds", *Review of Derivatives Research*, 2, 99-120.

Laurent, J-P and J. Gregory, (2003), "Basket Default Swaps, CDO's and Factor Copulas," *Working Paper*, ISFA Actuarial School, University of Lyon.

Lawrence, Edward C. and Nasser Arshadi (1995), "A Multinomial Logit Analysis of Problem Loan Resolution Choices in Banking", *Journal of Money, Credit and banking* 27, 202-216.

Leland, Hayne E., (1994), "Corporate Debt Value, Bond Covenants and Optimal Capital Structure", *Journal of Finance*, 49 (4), September, 1213-52.

Leland, H. E. and K. B. Toft, (1996), "Optimal Capital Structure, Endogenous Bankruptcy and the Term Structure of Credit Spreads", *Journal of Finance*, 51 (3), July, pp. 987-1019.

Li, D.X., (2000), "On Default Correlation: A Copula Approach," *Journal of Fixed Income*, Vol 9, March, p. 43-54.

Longstaff, F. and E. Schwartz, (1995), "A Simple Approach to Valuing Risky Fixed and Floating Rate Debt", *Journal of Finance*, Vol 50, No. 3., 789-819.

Madan, D. B., Senata, E. (1990), "The Variance Gamma (VG) Model from Share Market Returns", *Journal of Business* 63:511-524.

Madan, D. and H. Unal, (1993), "Pricing of risks of Default", *Working paper*, College of Business, University of Maryland.

Mandelbrot, B. (1963), "The Variations of Certain Speculative Prices", *Journal of Business* 36:394-419.

Merton, Robert C., (1974), "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates", *The Journal of Finance*, 29, May, 449-70.

Merton, Robert C., (1977), "On the Pricing of Contingent Claims and the Modigliani-Miller Theorem", *Journal of Financial Economics*, 5, 241-9.

Newey, Whitney and Kenneth D. West, (1987), "Hypothesis Testing with Efficient Method of Moments Estimation", *International Economic Review*, Vol28, No.3.

Pesaran, M.Hashem, Til Schuermann, Bjorn-Jakob Treutler and Scott M. Weiner (2005), "Macroeconomic Dynamics and Credit Risk: A Global Perspective." Forthcoming, *Journal of Money, Credit and Banking*; available at Wharton Financial Institution Centre Working Paper #03-13B.

Quandt, R. E, "The Estimation of the Parameters of a Linear Regression System Obeying Two Separate Regimes, "*Journal of the American Statistical Association,* 53,873-80.

Schunbucher, P., (1997), "The Term Structure of Defaultable Bond Prices", *Working Paper*, University of Bonn.

Schunbucher, P., (2003), *Credit Derivative Pricing Models*, John Wile & Son.

Singleton, K. (2001), "Estimation of Affine Pricing Models Using the Empirical Characteristic Function", *Journal of Econometrics*, 102,111-141.

Spendley, W., Hext, G., and Himsworth, F., (1962), "Sequential application of simplex design in optimization and evolutionary operation", *Technometrics*.

Vasicek, O. (1987), "Probability of Loss on Loan Portfolio", KMV Corporation.

Vasicek, O. (1991), "Limiting Loan Loss Probability Distribution", KMV Corporation.

Zhang, B., Zhou, H. and Zhu, H. (2005) ," Explaining Credit Default Swap Spreads with Equity Volatility and Jump Risks of Individual Firms", *Working Paper*, KMV Corporation.

Zhou, Chunsheng, (1997), "A Jump-Diffusion Approach to Modeling Credit Risk and Valuing Defaultable Securities", *Working Paper*, Federal Reserve Board, Washington, 47pp.

Zhou, C., (2001a), "The Term Structure of Credit Spreads with Jump Risk", *Journal of Banking and Finance*, Vol.25, 2015-2040.

Zhou, C., (2001b), "An Analysis of Default Correlations and Multiple Defaults", *Review of Financial Studies*, Vol. 14, No. 2, 555-576.