“Demand system estimation with panel data “

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1- Introduction

Analysis of the allocation of family incomes to goods and services is of interest to economists and policy makers. Estimation of the good and services demand has a great importance in identifying the preferences and future needs. Studying the allocation of income to goods and services will clarify the impact of important factors like income and prices with introducing price and income elasticities. Identifying the impressive factors and their magnitude on demand will give the producers the ability to forecast the market demand and will aid the government to select appropriate fiscal policy. In addition, analyzing the expenditure pattern of families in duration of time can demonstrate the changes of society welfare.

In this research, we study the expenditure pattern of Iranian families during 1992-1995 with using the Almost Ideal Demand System(AIDS). The main aim of this study is as below:

- Estimation of demand system and calculating the price and income elasticities.
- Using the panel data econometrics methods for estimation.
- Testing the theoretical restrictions of consumer theory
- Calculating the living indices for Iranian families..

The plan of paper is as follows. After the introduction, in the 2nd part we introduce the Almost Ideal Demand System. In 3rd part the panel data econometrics method will be explained. In the 4th part, we will show the results of estimation, elasticity, and the role of social characteristics of family in demand. In addition, we will use the results, for clarifying the welfare status of families.

2- The Demand System

Demand studies could be classified into two broad categories. One group of studies is concerned with finding a model explaining the relationship among the quantity consumed of a single commodity, its price and the prices of its related commodities, and total income. Such a demand model is considered in some of those studies as a part of a system of demand functions, and tests of theoretical properties such as homogeneity of degree zero in prices and income are conducted. The other group of demand studies is concerned with the problem of allocating total expenditure to an exhaustive set of different commodities. These studies usually assume that the problem of deciding how much to consume at any given time point has been solved, and concentrate on the problem of allocation. In this paper we shall not be concerned with demand models for a single commodity, but with demand systems designed to explain the allocation problem of rational consumer. Below, we will introduce the Almost Ideal demand System (AIDS).

- The Almost Ideal Demand System (AIDS)

The unfortunate named Demand system, AIDS was developed by Deaton and Muellbauer [1980] builds on a model by working [1943] and leser [1963]. Their model expresses the ith budget share, w_i, as a function of logx that is:

$$Wi = \alpha_i + \beta_i \log x \qquad (1)$$

where w_i and x are the ith budget share and total per capita, expenditure, respectively. The working - laser model was extended by Deaton and Mullbauer to include the effect of prices, the resultant demand system was derived, by use of duality concepts, from a particular cost function. The AIDS cost function belongs PIGLOG class which are defined by

$$\log c(u,p) = (1-u)\log\{a(p)\} + u\log\{b(p)\} \qquad (2)$$

which u lies between 0 (Subsistence) and 1 (bliss) so that the positive linearly homogeneous functions a(p) and b(p) can be regarded as the costs of subsistence and bliss, respectively. If we take

$$\log a(p) = \alpha_0 + \sum_K \alpha_K \log P_K + \frac{1}{2}\sum_K \sum_J \gamma^*_{KJ} \log P_K \log P_J \qquad (3)$$

$$\log b(p) = \log a(p) + \beta_0 \prod_K P_K^{\beta_K} \qquad (4)$$

then we have the AIDS cost function as below:

$$\log C(u,p) = \alpha_0 + \sum_K \alpha_K \log P_K + \frac{1}{2}\sum_K \sum_J \gamma^*_{KJ} \log P_K \log P_J + U\beta_0 \prod_K P_K^{\beta_K} \qquad (5)$$

where α_i , β_i and γ_{ij}^* are parameters.

By using shepherd lemma, we can extract the budget share as a function of prices and utility:

$$W_i = \alpha_i + \sum_j \gamma_{ij} \log P_J + \beta_i \log(\frac{x}{p}) \qquad (6)$$

where

$$\gamma_{ij} = \frac{1}{2}(\gamma^*_{ji} + \gamma^*_{ji}) \qquad (7)$$

and

$$\log P = \alpha_0 + \sum_K \gamma_K \log P_K + \frac{1}{2}\sum_j \sum_K \gamma_{Kj} \log P_K \log P_j \qquad (8)$$

The conditions which are required to make the model consistent with the theory of demand are:

- Adding-up: $\sum_i \alpha_i = 1$; $\sum_{\mathrm{i}} \gamma_{\mathrm{ij}} = 0$; $\sum_{\mathrm{i}} \beta_i = 0$ (j=1, ..., n); (9)
- Homogeneity: $\sum_j \gamma_{ij} = 0$ (i=1, ..., n); (10)
- Symmetry: $\gamma_{ij} = \gamma_{ji}$ (i≠j , i,j=1, ..., n); (11)

Conditions (10) and (11) are linear restrictions which may be tested by standard techniques, whereas condition (9) is imposed by the model and so is not testable. The negativity condition implies that the matrix of Hicksion price effects must be negative semi definite.

The aggregation theory developed in Muellbauer [1975, 1976] implies that exact aggregations is possible if, for an individual household h, behavior is described by the generalization of :

$$W_{ih} = \alpha_i + \sum_j \gamma_{ij} \log P_j + \beta_i \log(\frac{x_h}{K_h P}) \qquad (12)$$

The parameters K_h can be interpreted as a sophisticated measure of household size which, in principle, could take account of age composition, other household characteristics, which is used to deflate the budget x_h to bring it to a “needs corrected” per capita level. Using the model which its budget is deflated would give better estimates.

3- Panel data econometrics

A system which contains M equation, with panel data on N Individuals over T periods that can be shown as follows:

$$\begin{aligned} Y_1 &= Z_1\delta_1 + W_1 \\ &\vdots \\ Y_M &= Z_M\delta_M + W_M \end{aligned} \qquad (13)$$

Which Y_j is a NT × 1 vector of Jth dependent variable, Z_j a matrix of dimension NT × K_j and $\delta'_j = (\alpha'_j, \beta'_j)$ is a K_j × 1 Vector which $K'_j = K_j + 1$ the error term can be defined as below:

$$W_j = Z_u U_j + \varepsilon_j \qquad (14a)$$

$$\begin{aligned} \varepsilon'_j &= (\varepsilon_{11j}, \varepsilon_{12j}, \ \dots \ , \ \varepsilon_{1Tj}, \dots, \ \varepsilon_{N1j}, \varepsilon_{N2j}, \ \dots \ , \ \varepsilon_{NTj}), \\ U'_j &= (U_{1j}, U_{2j}, \ \dots \ , U_{Nj}), Z_u = I_N \otimes I_T \end{aligned} \qquad (14b)$$

with the following covariance matrix:

$$E(w_j w'_j) = E\left[\begin{pmatrix} u_j \\ \varepsilon_j \end{pmatrix} \begin{pmatrix} u'_l \varepsilon'_l \end{pmatrix}\right] = \begin{bmatrix} \sigma^2_{U_{jl}} I_N & 0 \\ 0 & \sigma^2_{\varepsilon_{jl}} I_{NT} \end{bmatrix} \quad j, l = 1, \dots, M \qquad (15)$$

we can deduct from (3) the follwing expression:

$$u \ \approx \ (0, \Sigma_u \otimes I_N) \qquad (16a)$$

$$\varepsilon \ \approx \ (0, \Sigma_\varepsilon \otimes I_{NT}) \qquad (16b)$$

$$\begin{aligned} &\Sigma_e = [\sigma^2_{\varepsilon\ jl}\ ], \Sigma_u = [\sigma^2_{u\ jl}\ ], u' = (u'_1, \ \dots \ , \ u'_M), \varepsilon' = (\varepsilon'_1 \ \dots \ \varepsilon'_M) \\ &j, l = 1, 2, \dots, M \end{aligned} \qquad (16c)$$

Now we can write the covariance matrix for M equation as below:

$$\Omega = E(ww') = \Sigma_u \otimes (I_N \otimes l_T l'_T) + \Sigma_\varepsilon \otimes (I_N \otimes I_T) \qquad (17)$$

$$w' = (w'_1, w'_2, \ \dots \ , \ w'_M) \qquad (18)$$

The Ω matrix can be defined as below too:

$$\Omega = (T\Sigma_u + \Sigma_\varepsilon) \otimes (I_N \otimes \bar{J}_T) + \Sigma_\varepsilon \otimes (I_N \otimes E_T) =$$

Or

$$\Omega = \Sigma_1 \otimes P + \Sigma_\varepsilon \otimes Q \qquad (19)$$

Which

$$\Sigma_1 = T\Sigma_u + \Sigma_\varepsilon \qquad (20)$$

$$P = I_N \otimes \bar{J}_T \qquad (21)$$

$$Q = I_{NT} - P \qquad (22)$$

$$\bar{J}_T = \frac{l_T l'_T}{T} \qquad (23)$$

$$E_T = I_T - \bar{J}_T \qquad (24)$$

Baltagi [1980] has shown that, $\Omega^{-\frac{1}{2}}$ can be written like this:

$$\Omega^{-\frac{1}{2}} = \Sigma_l^{-\frac{1}{2}} \otimes P + \Sigma_\varepsilon^{-\frac{1}{2}} \otimes Q \qquad (25)$$

For estimation of $\Sigma_\in$ and Σ_l we use respectively from $\frac{w'Qw}{N(T-1)}$ and

$\frac{w'pw}{N}$ Which W is defined in (4a). Using the error terms of least square with dummy variable (LSDV) leads to asymptotically efficient estimators for Σ_ε and Σ_L unlike to using ordinary least square which does not guarantee efficient estimates for Σ_ε and Σ_l .

4) Data and results:

- Data

The theory presented in section 2 provides a framework in which can be organized and interpreted.

To estimate demand parameters, we use data reported in the “Household social and economic characteristics Survey of Iran” a panel data for the years 1992-1995. The survey, based on a national sample, covered 443 households and a comprehensive list of commodities. 287 households in this data, lived in cities and 156 households were living in rural areas. For the purpose of this study, total household expenditure was broadly categorized into 8 distinct groups: 1) food 2) clothing 3) housing 4) Durable goods 5) treatment 6) Transportation 7) Recreation 8) other goods and services. As the model is a conventional static utility-maximizing model, we omit the expenditure of Durable goods, as it needs the assumption of weak intertemporal separability in the intertemporal utility function. For the sake of degrees of freedom in estimation we add the groups 5,7,8 to a group named others. So we use these 5 expenditure groups: 1) Food 2) clothing 3) Housing) transportation 5) others.

In table (1) the average share of expenditure groups are shown.

Year	**Food**	**clothing**	**Housing**	**transportation**	**Others**
			Urban		
1371	0.3561	0.1658	0.3066	0.0770	0.0940
1372	0.3522	0.1281	0.3368	0.0789	0.1037
1373	0.3554	0.1219	0.3437	0.0725	0.1062
1374	0.3927	0.1174	0.3066	0.0782	0.1049
			Rural		
1371	0.5194	0.2185	0.1294	0.0640	0.0680
1372	0.4944	0.1886	0.1482	0.0630	0.1049
1373	0.5049	0.1870	0.1482	0.610	0.0985
1373	0.5263	0.1783	0.1402	0.0620	0.0921

Table 1- Average shares of expenditure in urban and rural households

from Table (1) we can deduct the following results:

1) The shares of food and housing have not been changed too much but there is a great difference between urban and rural households expenditures in these 2 groups.

2) The share of clothing has decreased gradually.

3) The share of transportation was somehow constant and the same in both rural and urban households.

4) The share of “Others” expenditure has not changed in urban but has been increased in rural households.

- Results and Analysis

In this section we present the results of estimation via various methods. At first we will show the results of ordinary SUR and then we will present the results of variance component one-way panel data method. For estimating the parameters, a program is written in MATLAB and GUASS.

At first we will compare the results of two methods. Then we will present the results in which the role of family size is considered. In 3rd step we will test the theoretical restriction of consumer behavior and in the end we compare the changes of the welfare status of households during the period of 1992-1995.

a) The results of ordinary SUR:

This method is accomplished with imposing the theoretical restrictions (homogeneity, summitry, adding-up). The estimated parameters are displayed in Table 2. The price elasticities and income elasticities are shown in Table 3 and Table 4, respectively.

	α_i	γ_{i1}	γ_{i2}	γ_{i3}	γ_{i4}	γ_{i5}	β_i
Food	1.5116	0.0719	0.0200	-0.0165	0.0704	-0.1467	-0.1046
Clothing	-0.1213 (-2.45)	0.0200	0.0374 (0.32)	0.0019 (0.10)	0.0284 (0.47)	-0.0877 (-1.30)	0.0269 (5.76)
Housing	0.1946 (3.28)	-0.0166	0.0019 (0.10)	0.0874 (4.12)	-0.1051 (-7.16)	0.0324 (2.13)	0.0077 (1.37)
Transportation	-0.4475 (-10.7)	0.0705	0.0284 (0.47)	-0.1052 (-7.16)	-0.0489 (-1.271)	0.0552 (1.50)	0.0482 (12.23)
Others	-0.1374 (-3.47)	-0.1467	-0.0877 (-1.3)	0.0324 (2.13)	0.0552 (1.5)	0.1468 (2.98)	0.0218 (5.79)

Table 2- estimated parameters using ordinary SUR (The number in parentheses are t-ratio)

	Food	**Clothing**	**Housing**	**Transportation**	**Others**
Food	-0.4464	0.4385	0.3679	0.5184	0.0107
Clothing	0.1915	-0.7359	0.1516	0.0729	-0.6171
Housing	-0.0487	0.0246	-0.6682	-0.4800	0.0823
Transportation	1.4050	0.8281	-0.8031	-1.3920	0.8513
Others	-1.3954	-0.7896	0.5527	0.3760	0.5215

Table 3- Price elasticity (using Ordinary SUR)

Commodities	**Income Elasticity**
Food	0.7486
Clothing	1.1747
Housing	1.0297
Transportation	1.6697
Others	1.2213

Table 4- Total expenditure elasticity (using Ordinary SUR)

13 parameters from 24 have the t-statistics of over 2 and 6 parameters have t- ratio between 1 and 2 Food, clothing and Housing are price inelastic and just transportation is price elastic (table -3) The price elasticity of others is positive. We can explain the positivily of this price elasticity in two ways: 1) We should claim that this group is a giffen good or 2) because of estimation bias, this estimate is not correct. From the income elasticity (Table - 4) we can say that except Food, the remaining goods are luxury goods.

b) The results of Variance component SUR:

This method is also done with imposing the theoretical restrictions. A program is written in MATLAB and GUASS for estimation of parameters.

	α_i	γ_{i1}	γ_{i2}	γ_{i3}	γ_{i4}	γ_{i5}	β_i
Food	1.1426	0.1203	0.1087	-0.0551	-0.1082	-0.0657	-0.0694
Clothing	-0.4410 (-8.52)	0.1087	-0.0696 (-0.63)	0.316 (1.79)	0.0599 (1.03)	-0.1306 (-2.02)	0.0581 (11.88)
Housing	0.9773 (16.38)	-0.0551	0.0316 (1.78)	0.0438 (3.29)	-0.0237 (-1.74)	0.0034 (0.24)	-0.0681 (-12.0)
Transportation	-0.5193 (-11.8)	-0.1082	0.0599 (1.03)	-0.0237 (-1.74)	0.0359 (-0.92)	0.1079 (2.96)	0.0558 (13.43)
Others	-01596 (-3.74)	-0.0657	-0.1306 (-2.02)	0.0034 (0.25)	0.1079 (2.96)	0.0850 (1.80)	0.0236 (5.85)

Table 5- Estimated parameters using variance component SUR

	Food	**Clothing**	**Housing**	**Transportation**	**Others**
Food	-0.5208	0.4740	0.0735	-0.0877	0.0100
Clothing	0.8902	-1.2809	0.4387	0.4714	-0.7824
Housing	0.0506	0.4096	-0.5765	0.1388	0.2370
Transportation	-1.0458	1.5056	0.2605	-1.1035	1.6199
Others	-0.6010	-1.1530	0.1685	1.01189	-0.0994

Table 6- Price elasticities (using variance component SUR)

Commodities	Income Elasticity
Food	0.8330
Clothing	1.3765
Housing	0.7374
Transportation	1.7761
Others	1.2404

Table 7- Total expenditure elasticity (using variance component SUR)

Comparing the results of Table 5- with Table 2 shows that the number of significant parameters is increased with using the new method (Table 6). All of the own-price elasticities are negative that is more compatible to theory unlike the previous method that brings a positive own - price elasticity. The sensitivity of Food, Housing and others to price is low, unlike clothing and transportation which are more sensitive to their prices.

Food and Housing are necessities (Table 7) while clothing , transportation and others are luxury goods. These results are different, respect to ordinary SUR Results; and are more compatible to consumer theory.

C) The results of variance component SUR with considering family size:

In the previous 2 methods we had considered K_h=1 in (12). As the theory says, considering the social characteristics of family will present a model closer to reality. In this section we present the results which the impact of family size is also considered. The related estimated parameters are shown in Table 8 and price and total expenditure elasticities in table 9 and table 10, respectively, comparing the results to part (b), does not demonstrate difference just in the magnitude of elasticities. The sensitivity of households towards price changes are more in Housing and clothing expenditure and less in Food.

	α_i	γ_{i1}	γ_{i2}	γ_{i3}	γ_{i4}	γ_{i5}	β_i
Food	1.09	0.1292	0.1214	-0.055	-0.1262	-0.0694	-0.0773
Clothing	-0.2618 (-5.34)	0.1214	-0.0799 (-0.62)	0.0330 (1.61)	0.0580 (0.84)	-0.1325 (-1.76)	0.0496 (8.87)
Housing	0.7068 (12.41)	-0.055	0.9330 (1.61)	0.0382 (2.46)	-0.0192 (-1.22)	0.0030 (0.19)	-0.0512 (-7.89)
Transportation	-0.3996 (-9.71)	-0.1262	0.0580 (0.84)	-0.0192 (-1.22)	-0.0272 (-0.59)	0.1146 (2.67)	0.0536 (11.46)
Others	-0.1354 (-3.47)	-0.0694	-0.1325 (-1.76)	0.0030 (0.19)	0.1146 (2.66)	0.0843 (1.54)	0.0253 (5.79)

Table 8 - Estimated parameters using variance component SUR with considering family size

	Food	**Clothing**	**Housing**	**Transportation**	**Others**
Food	-0.4861	0.5178	0.0846	-0.1180	0.0142
Clothing	0.8999	-1.4295	0.3540	0.3788	-0.8751
Housing	-0.0621	0.3013	-0.7145	0.0362	0.1144
Transportation	-1.3740	1.3783	0.1914	-1.0876	1.6051
Others	-0.6260	-1.1680	0.1505	1.0795	-0.1102

Table 9 - Price elasticities (using variance component SUR with considering family size)

Commodities	**Total Expenditure elasticity**
Food	0.8129
Clothing	1.3214
Housing	0.8028
Transportation	1.7458
Others	1.2621

Table 10 - Total expenditure elasticity

- Testing the theoretical restrictions:

The theoretical restrictions (Homogeneity and Symmetry) have been tested with using ML test. The results are shown in table 11 and 12.

Method	**Family size consideration**	**ML Statistics**	**Critical value α = 0.005**	**Result of test**
Ordinary	yes	2.3	14.86	Accepted
Ordinary	no	4.2	14.86	Accepted
Panel	yes	4.0	14.86	Accepted
Panel	no	4.0	14.86	Accepted

Table 11- Homogeneity tests

Method	**Family size consideration**	**ML Statistics**	**Critical value α = 0.005**	**Result of test**
Ordinary	yes	465.28	18.55	Rejected
Ordinary	no	586.8	18.55	Rejected
Panel	yes	104.0	18.55	Rejected
Panel	no	108.0	18.55	Rejected

Table 12 - Symmetry tests

In all of the equations the Homogeneity have been accepted unlike summetry which is rejected. These results are somehow in accordance with previous studies (Deaton [1980]). Rejecting symmetry means rejecting the axioms of well-behaved preferences. But for clarifying this result we should take into account the assumptions we had accepted in empirical research. The following assumption are the ones which are far from reality and make symmetry condition to fail in tests;

1) Aggregation among commodities
2) Considering income as exogenous
3) Considering prices as exogenous
4) Static utility maximization

- Changes of the welfare status of households

Calculating the changes of welfare status of households is of interest to economists. Laspeyers is one of the ordinary indices of this variable. It measures the incomes for obtaining a specified basket of good in various price situations. Estimating a demand system puts forward an index that measures the income necessary for obtaining a specified utility in various price conditions (Deaton [1981]). As it considers the utility as determined variable, it is a better index than laspeyers. This index is defined as below:

$$S = \frac{C(P^2, \overline{U})}{C(P^1, \overline{U})} \tag{26}$$

We have calculated these indices during 1992-1995 for various categories of people. They are normalized to year 1992 and are shown in table 13.

The groups categorized based on urban-rural, income and family size. The families with under mean income is called low-income (LI) and above mean income is called high-income (HI). The families with family size below 6 is called low-family-size (LFS) and above 6 is called high-family-size (HFS).

Urban	**Income**	**Family**	**1992**	**1993**	**1994**	**1995**

		size				
Total	-	-	100	122.99	172.77	261.69
Rural	-	-	100	123.44	177.28	269.80
Urban	-	-	100	121.97	166.5	249.52
Rural	LI	-	100	122.69	174.44	263.55
Rural	HI	-	100	123.92	179.09	273.82
Urban	LI	-	100	121.93	165.33	247.24
Urban	HI	-	100	122.00	167.25	250.94
Rural	LI	LFS	100	122.59	174.09	262.78
Rural	LI	HFS	100	122.72	174.58	263.84
Rural	HI	LFS	100	123.79	178.63	272.79
Rural	HI	HFS	100	123.94	179.14	273.94
Urban	LI	LFS	100	121.93	165.21	247.03
Urban	LI	HFS	100	121.93	165.43	247.45
Urban	HI	LFS	100	121.99	167.12	250.69
Urban	HI	HFS	100	122.00	167.33	251.10

Table 13 - Necessary total expenditure for acquiring the same utility at 1992.

In Average, The utility of 100 units of expenditure at 1992 was equal to 261.69 units of money in 1995. For rural, specifically, it was 269.80 while for urban it was 249.52. So the people in villages were to acquire more income to stay the same position than who were living in cities. The price changes was also better for high income people than low incomes both in rural and urban. We have found in Data, the total expenditures of households during the period. Comparing these data with the indices in table 13, we calculate the deviation from indices. the results are shown in table 14 in percent.

Urban Rural	**Income**	**Family size**	**1992**	**1993**	**1994**	**1995**

Total	-	-	0	-17.1	-25.2	-20-7
Rural	-	-	0	-19.1	-22.3	-15.3
Urban	-	-	0	-15.7	-24.1	-19.7
Rural	LI	-	0	-16.1	-19.6	-16.4
Rural	HI	-	0	-20.4	-23.6	-15.3
Urban	LI	-	0	-5.7	-10.6	-12.8
Urban	HI	-	0	-19.5	-29.3	-22.5
Rural	LI	LFS	0	-12.3	-14.4	-20.9
Rural	LI	HFS	0	-17.4	-21.3	-14.3
Rural	HI	LFS	0	-31.9	-24.5	-19.2
Rural	HI	HFS	0	-19.2	-23.6	-14.4
Urban	LI	LFS	0	-3.6	-10.0	-11.4
Urban	LI	HFS	0	-6.9	-10.2	-12.9
Urban	HI	LFS	0	-14.6	-26.6	-14.8
Urban	HI	HFS	0	-23.0	-31.4	-27.6

Table 14 - The difference between actual expenditure and index (percent)

In Average, the expenditure of people of Iran were in 1995 20 percent below the needed expenditure to maintain the same utility at 1992. And the rural were 15.3 below and urban 19.7 percent. In the categories of HFS in cities, the people suffered more pressure than LFSs. While in rural the situation was inverse. We can deduce that high family size in rural protect the families from economic fluctuations, unlike urban in cities. In cities normally, just the head of family can work and compensates the losses of price changes. But in rural, the children can help in making money and the families can tolerate the bad economic situations. So it may be the reason of greater high birth rates in rural area than urban area in Iran.

5- Conclusion

We have estimated on almost Ideal Demand System (AIDS), by utilizing a panel Data set for 443 Iranian households over 1992-1995. Careful specification analysis indicated that the so-called random-effects model is the appropriate framework for empirical analysis and the comparison between the results of this method and simple SUR, has been conducted. The hypotheses of consumer theory tested, and demand elasticities computed. Test results showed that the homogeneity is accepted while the symmetry is rejected, and its reason have been explained. We presented the living indices over 1992-1995 for Iranian families with using AIDS cost function. These indices have shown that Iranian family welfare's have decreased in these years, especially for families who were living in rural area.

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